PHYSICS OF TECHNOLOGY — A Modular Approach

THE ELECTRIC FAN

A MODULE ON RIGID BODY ROTATION

PRINCIPAL AUTHORS: **John W. McWane,**
Technical Education Research Centers
Malcolm K. Smith,
Lowell Technological Institute

TERC
Technical Education Research Centers

Coordinated by the American Institute of Physics

PROJECT DIRECTOR: John W. McWane

PHYSICS CONSULTANTS: Nathaniel H. Frank,
Massachusetts Institute of Technology

Ernest D. Klema,
Tufts University

APPARATUS DESIGN: Richard R. Lewis

GRAPHIC COMPOSITION: Mary A. Leonard

ILLUSTRATION: John W. Saalfield

The materials contained herein were developed pursuant to a contract with the National Science Foundation, Grant Number GY - 9317.

The materials herein produced at the:

TERC Curriculum Development Laboratory
575 Technology Square
Cambridge, Massachusetts
02139

Printed: December, 1973

PREFACE

About The Physics Of Technology Modules:

The physics of Technology modules provide a course in experimental physics primarily for the technical student. The methods of presentation therefore differ from those of standard materials. This preface highlights some of the features of the module so that you can use it effectively and efficiently.

THE TITLE:

Each PoT module is centered on a device or system that is familiar to you or that you may meet in a later work situation. The operation of the device will generally depend on some area of physics that is relevant to the technology involved. This area of physics is listed in a secondary title for the module.

THE INTRODUCTION:

A brief introduction explains why we have chosen this particular device and what principles of physics we expect you to learn from it. Several examples are given to show why these principles are important and where else you may encounter them.

Objectives are then given for the module. These include a general discussion of the goals of the module as well as specific things you should be able to do to demonstrate that you have achieved these goals. After each objective is listed the pages in the text where it is discussed, and *Problems* and *Questions* that you can do to test whether you have achieved the objective.

Prerequisites to the module indicate the skills and knowledge you will use in the module, but will not be discussed. Lacking one or two of these should not prevent you from doing the module since they can be learned as you go along. However, if you lack more than that you might begin with another module whose prerequisites you have or take time out to learn the missing ones.

THE THREE PARTS:

The module is divided into three self-contained parts. Each part is designed to be completed in about one week's time.

PART I is usually devoted to familiarizing you with the *device*, the *instrumentation* you will use to measure its performance, and the *terms* that describe its characteristics. Often this will mean learning how to use a new measuring instrument or transducer.

PART II generally focuses on an experiment involving some specific behavior of the device. The laboratory instructions are explicit as to experimental procedures. They tell you in detail how to take, graph, and analyze the data. This is to familiarize you with the *experimental methods* involved with this area of physics.

PART III generally will involve doing some additional studies of the *behavior of the device*. You will use the instrumentation learned in the first week and the methods of data taking and analysis learned in the second week.

THE ROLE OF THE LABORATORY:

Since this is primarily a program in experimental physics, the material emphasizes the experimental activities. A short introduction will orient you to the experiment you will do, and to the important physics principles or skills that you should learn. The experiment, including the set-up and data taking procedures, then follows immediately.

The principal experiment is often preceded by some simple ones. These are included to give you a feel for what you will be doing in the main experiment. They generally can be done quickly and do not require extensive data taking. The principal experiment, however, should be done carefully since the remainder of the material will discuss this data.

Once your experimental work has been completed and all of your data taken, you can leave the laboratory. The remainder of the material is devoted to helping you graph and analyze your data while explaining the physics involved. This work can be done either as homework or in class with your instructor. Tear out pages for data and graphs are provided in the module as they are needed.

In reading the material be sure to keep in mind the module objectives. Although the material generally goes beyond what is needed to achieve them, your test at the end of the module will be on those objectives.

Finally, the module has been designed to provide you with an understanding of physics that will be useful to you. Within the text you will find conversion table, methods for calibrating transducers, explanations of physical terms, comparisons of ways to measure a physical parameter, and so on. These could be of use to you sometime and you may want to tear them out to keep as reference material.

I hope that the material in these Physics of Technology modules will provide you with the skills of experimental science and insight into the physical prinicples underlying your technical field.

