HECKATHORN

PHYSICS OF TECHNOLOGY — A Modular Approach

THE TORQUE WRENCH

A MODULE ON FORCES, TORQUES AND ELASTICITY

PRINCIPAL AUTHORS: John P. Ouderkirk
SUNY ATC-Canton

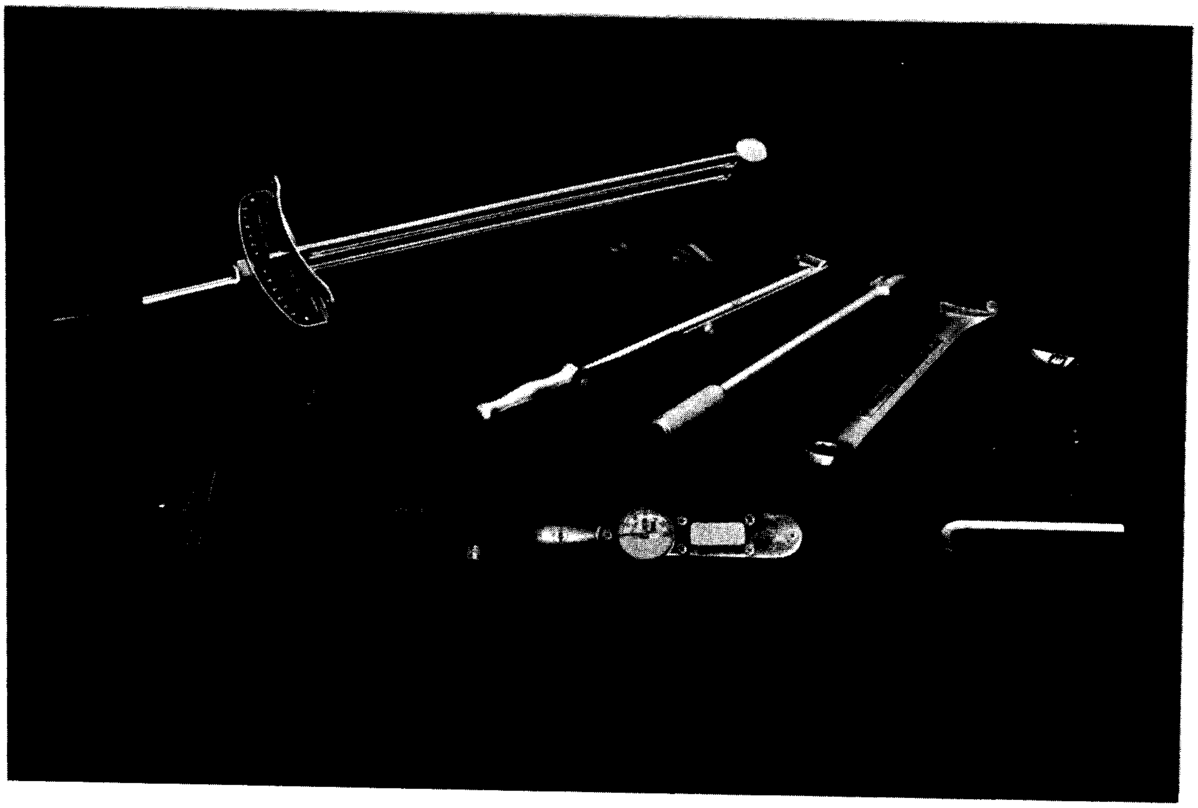
Bruce B. Marsh
SUNY-Albany

PROJECT DIRECTORS: Carl R. Stannard
Bruce B. Marsh

SUNY
State University of New York at Binghamton

Coordinated by the American Institute of Physics

Torque Wrench



Introduction

Wrenches come in a large variety of sizes and shapes. Despite the differences in appearance, all wrenches have two things in common. (1) All wrenches tend to rotate the objects to which they are applied; that is, they produce a torque. (2) All wrenches bend or flex while being used. These two characteristics will be explored in this module on the torque wrench. Although the device is a simple one, the principles associated with its design and operation are fundamental to many areas of science and technology.

Let's consider the first point of similarity among wrenches, "All wrenches produce torque." The boxend wrench, pipe wrench, crescent wrench, and Allen wrench don't look alike yet each of them is used to develop a torque or twisting effect. They all have handles of some kind. They differ in how they grip a nut or bolt, and in the shape of the handle or lever part of the wrench. In each case a force is applied to a handle or lever and a twist or torque is applied to a nut bolt. Why don't we call these wrenches by their proper names like the boxend torque wrench or the pipe torque wrench? Why reserve the name "torque wrench" for only one member of the wrench family? Probably no one gave careful thought to a naming procedure. If they had, what would you call this device that we now call a torque wrench? I suppose that it could be called a "torque measuring wrench" since its only claim to being different is that it can measure torque.

The ability of the torque wrench to measure torque is a result of the second characteristic. The handles of all wrenches flex. Most wrenches have such strong handles that the flexing isn't noticed and indeed that's the way they are designed. It would be inconvenient to use a wrench with a flimsy handle. However, one kind of wrench actually makes use of the flexing in the handle--the torque wrench! As you can see in Figure 2, the torque wrench has a scale and pointer that indicate the twisting effect (torque). The figure shows some other scales that are commonly used on torque wrenches. The fact that the handle of the torque wrench flexes and then returns to its original shape makes this use of a wrench possible. The property of metal that allows the wrench to be deflected and then return is called elasticity. As a result of this property, the applied torque can be measured and the mechanic will know when he has tightened a bolt to the manufacturer's specification.

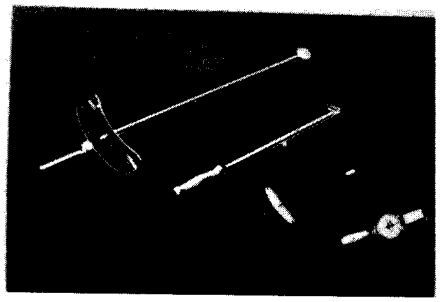


Figure 2

This section can be summarized briefly by stating the general properties of wrenches and the special feature of the torque wrench. We have seen that all wrenches produce torque and flex (elastically) when used. The torque wrench is unique in that its output torque is shown by a built-in scale which is activated as the torque wrench flexes elastically.

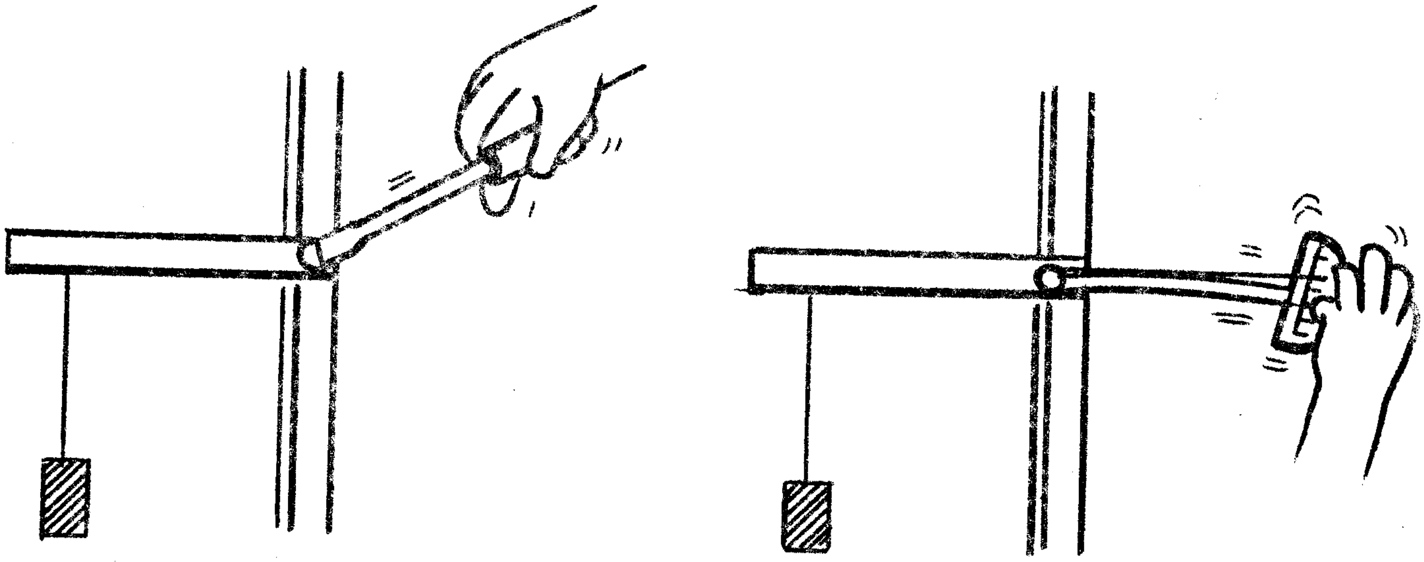
These features will be developed in the study of torque, equilibrium, and elasticity in subsequent sections of the module.

Torque

Torque is the technical name for the twisting effect produced by the wrenches that we have discussed. Wrenches aren't the only devices that produce torque, but we'll stick to them for the moment. The first experiment is designed to show how the torque depends on three factors: (1) the force applied to the handle; (2) the distance from the axis to the point of application of the force; (3) the angle between the handle and the line of action of the force.

Experiment 1 -- Torque

(A) Hang a mass from the end of the load arm as shown in Figure 3. Use a screw driver to feel how much torque is required to hold the arm horizontal. Just from the feeling you can get some idea of the torque produced by the hanging mass. To get a number for the torque, use the torque wrench as shown in Figure 4.



Suspend several different weights from the end of the arm and record the scale reading for each weight.

Question 1: If your results agree with one of the following statements, underline it and cross out the others. (Otherwise call for help.)

a) The torque is proportional to the square of the applied force.

b) The torque is proportional to the applied force.

c) The torque is inversely proportional to the applied force.

(B) With the same weight each time, and with the load arm horizontal, vary the distance from the axis to the point where the force is applied. Measure the torque and record your results for several values of distance.

Question 2: Underline the statement consistent with your results and cross out the others.

a) The torque is inversely proportional to the distance from the axis to the point of application of the force.

b) The torque is proportional to the distance from the axis to the point of application of the force.

c) The torque is proportional to the square of the distance from the axis to the point of application of the force.