January 1973

TERC

Cambridge, Massachusetts

John W. McWane
Project Director

CONTENTS

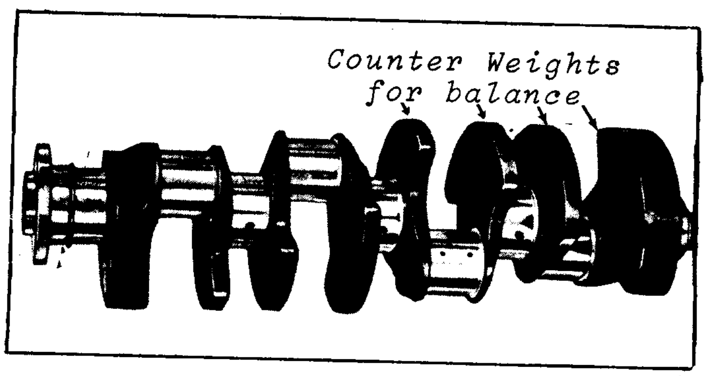
INTRODUCTION

WHY STUDY THE ELECTRIC FAN?

In this module you are going to study the electric fan because it can be used to illustrate some important principles of physics. We all have used fans to stir up a breeze and help us keep cool on a hot day. But in this module you will not study air motion or the action of the fan blades. Instead, our interest is in rotation - the behavior of spinning bodies, automobile tires, gyros, and many other rotating objects. We will use the electric fan because it is inexpensive, convenient to instrument and its rotational speed can be easily controlled.

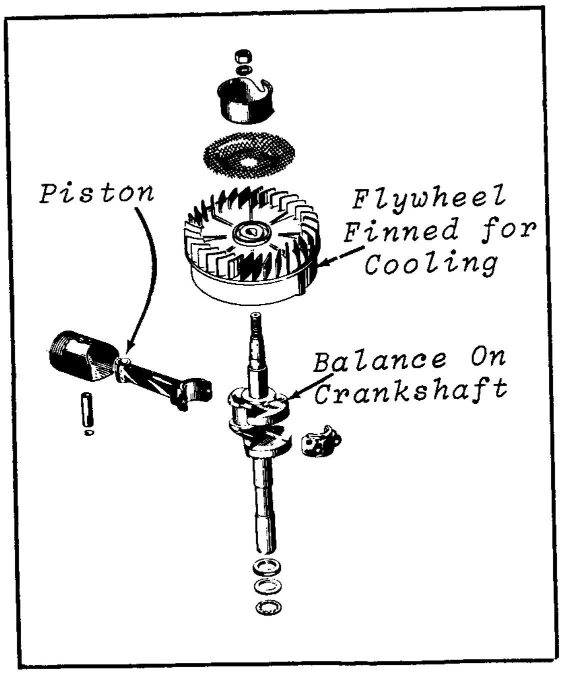
THE PHYSICS OF ROTATION HAS MANY APPLICATIONS:

<u>Balancing Automobile Wheels</u>: Automobile wheels need to be properly balanced. The machine shown is a static balancer, which means that the wheels will be in balance as they stand or at low speeds. Usually this is adequate. Sometimes, however, wheels may be *statically* balanced, but out of balance *dynamically*, that is unbalanced at high rotational speeds.



<u>Balancing Crankshafts</u>: Automobile engine crankshafts must be dynamically balanced with great care. The crescent shaped pieces opposite are counterweights, carefully located and of just the right mass. Dynamic balance is attained by the placement of weights all along the crankshaft. Unbalance in a modern, high-speed engine could quickly destroy it.

Storing of Energy: The flywheel shown (finned to act as a cooling fan) is bolted onto the end of a lawn mower engine crankshaft. Flywheels are particularly important in one cylinder engines like this since there is only one power impulse in every two revolutions. The flywheel stores the energy of the power stroke until the next one occurs. This stored energy keeps the engine and its parts moving smoothly and at a steady angular speed.



IN THIS MODULE YOU WILL LEARN:

PART I THE DESCRIPTION OF ROTATION: KINEMATICS. Kinematics is concerned with how motions take place: how fast? where? when?. Rotational kinematics is described by angular speed, such as the rpm of an engine, and by the angle turned, such as when a building crane swings from one position to another. Often the relation between straight line motion and rotation is what counts. For example an automobile odometer reads distance traveled as the wheels turn. In Part I of this module you will study the behavior of the electric fan to learn about its rotational kinematics.

PART II THE CAUSES OF ROTATION: DYNAMICS. Dynamics tries to answer such questions as: what caused a wheel to slow down? how much torque can an engine exert? why do gasoline engines need flywheels and how should they be made?. Our main concern in Part II will be with the torques that cause changes in rotation, and with the rotating behavior of bodies of various sizes and shapes.

PART III STATIC AND DYNAMIC BALANCE. Balance means much more than just two kids at either end of a see saw. The principles of balance tell how an automobile engine crankshaft must be built, if the engine is to run smoothly and not vibrate excessively. Every turning wheel must meet the special conditions called dynamic balance or great strain will be put on the shaft and perhaps the bearings will be destroyed. In Part III you will learn the the difference between static and dynamic balance and methods for achieving them for a rotating body.

MODULE OBJECTIVES

The general goal of this module is to give you an understanding of the important features of rotational motion. You will see how these principles are applied in real rotating devices.

This involves a knowledge of:

* The kinematics of rotation; that is, the description of angular motion in terms of angles, angular speed, and their relation to linear motion.
* The dynamics of rotation: including the relation between torque, moment of inertia, and change of angular speed.
* Instruments used in observing and measuring rotational motion such as tachometers, revolution counters, and stroboscopes.
* Static and dynamic balance, and methods of statically and dynamically balancing a rotating body.

At the end of this module you should be able to demonstrate your understanding of its objectives by doing the following:

(PART I)	Pages Where Discussed	Problems and Questions
1. Explain how to use a stroboscope to measure angular speed.	10,11,13	Q - 1 P - 1
2. Use a stroboscope to make a calibration graph for a tachometer generator.	15,17,18	Q - 2 P - 2,3
3. Make a graph of angular speed against time and use the area under the curve to calculate the total number of revolutions.	24,25	P - 3

	Pages Where Discussed	Problems and Questions
4. Use the angle turned to calculate the linear distance moved by a point on a rigid body.	19,20,23	Q - 3,4 P - 4,5,6 7 & 8
5. Use the common units of angle measure, degrees and radians, in calculations and explain why radian measure is often preferred.	19,22,23	Q - 3,4
(PART II)		
6. Explain the concept of torque, including stalling torque and dynamic torque.	38,44	Q - 1,4 P - 1,2
7. Explain moment of inertia, I, and calculate it for bodies of various shapes.	39,41,42	Q - 2,4,5 P - 3,4
8. Use the basic rotational dynamic equation: $\tau = I\frac{\Delta\omega}{\Delta t}$, to analyze changes in rotational motion.	43,44	Q - 4 P - 5,6,7
9. Use the dynamic equation to determine the moment of inertia, I, of a rotating body from its behavior under constant torque.	43,44	
10. Explain the purpose and design of flywheels in terms of rotational kinetic energy.	45,46	Q - 2,5 P - 5,8
(PART III)		
11. Statically balance a rotor when provided with a simple balancing machine.	57	Q - 1 P - 1

	Pages Where Discussed	Problems and Questions
12. Dynamically balance a simple rotor using a dynamic balancing table.	60,61	
13. Describe static balance in terms of center of mass.	63,65,66	Q - 2 P - 2
14. Explain reaction force and calculate its magnitude.	63,65,68	Q - 3,4 P - 3,4
15. Describe dynamic unbalance in terms of reaction force and wobble torque.	68	Q - 5 P - 5

PREREQUISITES:

Before begining this module you should already have the following skills, since they will be used, but not described, in the module:

1. graphing data on linear graph paper;
2. using scientific notation;
3. solving simple algebraic equations;
4. reading meters and using a stop watch;
5. using elementary geometry; measuring angles and finding areas of rectangles and triangles;
6. using the idea of rate; distance or revolution, or some other quantity per unit time.

If you are unsure whether you have these prerequisites, ask your instructor to give you a prerequisites test.

If you find you do not have a certain prerequisite, ask your instructor to give you material to help you learn the needed skill; or have someone help you learn it as you need it in the module.

PART I

ROTATIONAL KINEMATICS

In this module, we want to talk about rotation in a useful way, and that means in a quantitative way. That is, we must describe rotation using numbers; measurements that tell how fast, how far, how much a rotating body has moved.