(C) In this part you are to determine what happens if the arm is not horizontal. To discuss the effect some new terms are needed. In Figure 5 the vertical line is an extension of the

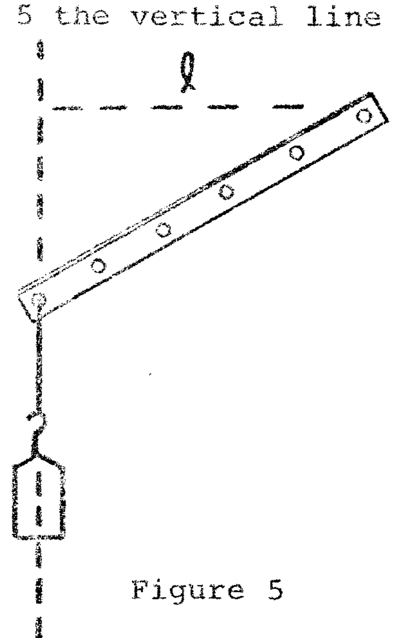


Figure 5

line along which the force acts. This is called the line of action of the force. The line labelled "ℓ" in the figure is the perpendicular from the axis to the line of action of the force. It is called the lever arm, or torque arm.

With the same weight each time, located at the end of the load arm, vary the length of the lever arm by changing the angle of the lever arm. Measure the torque for each length of the lever arm and record your results.

Question 3: Underline the statement consistent with your results and cross out the others.

a) The torque is proportional to the lever arm.

b) The torque is proportional to the square of the lever arm.

c) The torque is inversely proportional to the lever arm.

It would be nice to write a mathematical equation to represent the results of the above measurements. To do this we will use the following symbols for the physical quantities.

τ: torque

F: applied force

ℓ: length of torque arm

Question 4: Underline the correct equation and cross out the others.

a) $\tau = F\ell$

b) $\tau = F\ell^2$

c) $\tau = F/\ell$

Question 5: Are the units on the torque wrench consistent with your results? Can you think of other units which might be appropriate for torque?

In Experiment 1 you were able to show that

$$\tau = F\ell \tag{1}$$

τ is the torque, F is the applied force and ℓ is the lever arm as indicated in Figure 6. Actually torque is defined by the above. The torque wrench which you used was calibrated so as to agree with the definition.

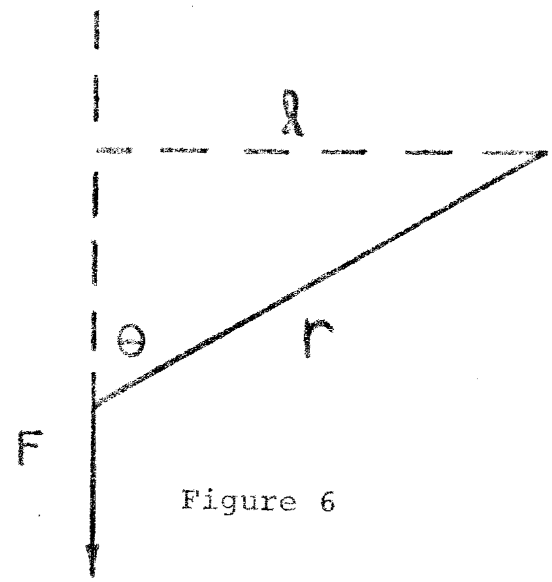


Figure 6

Sometimes it is convenient to express the torque in terms of the distance from the axis to the point of application of the force. That distance is labelled r in the figure. The angle between r and F is labelled θ. Using the definition of the sine of an angle,

$$\ell = r \sin \theta \tag{2}$$

Therefore

$$\tau = Fr \sin \theta \tag{3}$$

Note that the units for torque must be the units for force times the units for length. The most common units in engineering and technology in this country are pound-foot, pound-inch, or ounce-inch. Most other countries and many scientists in this country use the metric system in which the unit for torque is newton-meter.

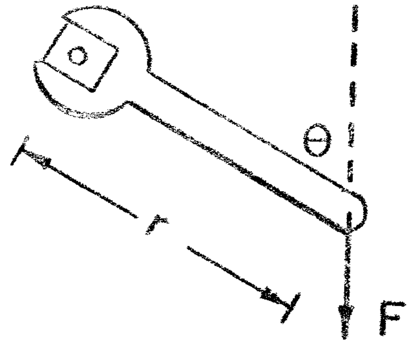


Figure 7

Example 1: Determine the torque applied to the nut for each of three values of θ:
(a) $\theta = 90^\circ$, (b) $\theta = 30^\circ$, (c) $\theta = 0^\circ$.
The value of r is 12 inches and the applied force is 10 lbs.

Solution: The torque can be obtained by application of Equation 1:

$$\tau = F\ r\ \sin$$
$$= (10\ \text{lb})(12\ \text{in})(\sin\theta)$$

Inserting the different values of θ,

(a) $$\tau = (10\ \text{lb})(12\ \text{in})(\sin 90^\circ)$$
$$= (10\ \text{lb})(12\ \text{in})(1)$$
$$= 120\ \text{lb-in} = 10\ \text{lb-ft}$$

(b) $$\tau = (10\ \text{lb})(12\ \text{in})(\sin 30^\circ)$$
$$= (10\ \text{lb})(12\ \text{in})(0.5)$$
$$= 60\ \text{lb-in} = 5\ \text{lb-ft}$$

(c) $$\tau = (10\ \text{lb})(12\ \text{in})(\sin 0^\circ)$$
$$= (10\ \text{lb})(12\ \text{in})(0)$$
$$= 0$$

Note that as the angle θ decreases from 90° the torque decreases. This should be apparent because the lever arm is decreasing. The perpendicular distance from the axis to the line of action of the force has these values for the three cases:

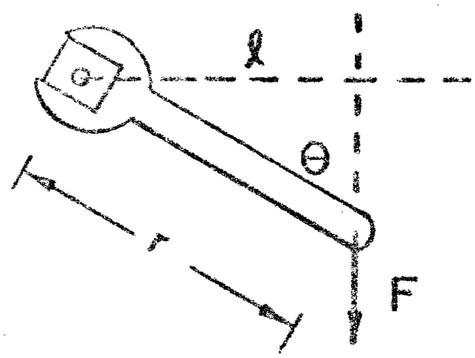


Figure 8

(a) $\ell = r = 12$ in

(b) $\ell = r \sin 30^{\circ} = 6$ in

(c) $\ell = r \sin 90^{\circ} = 0$

Now use equation 2 to check the values of torque calculated above.

Problem 1: In a situation such as Example 1, if it is necessary to apply a torque of 100 lb-in with a force of 8 lb when $\theta = 60^{\circ}$, how long must the wrench handle be?

Static Equilibrium

Although the torque wrench is used to produce turning, its most important position is when it is stopped. That is when the operator has reached the desired value of torque. We are going to study the relations among torques and forces on objects which are not moving. Such objects are said to be in static equilibrium.

(The term "equilibrium" applies to a more general situation. An object is in equilibrium if it is not being accelerated, even though it may be moving. Thus static equilibrium is a special case of equilibrium in general. However, the relations we will develop for forces and torques on objects in static equilibrium will also apply to the more general case of objects which are moving with no acceleration.)

What is involved in static equilibrium or the state of being stopped? Consider Figure 9 and see if you can tell if the situations shown are in equilibrium.

Experience tells us that the fish and scale are not going anywhere and that the scale pulls up on the fish with a force of 10 pounds and the fish is pulled down with a weight (gravitational force) of 10 pounds. It is important to note that the force exerted by the scale on the fish and the weight of the fish act through a single point. Forces that act through a single point are called concurrent forces.

The second part of Figure 9 shows a situation in which the forces do not act through a single point. These forces are called nonconcurrent. It seems clear that the arrangement will balance. If the bar weighs one pound, is the reading on the scale what we might expect? The equilibrium of this situation can be upset by moving either weight along the bar or changing the position of the scale. If this is done, what kind of motion results? Do the weights develop a torque about the spring scale?

In thinking about the situations described above, you probably realize that for an object to be in equilibrium the forces acting on the object must somehow cancel. Likewise, the torques acting on the object must cancel. We will find