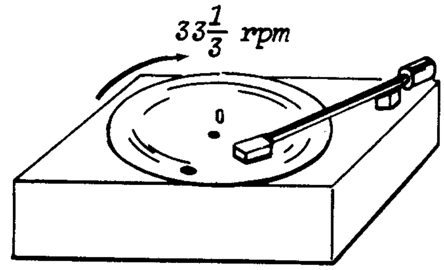
The first part of the module is concerned with explaining the definitions of these quantities, and how they are measured.

WHAT IS ANGULAR SPEED?

The performance of a record player turntable is measured by its ability to rotate. It must also maintain this rate regardless of the number of records it must turn. The rate of rotation is 33-1/3 revolutions per minute (rpm). *Angular speeds* are expressed as the number of revolutions (or the angle turned) per unit time.

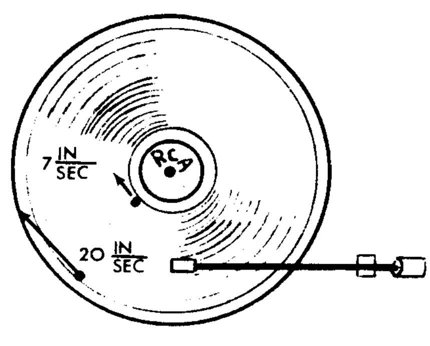
All points on a record (or any rotating rigid body) move at the same angular speed. When the body has turned through one revolution, all points on it have also turned through one revolution.

But what about the *linear speed* of the points? Linear speed means the linear distance traveled per unit time, for example inches per second. A point near the edge of a record covers a much greater linear distance in one revolution than a point near the center. For example, the needle on the record moves along the grooves at the linear speed. When the needle is in a groove out



All points move at same angular speed

but linear speed depends on the distance from center.

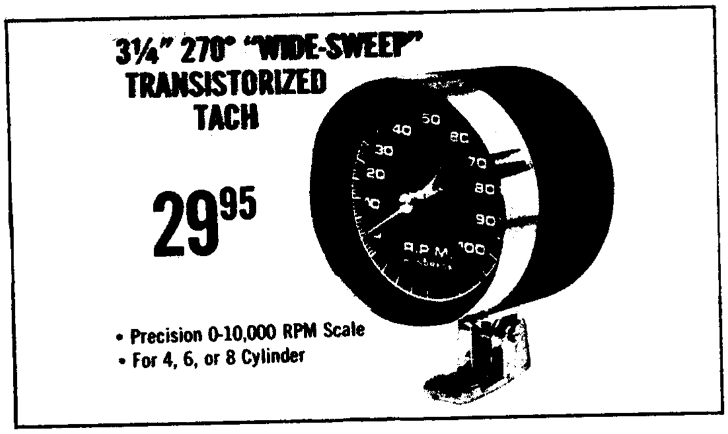


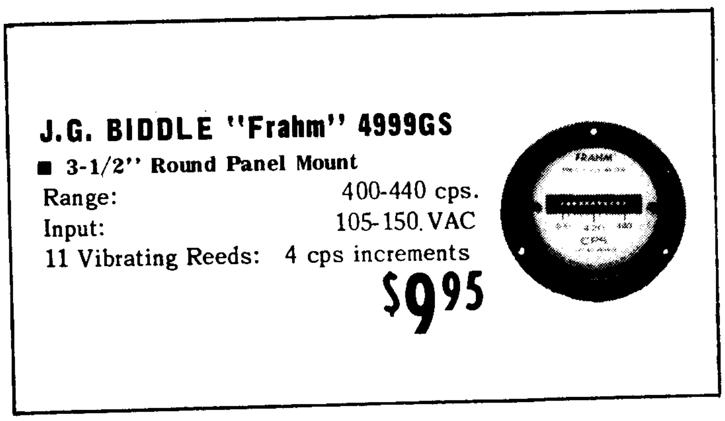
near the edge, its linear speed is about 20 inches per second, while near the center, it is only about 7 inches per second even though the angular speed is constant at 33-1/3 rpm.

This notion is particularly important for airplane propellors since it sets an upper limit to its diameter for a given rpm. If the diameter is too great, the linear speed of a point on the tip may exceed the speed of sound with disastrous results.

MEASURING ANGULAR SPEED: TACHOMETERS

With many rotating devices, it is essential to know the anglular speed at all times. The engines of ships and aircraft and high performance cars must be operated within a certain range of angular speeds. Electric generators must turn at precisely the right speed. Angular speeds are measured by instruments called *tachometers*, or *tachs* for short.





Electronic Tachometers: Most automobile and boat tachometers are now operated by electrical pulses from the ignition system. An electronic circuit counts the number of pulses per minute and displays the count in-rpm on a meter dial.

Vibration Tachometers: Vibration tachs are metal strips (called reeds) that vibrate strongly at a particular rate of vibration. Such tachs are often used for extremeley accurate measurements over a very narrow range. For example, portable electric generators must be operated at just the right angular speed to give a.c. voltage of 60 cycles per second. A vibration tach is usually built into such a generator. You will see how these vibrating reed tachs work in your experiment.

<u>The Tachometer Generator</u>: A dc electric generator can be made into a tachometer. The armature of a dc generator is a coil of wire wound on an iron core. It rotates in the magnetic field of a magnet. As the armature turns an electric voltage is produced in the wires which depends on the speed of rotation. This voltage can be a linear function of its angular speed. That is, the voltage measured on the coil is directly proportional to its speed of rotation. If the generator is attached to some rotating device, its output voltage will then indicate the angular speed of the device.

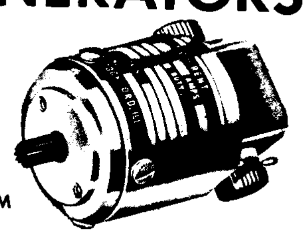
A dc motor has the same basic design as a dc generator. If its armature is rotated, a voltage will be generated at the motor terminals that depends on the angular speed of rotation. In the first experiment you will convert a simple dc motor into a tach generator. Later you will use it to measure the dynamic behavior of the fan.

DC PM TACHOMETER GENERATORS

TYPE
BYLM
FYLM
Tachometer Generator

• Designed to meet MIL-M-8609 requirements

FRAME SIZE	*MOMENTS OF INERTIA*
BYLM 4	8.32×10^{-3} lb-in^2
BYLM 7	12.53×10^{-3} lb-in^2
BYLM 9	17.97×10^{-3} lb-in^2



TYPICAL STANDARD UNITS

TYPE NUMBERS	*BYLM 43820-XX**	*BYLM 73820-XX**	*BYLM 93820-XX**	*FYLM 23920-XX**	*FYLM 43920-XX**	*FYLM 73920-XX**
Rated outputs (open circuit). *Note: Consult factory if application exceeds 100 volts or 7000 rpm.*	6 volts per 1000 rpm	12 volts per 1000 rpm	24 volts per 1000 rpm	3 volts per 1000 rpm	5 volts per 1000 rpm	10 volts per 1000 rpm
Rated output tolerance.	Calibrated within ± 1% in specified direction of rotation. Outputs in opposite directions will be equal to each other within 2%.					
Linearity error (above 500 rpm)	Per definition (1) above: ± 0.5% of output at 7000 rpm or ± 0.5 volts, whichever is smaller. Per definition (2) above: ± 1% between 500 and 10,000.					
rms ripple voltage.	3% maximum above 100 rpm.			3% maximum above 100 rpm		
Nominal armature resistance of generator having outputs quoted above.	80 ohms	110 ohms	160 ohms	70 ohms	90 ohms	130 ohms
Ambient temperature range.	−65°F to +200°F			−65°F to +200°F		
PRICE EACH	$ 92.70	101.30	107.50	79.10	79.10	81.50

CALIBRATING THE TACH GENERATOR. .

How do you determine the rpm corresponding to a particular voltage of a tach generator? Finding the relation is an example of the process called *calibration*. Thus we need some device whose angular speed we know, or another previously calibrated tachometer to compare it with. For your experiment you will use a stroboscope as the *calibration standard* for such a comparison. The stroboscope is still another type of device for measuring angular speed.