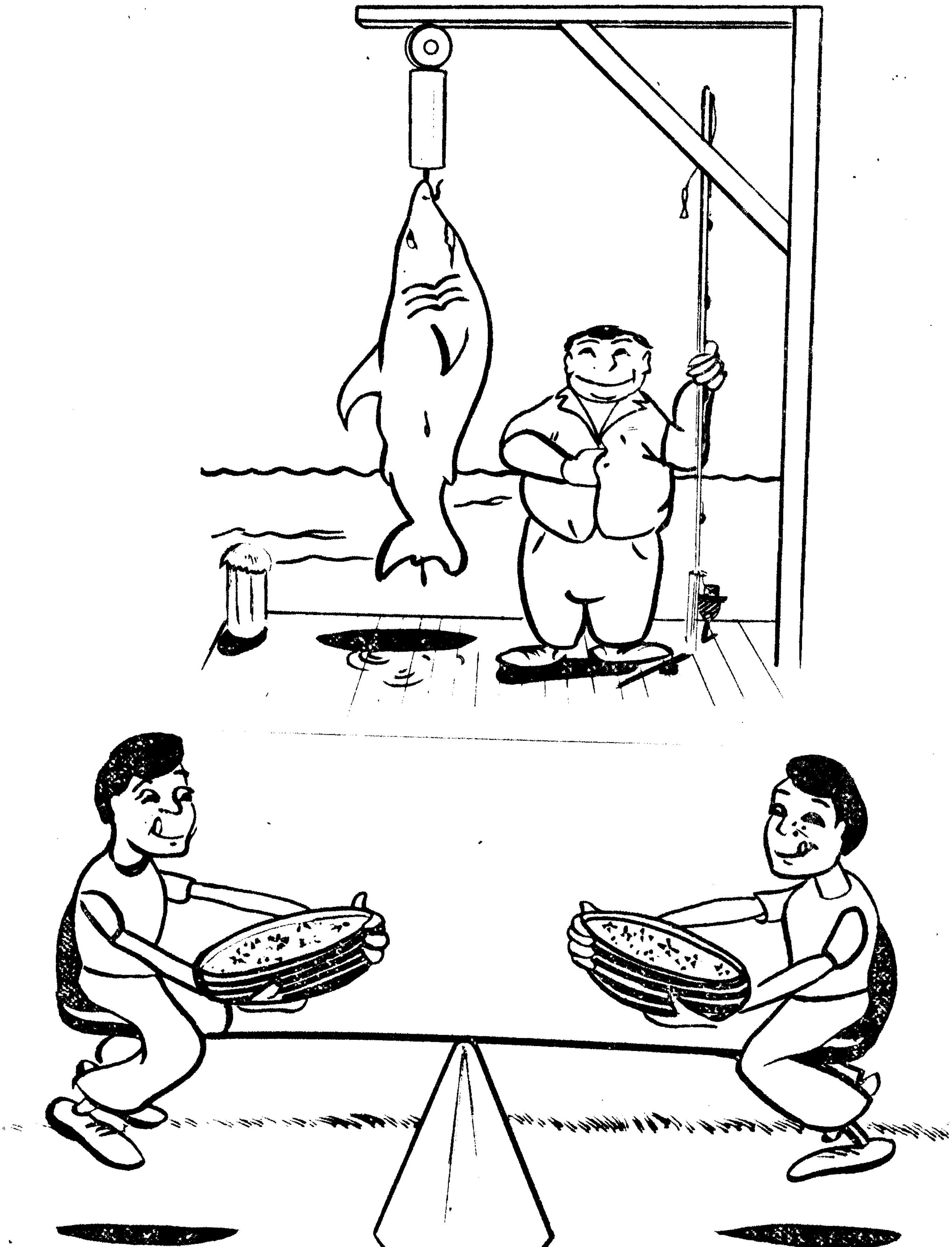


Figure 9

a way of stating more precisely what is meant when we say that the forces cancel and the torques cancel.

In the case of the fish hanging from the scale, the force of the spring on the fish is up and the force of gravity on the fish is down. The two forces have the same magnitude, and because of their opposite <u>directions</u> they cancel each other to give a resultant force of zero acting on the fish. For the object to be in equilibrium, the total upward force must equal the total downward force.

If there were horizontal forces acting a similar requirement would be in effect; for the object to be in equilibrium, the total force to the right must equal the total force to the left.

Although reference to up-down and right-left is good enough for many situations, it is convenient to have a more general way to refer to directions. This can be done by introducing a set of coordinate axes. In Figure 10 the X, Y, and Z

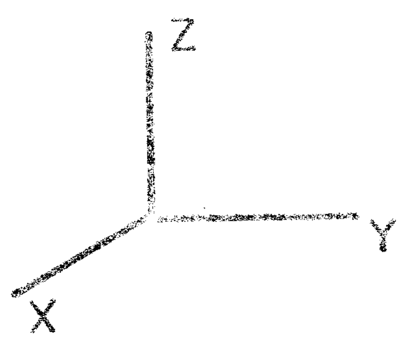


Figure 10

axes are perpendicular to each other. They provide a convenient way of referring to directions. We can now say that for a body to be in equilibrium the total force acting in the positive

X-direction must equal the total force acting in the negative X-direction. Corresponding statements apply to the y and z directions.

We will now agree to call forces positive if they act in the positive direction of the axis and negative if they act in the negative direction of the axis. With this convention it is possible to state the force condition for equilibrium. For an object to be in equilibrium, the sum of the x forces must equal zero (that is, combining positive and negative numbers), and likewise for the y and z forces. It is usually stated in somewhat brief form: <u>For an object to be in equilibrium the resultant force must be zero.</u>

This statement is often given in the form of an equation:

$$\Sigma\vec{F} = 0 \tag{4}$$

The symbol "Σ" is read "the sum of." The arrow over $\vec{F}$ is to remind us to consider the direction of the forces.

A similar statement can be developed for torques. Torques tend to produce either a clockwise or a counterclockwise rotation. For an object to be in equilibrium the total clockwise torque must equal the total counterclockwise torque. It is customary to call counterclockwise torques positive and clockwise torques negative. Then, the torque condition for equilibrium is: <u>For an object to be in equilibrium the resultant torque must be zero.</u>

$$\Sigma\tau = 0 \tag{5}$$

By applying these statements of the equilibrium conditions you will come to understand them better. You will have the opportunity to apply them in the examples and problems which follow and in laboratory exercises.

Example 2: A meter stick is suspended from a spring balance by means of a string attached to the 50 cm mark. M_1, a one kg object, is attached at the 10 cm mark. Another object with mass M_2 is attached at the 70 cm mark. The system is in equilibrium.

(a) What is the value of M_2?

(b) What is the reading on the spring balance in newtons?

(The weight of the meter stick is so small it can be ignored.)

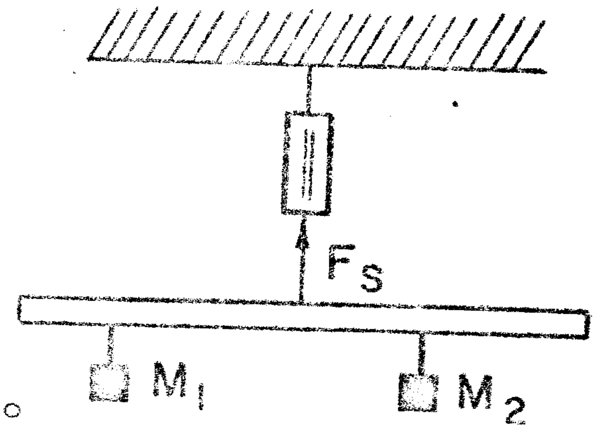


Figure 11

Solution: The two conditions for equilibrium must be satisfied. Considering the force condition first, it is clear that the upward force on the stick by the spring balance (the reading on the spring balance) must be just equal to the downward force exerted by the objects hanging from the meter stick. That is, the spring balance reading will be equal to the weight of the two objects. Recall that the weight of an object is equal to its mass times the acceleration due to gravity.

$$F_s = M_1 g + M_2 g$$

$$= (1\ \text{kg})(9.8\ \text{m/sec}^2) + M_2(9.8\ \text{m/sec}^2)$$

* If you are not familiar with the difference between weight and mass, read Appendix A.

But we cannot get both F_s and M_2 from this one equation. Therefore we will now consider the torque condition. First we must select a point about which to compute the torques. We can choose any point we want because the object is in equilibrium and the sum of the torques about <u>any</u> point must be zero. In this case it is convenient to choose the point at which the balance is attached; the 50 cm mark. With this choice, the torque produced by the spring balance is zero because the lever arm is zero. (The line of action of this force passes through the point selected for the torque calculation.) The force due to M_1 would tend to produce a counterclockwise (positive) rotation. The force due to M_2 would tend to produce a clockwise (negative) rotation. Applying the torque condition for equilibrium,

$$M_1 g \ell_1 - M_2 g \ell_2 = 0$$

where ℓ_1, the lever arm associated with M_1 has a value of 40 cm and ℓ_2, the lever arm associated with M_2, has a value of 20 cm. Solving for M_2,

$$M_2 = (M_1 g \ell_1)/(g \ell_2) = M_1 \ (\ell_1)/(\ell_2)$$

$$= (1 \text{ kg})(40 \text{ cm})/(20 \text{ cm}) = 2 \text{ kg}$$

This value of M_2 can be substituted into the equation obtained from the force condition.

$$F_s = M_1g + M_2g$$

$$= (1\ \text{kg})(9.8\ \text{m/sec}^2) + (2\ \text{kg})(9,8\ \text{m/sec}^2)$$

$$= 29.4\ \text{kg m/sec}^2 = 29.4\ \text{N}$$

Application of the equilibrium conditions has enabled us to determine the two unknowns.

Example 3: Compare the equilibrium situation shown in the figure with the preceeding example. Can you determine F_2 and F_s? (Assume that the weight of the odd shaped object can be ignored.)

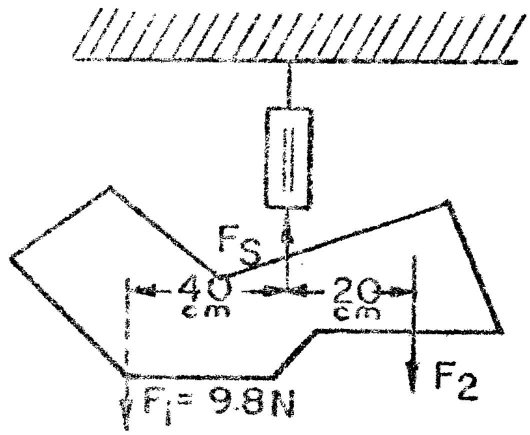


Figure 12

Solution: This is very similar to Example 2. F_1 has the same value as the weight of M_1 (9.8 N), and the lever arm is the same (40 cm). The lever arm for F_2 is the same as the lever arm for M_2. Therefore we can take over the results of Example 2. The force F_2 must have a value equal to the weight of M_2.

$$F_2 = (2\ \text{kg})(9.8\ \text{m/sec}^2)$$

$$= 19.6\ \text{kg m/sec}^2 = 19.6\ \text{N}$$

The reading on the spring balance will be the same as in Example 2.

$$F_s = 29.4\ \text{N}$$

Problem 2: For the equilibrium situation shown in Figure 13, determine the values of M and D. (The weight of the stick can be ignored.)

$F_1 = 30N$ $\quad$ $F_2 = 50N$

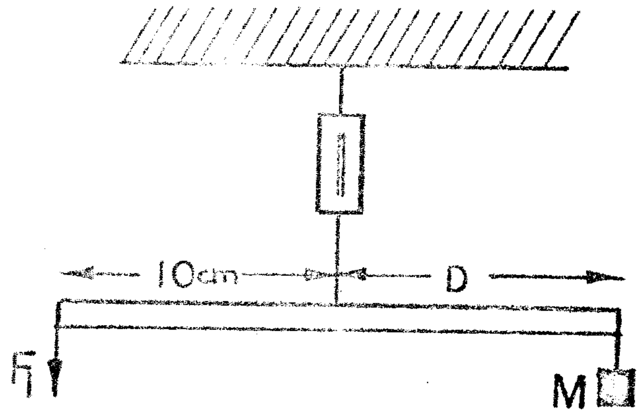


Figure 13

Example 4: In Figure 14 determine the value of F_1 needed to produce equilibrium. (Ignore the weight of the Stick.)

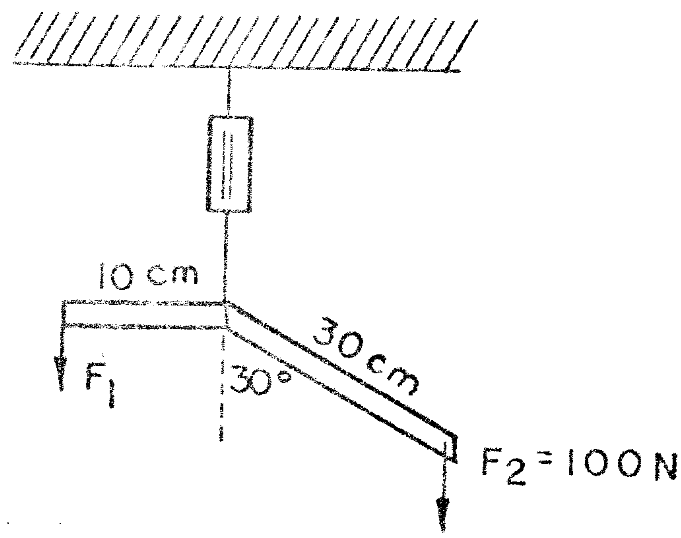


Figure 14

Solution: The torque about the point of attachment of the spring, due to F_2 is clockwise and therefore negative. Its magnitude can be calculated from Equation 1.

$$\tau_2 = -F_2 r_2 \sin\theta$$

$$= -(100N)(0.3m)(0.5)$$

$$= -15N\text{-}m$$

(This could also be calculated by use of Equation 2 if you first use trigonometry to determine the lever arm. You should try it.)

F_1 must produce a torque of the same magnitude, but counterclockwise.

$\tau_1 = -\tau_2$

$\tau_1 = F_1(0.1\ m)$

$F_1 = (15\ N\text{-}m)/(0.1\ m) = 150\ N$

Problem 3: For the equilibrium situation shown, determine the value of F_1. $F_2 = 50$ N

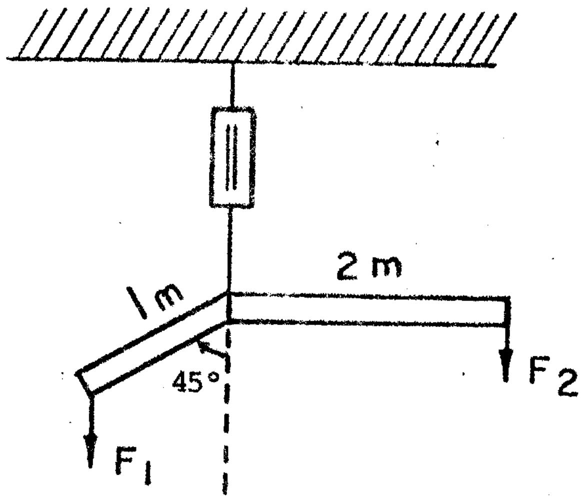


Figure 15

Experiment 2 -- Equilibrium

For each of the situations described, predict the values of the unknowns (indicated by blanks in the tables), then verify the predication with the experimental apparatus. Your predictions should be based on the two conditions for equilibrium; $\Sigma\vec{F} = 0$ and $\Sigma\tau = 0$.

For cases A through D, the arms are both horizontal as shown in the figure. W is the weight of the hub and arm assembly.

Case	M_1 (kg)	M_2 (kg)	r_1 (m)	r_2 (m)	$F_s - W$ (N)
A	0.20	0.40	0.30		
B		0.30	0.15	0.20	
C			0.16	0.24	9.80
D	0.50		0.15		7.35

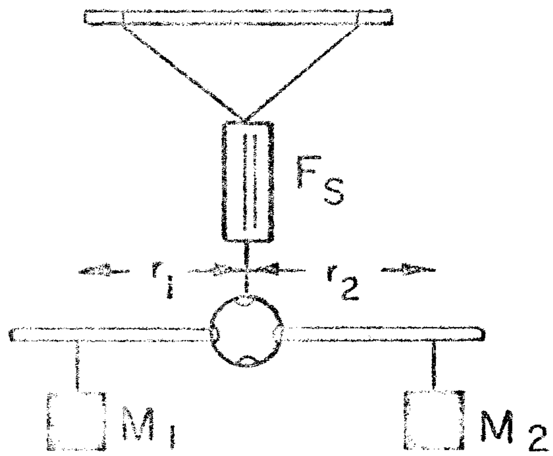


Figure 16

For cases E through G, the left arm is horizontal and the right arm is 45° below the horizontal, as shown in the figure.

Case	M_1	M_2	r_1	r_2	$F_s - W$
E		0.40	0.11	0.20	
F	0.30	0.60	0.20		
G			0.14	0.20	0.49

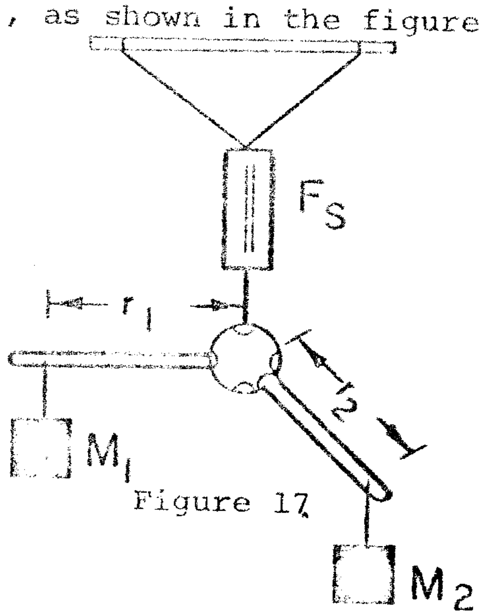


Figure 17

For cases H through J, the arms form a right angle, as in the figure. The angle θ which the right arm makes with the vertical varies from case to case.

Case	M_1	M_2	r_1	r_2	θ
H	0.50		0.10	0.22	30°
I	0.20	0.20	0.15		60°
J	0.55	0.28	0.12	0.24	

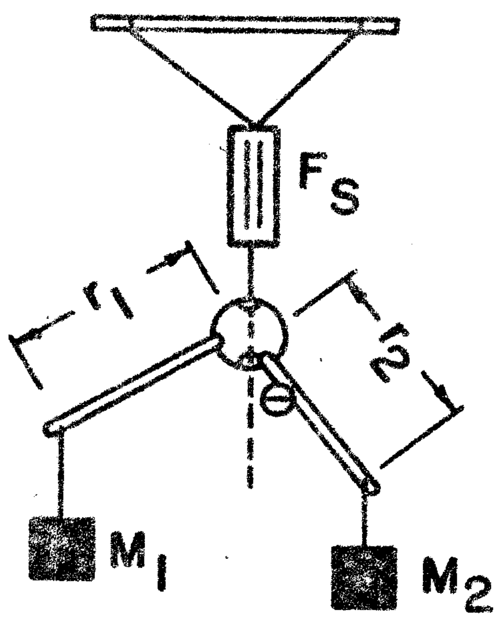


Figure 18

More Equilibrium

The equilibrium situations we have considered so far have had a simplifying feature; the forces were all vertical, either straight up or straight down. Horizontal forces add a minor complicating feature which can be handled easily.

Example 5: The board shown in the figure is in equilibrium as a result of the four forces indicated. The board is 2 ft wide and 4 ft high. If F_3 is 20 pounds, what are the values of F_1, F_2, and F_4?

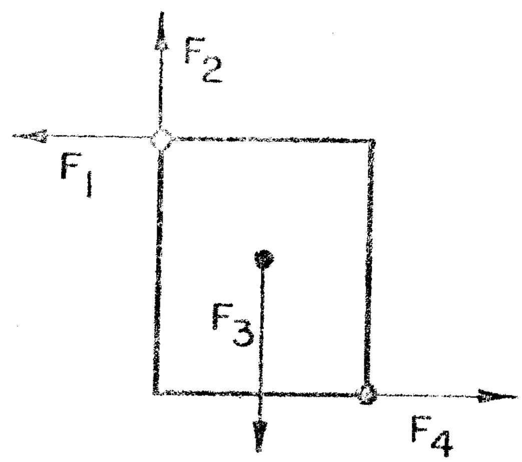


Figure 19

Solution: The problem isn't as difficult as it looks. The force condition for equilibrium tells us that the net vertical force must equal zero and that the net horizontal force must equal zero.

$$F_2 = F_3 = 20 \text{ lb}$$
$$F_1 = F_4$$

Now we must use the torque condition for equilibrium. We are free to use any point for calculating the torques. One convenient point is the upper left corner of the board, because with this choice neither F_1 nor F_2 produce a torque. The lever arm for F_3 is one ft and the lever arm for F_4 is four feet. So

$$F_4(4\text{ ft}) - (20\text{ lb})(1\text{ ft}) = 0$$

$$F_4 = (20\text{ lb-ft})/(4\text{ ft}) = 5\text{ lb}$$

We now know the forces we were asked to determine;

$$F_1 = F_4 = 5\text{ lb}$$

$$F_2 = 20\text{ lb}$$

Problem 4: Determine d, F_1, and F_2. See Figure 20.

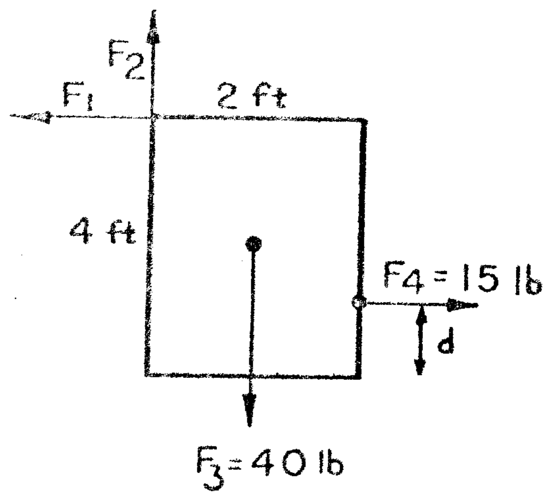


Figure 20

One more complication will now be studied; what if a force is neither vertical or horizontal, but somewhere in between? Such a situation can be handled by use of components of the force, which will now be described.

Figure 21 shows how to obtain the components of a force. The x-component of the force is the projection of the force on the X-axis. The y-component is the projection of the force on the y-axis. If F makes an angle of θ with the x-axis, then

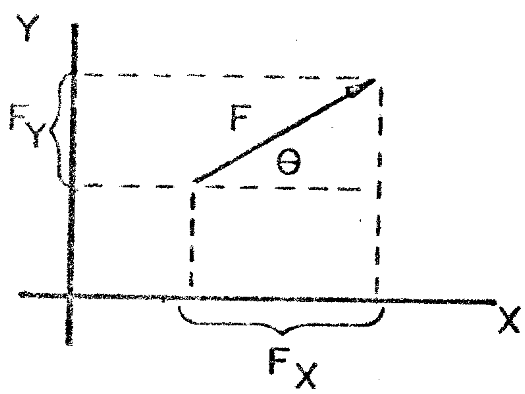


Figure 21

$$F_x = F\cos\theta$$

where F_x is the symbol for the x-component of F. Similarly

$$F_y = F \sin\theta$$

where F_y is the y-component of the force. The effect of the force F is exactly the same as the combination of F_x acting parallel to the x-axis and F_y acting parallel to the y-axis.

<u>Example 6</u>: If the object is in equilibrium, what are the values of F_1 and F_2?

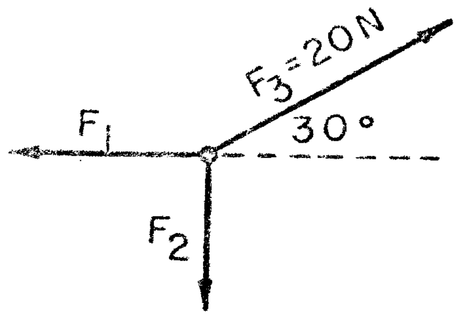


Figure 22

<u>Solution</u>: If we first replace F_3 by its horizontal and vertical components, we can then proceed as in previous examples. Let F_{3h} be the horizontal component of F_3, and F_{3v} be the vertical component.

$$F_{3h} = F_3 \cos 30^\circ = (20\ N)\cdot(0.866) = 17.3\ N$$

$$F_{3v} = F_3 \sin 30^\circ = (20\ N)(0.5) = 10\ N$$

From the force condition for equilibrium,

$$F_1 = F_{3h}$$

and $$F_2 = F_{3v}$$

Therefore,

$$F_1 = 17.3\ N$$

and $$F_2 = 10\ N$$

Example 7: A beam is supported as shown. The weight of the beam, F_2, can be treated as a single force acting at the center. Determine F_2, F_3, and F_4. F_1 = 40 N.

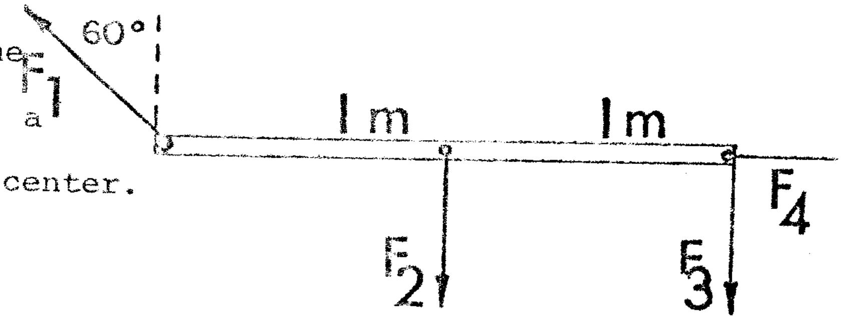


Figure 23

Solution: All forces except F_1 are either horizontal or vertical. Therefore let's resolve F_1 into its horizontal and vertical components.

$$F_{1v} = F_1 \cos 60^\circ = (40\ N)(0.5) = 20\ N$$

$$F_{1h} = -F_1 \sin 60^\circ = -(40\ N)(0.866) = -34.6\ N$$

The reason for the minus sign in the value of F_{1h} is that it acts to the left, and I am choosing directions to the right as positive. Applying the force condition for equilibrium,

$$F_4 = 34.6\ N$$

$$F_2 + F_3 = 20\ N$$

Take torques about the left end of the stick. Both F_1 and F_4 have zero torque arms. So

$$-F_2(1\ m) - F_3(2\ m) = 0$$

or

$$F_2 = -2F_3$$

Either F_2 or F_3 is negative! What does this mean? It means that the negative one is actually acting up, not down as shown in the figure.

To obtain the answer, note that we have two equations in two unknowns:

$$F_2 + F_3 = 20 \text{ N}$$

and $$F_2 = -2F_3$$

Solving,

$$-2F_3 + F_3 = 20 \text{ N}$$

$$F_3 = -20 \text{ N}$$

$$F_2 = -2F_3 = 40 \text{ N}$$

So F_3 actually acts up, not down.

Problem 5: A 3000 lb car is being towed as shown in the figure. The weight of the car acts as though it was all concentrated at a point 10 ft in front of the rear wheels. The tow chain is attached at a point 18 ft in front of the rear wheels and 3 ft off the ground. Determine F_1, F_2, and F_3.

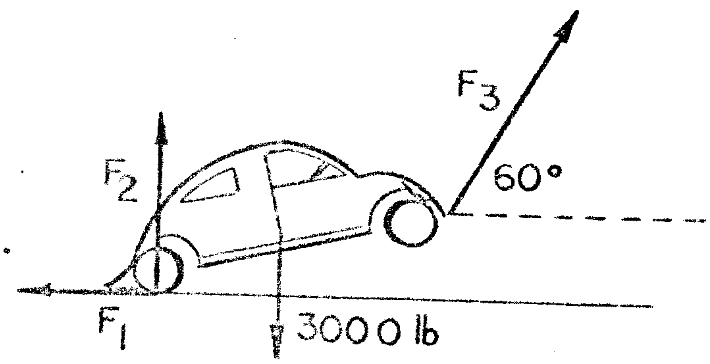


Figure 24

Example 8: A torque wrench is used to tighten a nut on a box as shown in the figure. Determine the maximum force, F, that can be applied without tipping the box. Also determine the torque associated with this force. The box

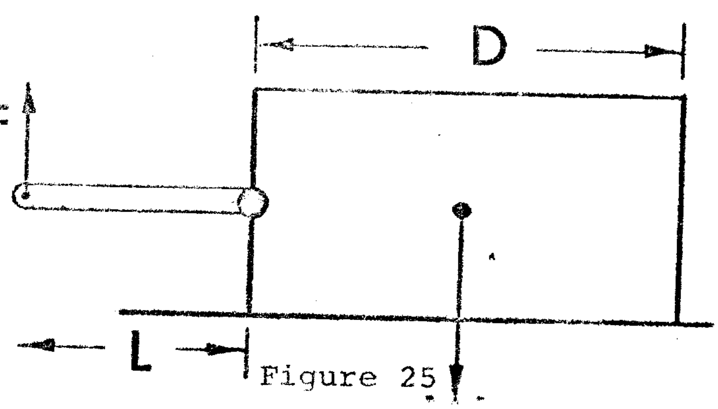


Figure 25

weighs 100 pounds and is 4 feet long. The wrench has a two foot handle. [Hint: When the box is about to tip, the force of the floor on the box will be acting on the right-hand edge of the box.]

Solution: If we take torques about the lower right hand corner of the box, then the force of the floor on the box will not appear in the torque equation. (The lever arm is zero.) The equation for the torque condition is

$$W(D/2) - F(D + L) = 0$$

Solving for F,

$$F = W[D/(2D+L)]$$

$$= (100\ \text{lb})[4\text{ft}/(8\text{ft}+2\text{ft})] = 40\ \text{lb}$$

The greatest force which can be applied is 40 lb. The torque applied to the nut by this force is readily obtained.

$$\tau = FL = 80\ \text{pound-feet}$$

Experiment 3 -- The Cantilever Beam

The flexing property of the torque wrench will be studied in this experiment. Instead of using real torque wrench handles (which often have somewhat complicated shapes), we will take measurements on uniform rectangular beams of metal. Beams which are held rigidly at one end and loaded along the length or at the other end are called cantilever beams. Thus in studying the behavior of the torque wrench handle you are also studying the cantilever beam.

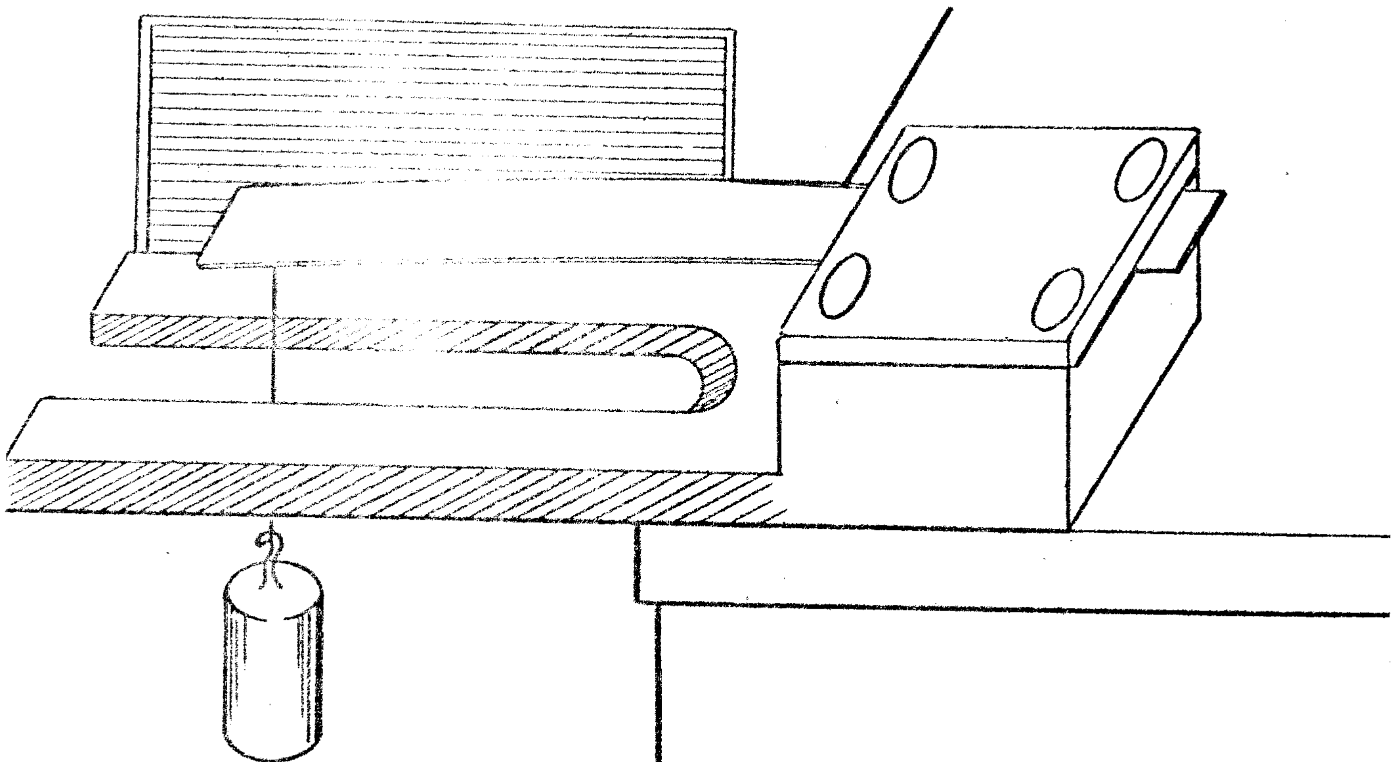


Figure 26

(A) Set up your apparatus as shown in Figure 26. Make the length of the beam 40 cm, measured from the holder to the end of the beam. Record the position of the end of the beam with no load attached. (In all of your measurements of position, sight over the top of the beam, with your eye at the same level as the top of the beam.) Now make measurements of the position of the beam for each of several weights at the end. Choose the weights to cover a range of deflections up to about 30 mm. (Some beams may be too stiff to get that much deflection.) Record the position for each value of load attached. Also record the breadth, B, and depth, D, of the beam. Subtract the no-load position from all your readings so that you will have the values of actual deflection. Plot your data, putting the deflection on the horizontal axis and the mass on the vertical axis. (Note that the applied force is mg, so you are plotting F/g versus deflection, not force versus deflection.)

With a ruler draw the straight line which best fits your data points. Determine the slope of the straight line. (Be sure to indicate the units for the slope.)

-(B) If your apparatus and procedure in part (A) were good, your data points were very close to a straight line. This means that the force is proportional to the deflection:

$$F = kx$$

The constant of proportionality, k, is called the <u>spring constant</u>, or <u>force constant</u>. Determine the value of k from the slope of your straight line. Be sure to include units for k.

-(C) The spring constant is a measure of the stiffness of the beam. From experience you know that, for beams of the same material, the stiffness depends on the dimensions, L, B, and D (length, breadth and depth). Make a guess (perhaps a wild guess) on how the spring constant depends on each of the three dimensions. (Clue: it depends on each dimension to some power; that is, D^n, where n is a positive or negative integer.) For example you might guess a direct proportion, or an inverse proportion, or proportional to the inverse square, etc. To check on your guess you could repeat the experiment you have done, using beams with different dimensions. However, you may be able to get the results which other students have obtained on different beams. If you pool your results, you should be able to figure out how the spring constant depends on each of the dimensions. For example you might have available the value of k for two beams which are identical except that one is twice as thick as the other. If the spring constant for the thick one is one quarter that of the thin one, this would suggest that the spring constant is inversely proportional to the square of the thickness.

If your pool of data does not contain enough information, fill it in by doing more experiments. It won't be necessary to completely repeat the process of part (A). You have already shown that

$$F = kx$$

Therefore

$$k = F/x$$

So one or two measurements of F and x should enable you to get the value of k for each beam. Eventually you should be able to write the spring constant in this form:

k = (a number) x (L to some power) x (B to some power) x (D to some power)

Hooke's Law

One of the results of Experiment 3 is that the deflection of a cantilever, or the flexing of a wrench handle, is proportional to the applied force:

$$F = kx$$

There are many other systems which behave this way. The force required to stretch a spring is proportional to the

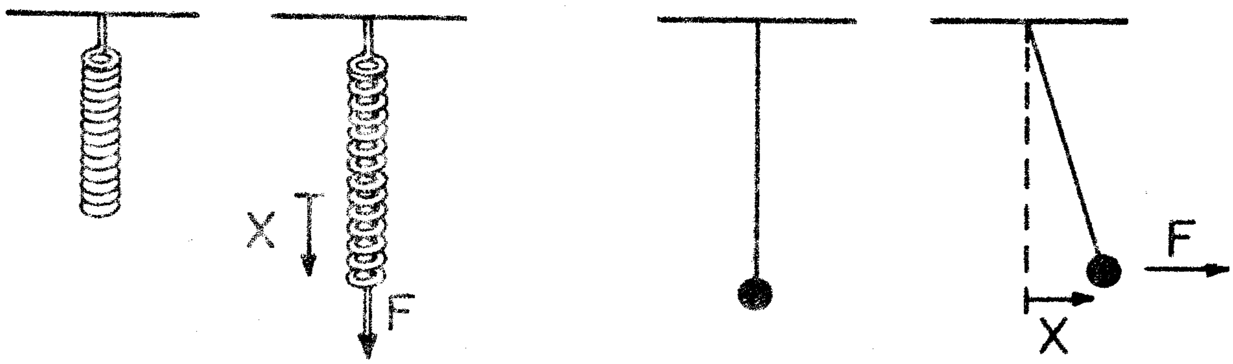


Figure 27

amount of stretching. If a pendulum is pushed a short distance from its equilibrium position, the force required to hold it there is proportional to the displacement. Systems which have this property are said to obey Hooke's Law: The force required to produce a displacement is proportional to the displacement.

Example 9: The graphs below show how the force depends on displacement for three different systems. Which one follows Hooke's Law? What is the value of k?

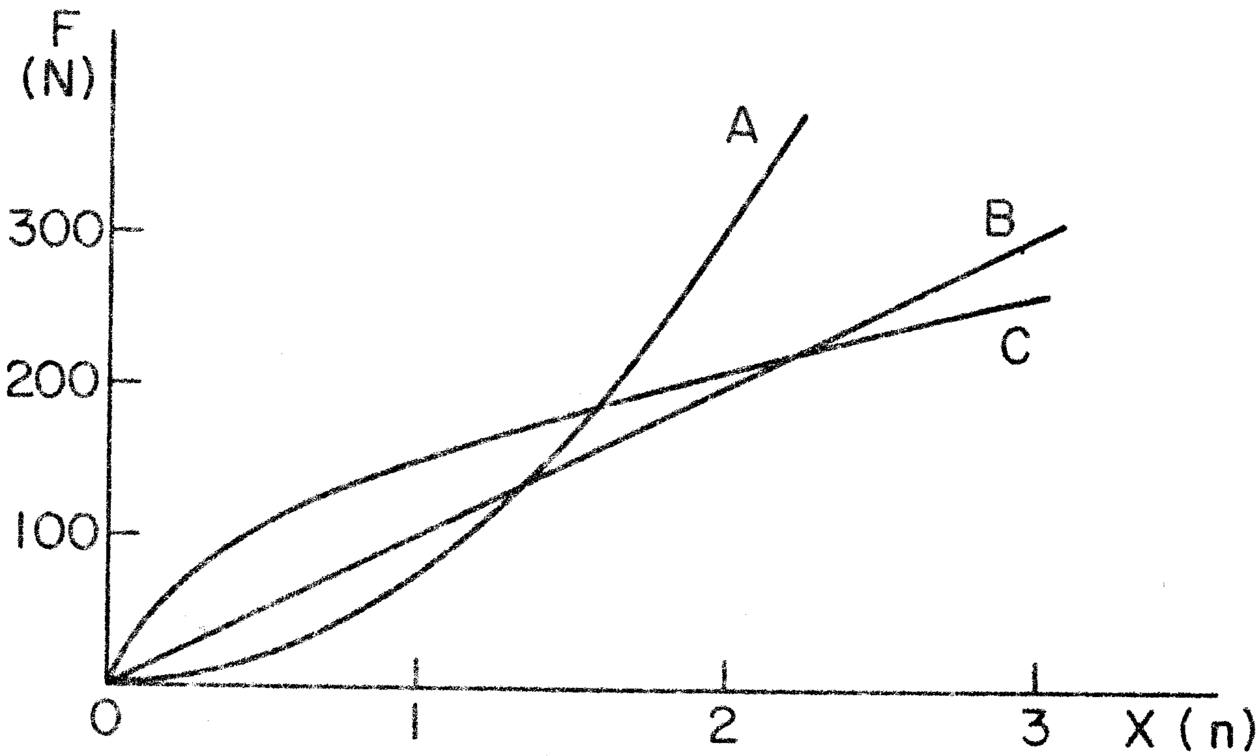


Figure 28

Solution: You may recall that F = kx is the equation of a straight line. So only in system B is the force proportional to the displacement. The slope of the straight line is the value of k. When x increases by one meter, F increases by 100 newtons. Thus the value of k is 100 newtons per meter. Notice that for system A, when the displacement is doubled, the force goes up by a factor of four. At one meter the force is 75 N while at two meters the force is 300 N. For system C, if the displacement is doubled, the force goes up less than a factor of two.

Problem 6: The table below is for a spring which follows Hooke's Law. Determine the spring constant and fill in the blanks.

Displacement (m)	0	0.05	0.1		0.7
Force (N)	0		20	100	

The Hooke's Law behavior of many complex systems can be explained starting from the behavior of simpler systems. For example, the bending of a torque wrench or cantilever beam is determined by the geometry and the properties of the metal. We will describe how the bending of a cantilever beam is related to the stretching of a piece of the same material.

If a weight is suspended by a rod as shown in Figure 30, the rod will stretch. When the weight is removed, the rod

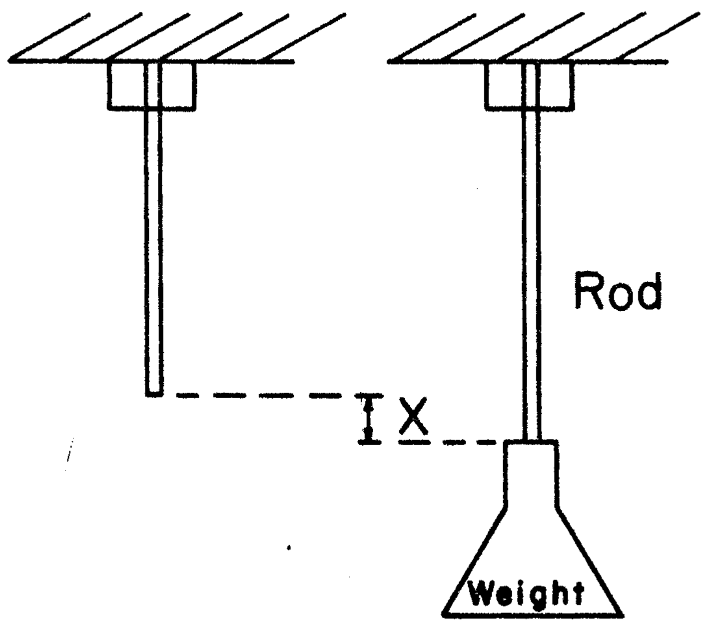


Figure 29

returns to its original length. Experiments show that the increase in length is proportional to the applied force. In other words, the rod obeys Hooke's Law. Figure 30 shows a typical result of such an experiment. Note the graph shows a straight line behavior until the force gets to a large value; then the graph curves. So it isn't quite correct to say that the rod obeys Hooke's Law. We should say that it obeys Hooke's Law if the force isn't too big. Actually this is true for any Hooke's Law system. There is a limit to how much you can stretch or bend or twist something. If you go beyond this limit, the displacement is no longer proportional to force, and the device will probably be permanently deformed. This limit is called the elastic limit. From now on we will restrict our attention to the straight line region, or Hooke's Law region.

The behavior of the rod can be described by the equation

$$F = kx$$

The constant of proportionality, k, is called the spring constant, or force constant. The form of this equation is the same as for the bending of the cantilever beam. However, in this case it refers to stretching, not bending, and the value of k will be different.

A similar experiment could be performed in which the rod is compressed instead of stretched. Such an experiment shows that not only is the compression proportional to the applied force, but the spring constant for compressing is the same as for stretching.

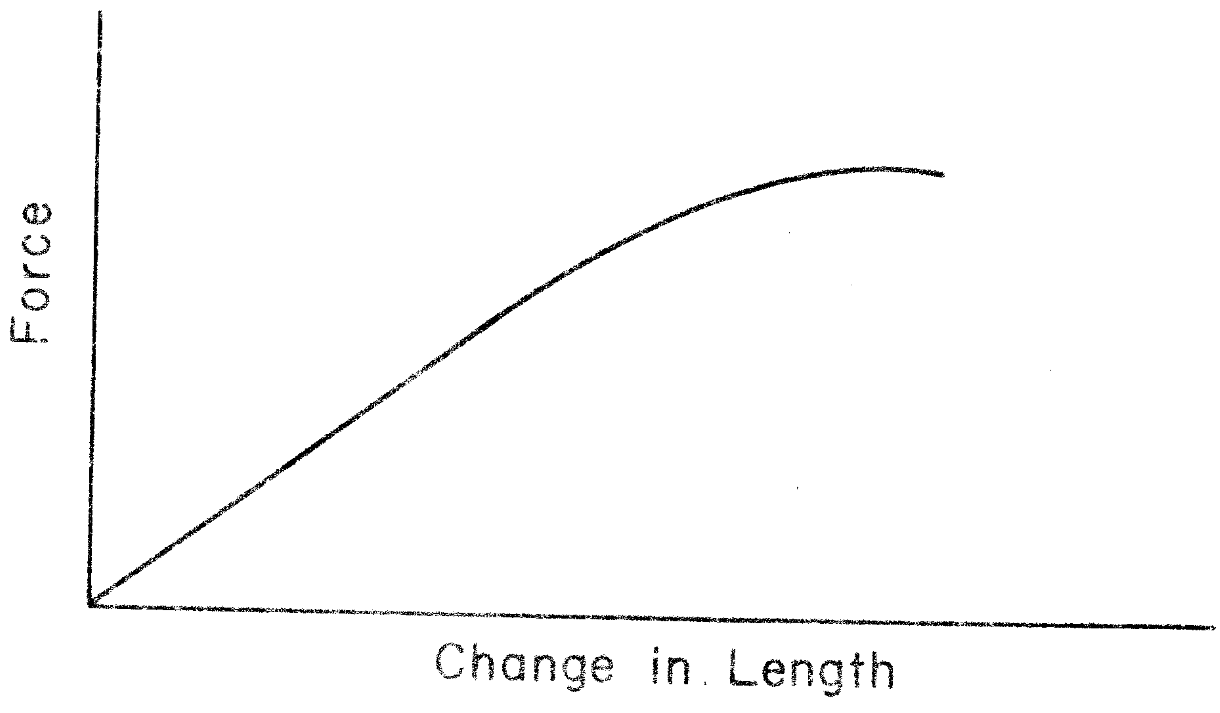


Figure 30

What determines the value of the spring constant? It depends on the material the rod is made from, the length of the rod, and the cross-sectional area.

The dependence of the spring constant on the cross-sectional area can be deduced with the aid of Figure 31.

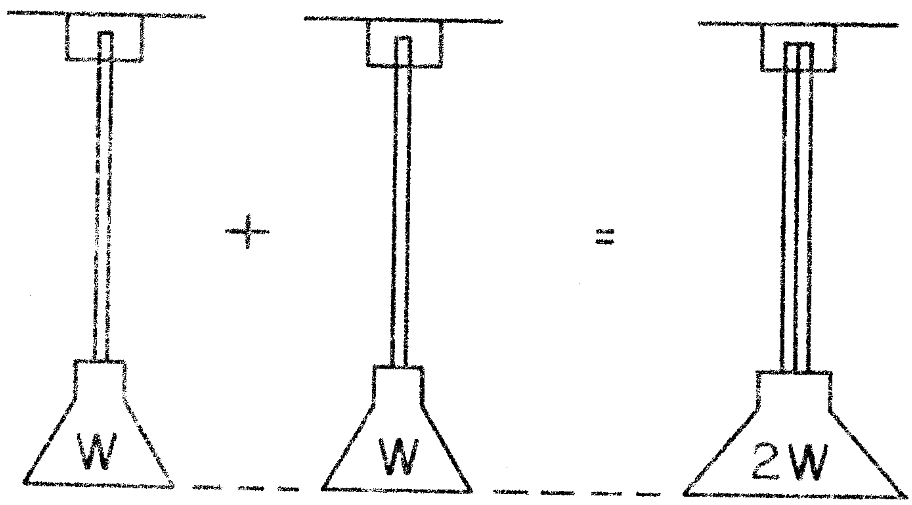


Figure 31

If the two rods are brought together, the new system will have the same length and the same amount of stretching, but twice the area and twice the applied force. That is, if you double the area you double the force required for a given increase in length. So the spring constant is proportional to the cross-sectional area.

The way k depends on length is illustrated by Figure 32. If the same load is applied in each of the two cases, the tension will be the same. Thus each unit of length will stretch the same amount, producing an increase in length which

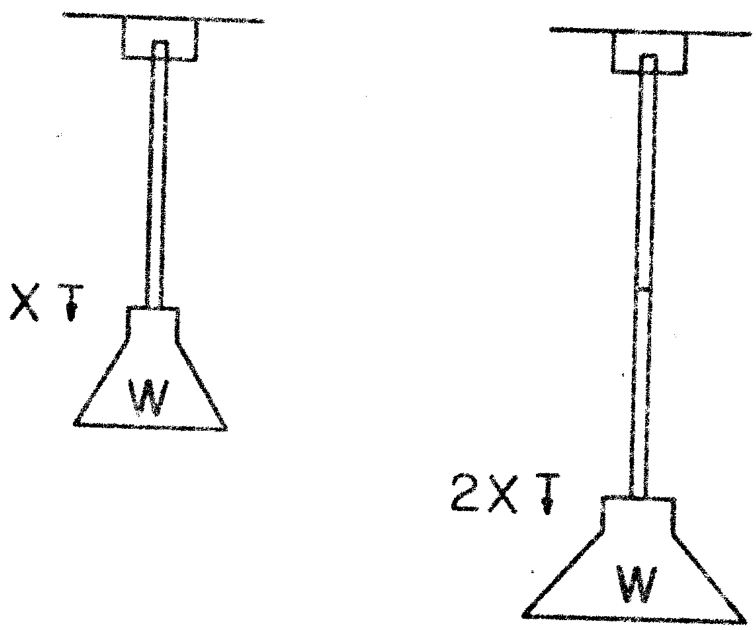


Figure 32

is proportional to the length. So for a fixed force the increase in length is proportional to the original length. This leads to the conclusion that the spring constant is inversely proportional to the length. Combining the dependence on length and cross-sectional area we have

$$k = Y\ (A/L)$$

where A is the cross-sectional area, L is the length, and Y is a constant for a particular material. The name for Y is Young's modulus. Its value can be determined experimentally by finding the value of k for a rod with known length and cross-sectional area. Table 1 gives the approximate value of Young's modulus for several materials. The precise value depends on how the metals are prepared.

Table 1

Material	Young's Modulus (N/m^2)
Aluminum	7×10^{10}
Brass	9×10^{10}
Cast iron	9×10^{10}
Copper	12×10^{10}
Steel	20×10^{10}

Example 10: A brass rod one square millimeter (10^{-6} m^2) in cross-sectional area and two meters long is used to support a chunk of material weighing 100 newtons. Determine the change in length caused by the load.

Solution: $F = kx$

and $k = Y(A/L)$

Therefore $x = F/k = FL/YA$

Use the data given in the problem together with the value of Young's modulus from Table 1.

$$x = \frac{(100\ N)(2\ m)}{(9 \times 10^{10}\ N/m^2)(10^{-6}\ m^2)} = 2.2 \times 10^{-3}\ m$$

$$= 2.2\ mm$$

Problem 7: A steel rod has a cross-sectional area of two square millimeters ($2 \times 10^{-6}\ m^2$) and a length of 10 meters. How much force is required to stretch it two millimeters?

It might seem that the bending of a cantilever is only remotely related to the streching and compressing of a rod. However there is a close relationship, as indicated by Figure 33. When the beam is bent down, the top parts are stretched

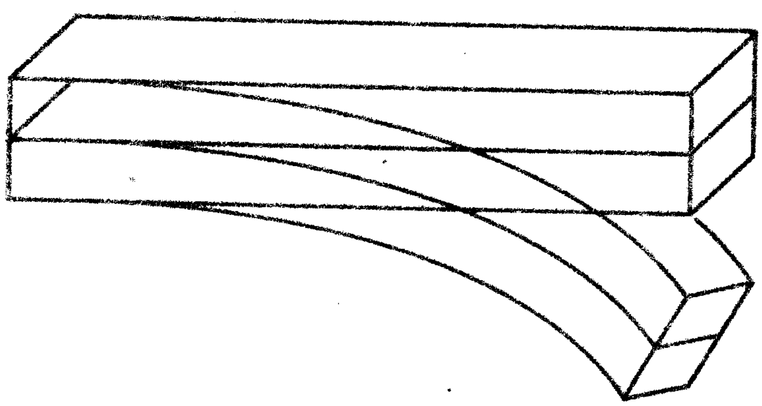


Figure 33

and the bottom parts are compressed. Therefore we might expect the force constant for the cantilever to depend on Young's modulus. It also depends on the length (L), the breadth (B), and the depth (D) of the beam, as you probably discovered in Experiment 3. A careful analysis (which we will omit) leads to this equation for the spring constant for a cantilever.

$$k = (Y/4)(BD^3/L^3)$$

The Y in this equation is the same Young's modulus that we met in the stretching and compressing of a rod. Thus the same physical property of the material enters in both stretching and bending.

Example 11: Determine the spring constant for a steel cantilever beam which is 40 centimeters long, one centimeter wide and three millimeters thick.

Solution: This requires a straight forward application of the equation for the spring constant for a cantilever beam:

$$k = (Y/4)(BD^3/L^3)$$

$$= \frac{20 \times 10^{10}\ \mathrm{N/m^2}}{4} \times \frac{(10^{-2}\ \mathrm{m})(3 \times 10^{-3}\ \mathrm{m})^3}{(0.4\ \mathrm{m})^3}$$

$$= (5 \times 10^{10})\ \frac{(10^{-2})(27 \times 10^{-9})}{(0.064)}\quad \mathrm{N/m}$$

$$= 211\ \mathrm{N/m}$$

Problem 8: Using your data from Experiment 3, determine Young's modulus for the material in one of the beams you used.

Appendix A

Weight and Mass

Weight and mass are two entirely different concepts. The weight of an object is the force of gravity on an object. For example, if you hold a chunk of iron in your hand, you can feel the weight, which is due to the gravitational attraction which the earth exerts on the chunk of iron.

The mass of an object is a measure of its resistance to change in its motion. You can feel this property when you throw an object; you must exert a force on the object to change its motion from a state of rest to one in which it is moving. Similarly, you feel it when you stop a moving object. Scientists have a precise definition for mass, but we won't introduce it here because we don't need it. At this point we only need to recognize the difference between weight and mass, and to establish how to work with them.

You know from experience that the heavier an object is, the harder it is to change its motion. In other words, the greater the weight, the greater the mass. One of the most remarkable aspects of nature is the very simple way in which weight and mass are related; the weight is proportional to the mass. If we let W represent the weight and M the mass, then

$$W = Mg$$

where g is simply the acceleration due to gravity. The value of g near the surface of the earth, in the metric system of units is 9.8 m/sec.^{2} Thus a one kilogram object has a weight of 9.8 newtons. In general, to find the weight in newtons, multiply the mass in kilograms by 9.8.

The relation between weight and mass emphasizes an important point; although the mass of an object does not depend on its location, the weight does. A one kilogram object will weigh 9.8 newtons on the earth, but only about one-sixth that much on the moon. So it would be easier to hold it on the moon (compared to the earth) but just as hard to throw it at some specified speed.

Sample Examination Questions

1. Define the following:

   (a) Torque wrench

   (b) Torque

   (c) Lever arm

   (d) Equilibrium

   (e) Static equilibrium

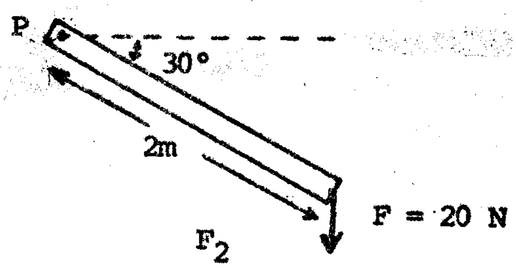
   (f) Spring constant

   (g) Cantilever beam

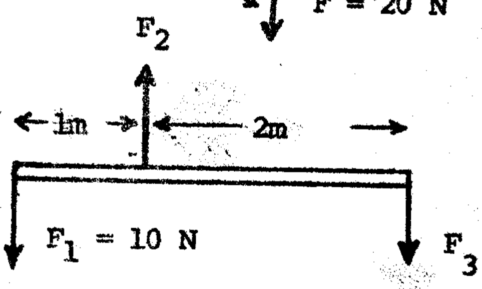
2. State the conditions which the forces and torques must satisfy for an object to be in equilibrium.

3. A 100 newton force makes an angle of 30° with the horizontal. Determine the horizontal and vertical components of the force.

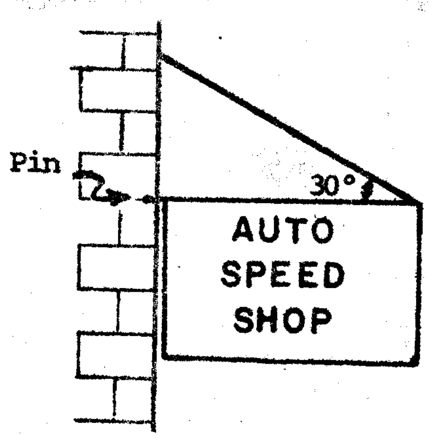
4. For the situation shown in the diagram, determine the lever arm and the torque associated with the force F. Use the point P for the axis.



5. The stick shown in the diagram is in equilibrium. Determine F_2 and F_3. (The weight of the stick can be ignored.)



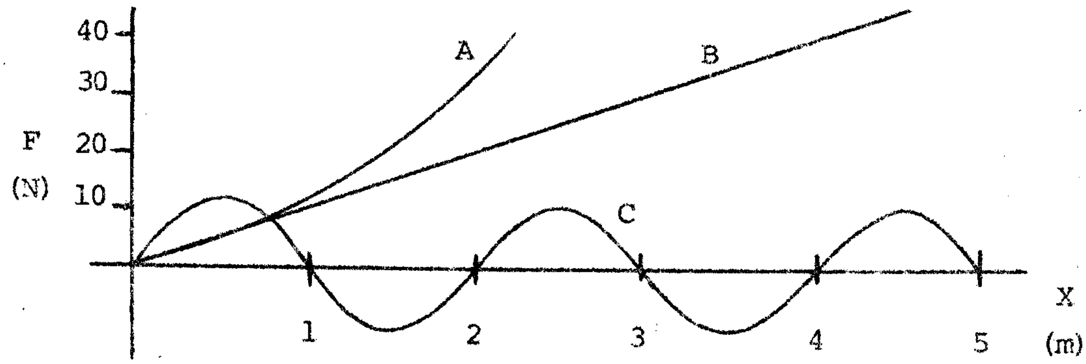
6. A 20 pound 8 feet long sign is held in equilibrium by means of a pin and a wire as shown. The weight of the sign acts through its



center. Determine the tension in the wire and the vertical and horizontal components of the force of the pin on the sign.

7. Describe the behavior of systems which obey Hooke's Law.

8. The graphs below show how the force depends on displacement for three different systems. Which one follows Hooke's Law? What is the value of the spring constant?



9. The equation for the spring constant for a contilever is

$$k = (Y/4)\ (BD^3/L^3)$$

If a steel beam two meters long and one centimeter thick is to have a spring constant of 5×10^{12} newtons per meter, how wide must it be? (Youngs modulus for steel is 20×10^{10} N/m^2)

Sample Examination Answers

1. (a) A torque wrench is a tool which can apply a torque to a nut or bolt and which has a scale which indicates how much torque is being applied.

   (b) Torque is the twisting effect produced by an applied force. It is defined by the equation

$$\tau = F\ell$$

   where τ is the torque, F is the applied force, and ℓ is the lever arm.

   (c) The lever arm is the perpendicular from the axis to the line of action of the force

   (d) An object is in equilibrium if it is not being accelerated; that is, both its translational speed and rotational speed must be constant.

   (e) An object which is not moving is in static equilibrium.

   (f) If a system responds in such a way that the force required to produce a displacement is proportional to the displacement, the constant of proportionality is called the spring constant.

   (g) Beams which are held rigidly at one end and loaded along the length or at the other end are called cantilever beams.

2. For an object to be in equilibrium, two conditions must be satisfied:

   (a) The resultant force must be zero,

   (b) The resultant torque about any axis must be zero.

3. Horizontal component = 86.6 N

   Vertical component = 50 N

4. Lever arm = 1.73 m

   Torque = 34.6 N-m

5. F_3 = 5N F_2 = 15N

6. Tension = 20 lb

   Horizontal force of pin = 17.3 lb

   Vertical force of pin = 10 lb

7. For systems which obey Hooke's Law, the force required to produce a displacement is proportional to the displacement.

8. System B obeys Hooke's Law, and has a spring constant of 10 newtons per meter.

9. 8 centimeters

Learning Objectives

This list of objectives indicates what you should be able to do after completing the study of this module.

1. Define in clear and precise sentences the following words: torque, torque wrench, lever arm, equilibrium, static equilibrium, spring constant, cantilever beam.
2. State the conditions which the forces and torques must satisfy for an object to be in equilibrium.
3. Given the magnitude and direction of a force, determine its components in a specified coordinate system.
4. Given the magnitude and direction of a force, and its point of application on an object, determine the lever arm and the torque for a specified axis.
5. Solve equilibrium problems of the type illustrated by the following examples in the text: 2, 3, 4, 5, 6, 7, and 8.
6. Describe the behavior of systems which obey Hooke's Law.
7. Given empirical information, either tabular or graphical form, on how a system responds to an applied force, determine whether it obeys Hooke's Law; if so determine the spring constant.
8. Given the equation for the spring constant for a cantilever, and the values for all but one of the physical quantities appearing in the equation, determine the unknown.

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New York City Community College
Brooklyn, New York

Address inquiries to:

Philip DiLavore, Project Coordinator
Tech Physics
Indiana State University
Terre Haute, Indiana 47809