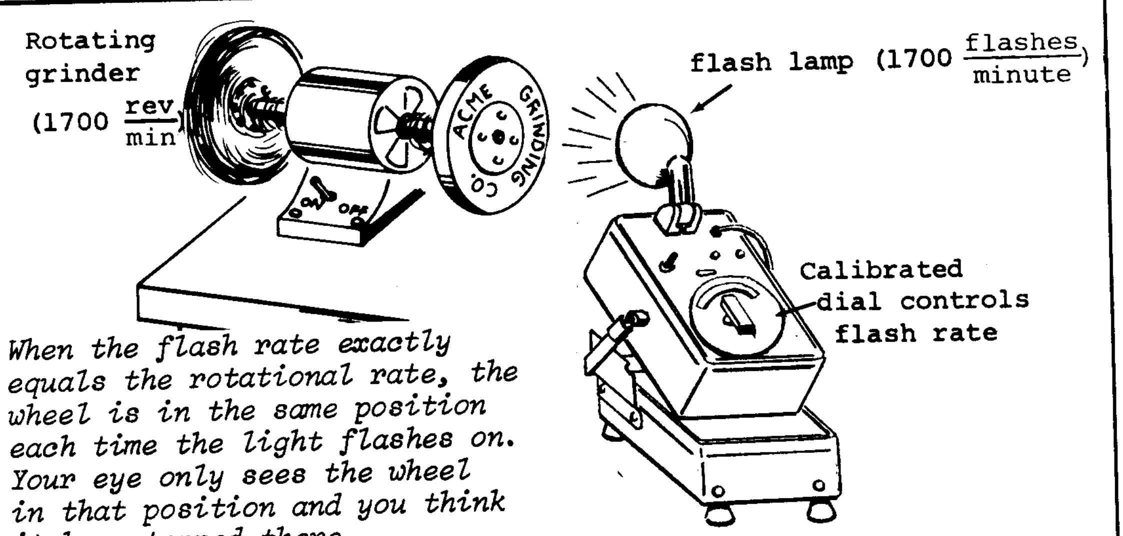
THE STROBOSCOPE

USED FOR "STOPPING" MOTION. . .

The stroboscope, or strobe, is an extremely useful tool because it provides a way of observing any kind of *repeating motion* as though it were stopped. It does this without making any connection to the moving object. An electronic strobe circuit produces very short and very bright flashes of light at regular intervals. A short flash is important because the object must move very little while the light is on; otherwise the image would be blurred.

The strobe is adjusted so that the time between flashes is exactly equal to one period of the object's motion, for example the time for one rotation. In this way, a point on the rotator is only "seen" at one place and appears "stopped" at that place.

Stroboscopes are widely used in industry to observe otherwise invisible details of the motion of high speed machinery. A simple kind os strobe is used, for example, to stop the motion of automobile engines in order to properly adjust the ignition.



When the flash rate exactly equals the rotational rate, the wheel is in the same position each time the light flashes on. Your eye only sees the wheel in that position and you think it has stopped there.

USED FOR MEASURING ROTATIONAL SPEEDS.

A strobe can be quite accurately calibrated electronically and thus is often used as a standard for calibrating other kinds of tachometers. Strobe dials are generally marked in rpm so that the dial setting that stops the motion gives the angular speed of the rotating device. This is particularly useful if the angular speed is constant.

If the speed is changing rapidly, however, the strobe will not keep pace with it. Then tachometers that read continuously are required. In the experiment you will use the strobe as a standard to calibrate the tach generator when the fan is rotating at constant speed. The tach will then be used to measure the speed when it changes rapidly.

HOW TO USE A STROBE:

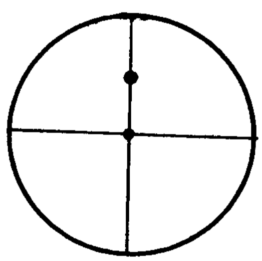
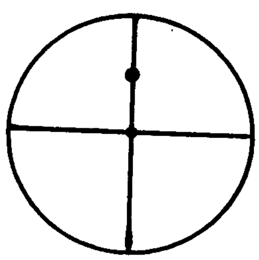
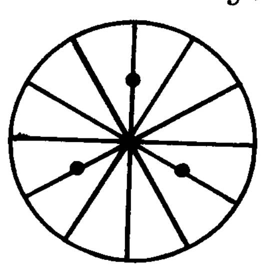
In using a stroboscope the first step is to make some easily distinguishable reference mark on the rotating system. A scratch, a paint spot, a nut, or any other similar mark will serve, but it is essential that it be the *only mark of its kind.*

When the system is rotating, the goal is to increase the strobe rate until that mark appears to be stopped and not rotating. It is worth noting that this stopped image may appear so real that you may actually believe it is not rotating. However, *do not stick your finger in the system to see if it is rotating.* Instead change the strobe rate to see if the mark moves.

There is more than one rate that appears to stop the motion. Rates that are fractions of the real rate will appear to stop the motion and multiples of the rate may stop the mark, which will appear superimposed on other marks. Can you figure out why? Thus we state:

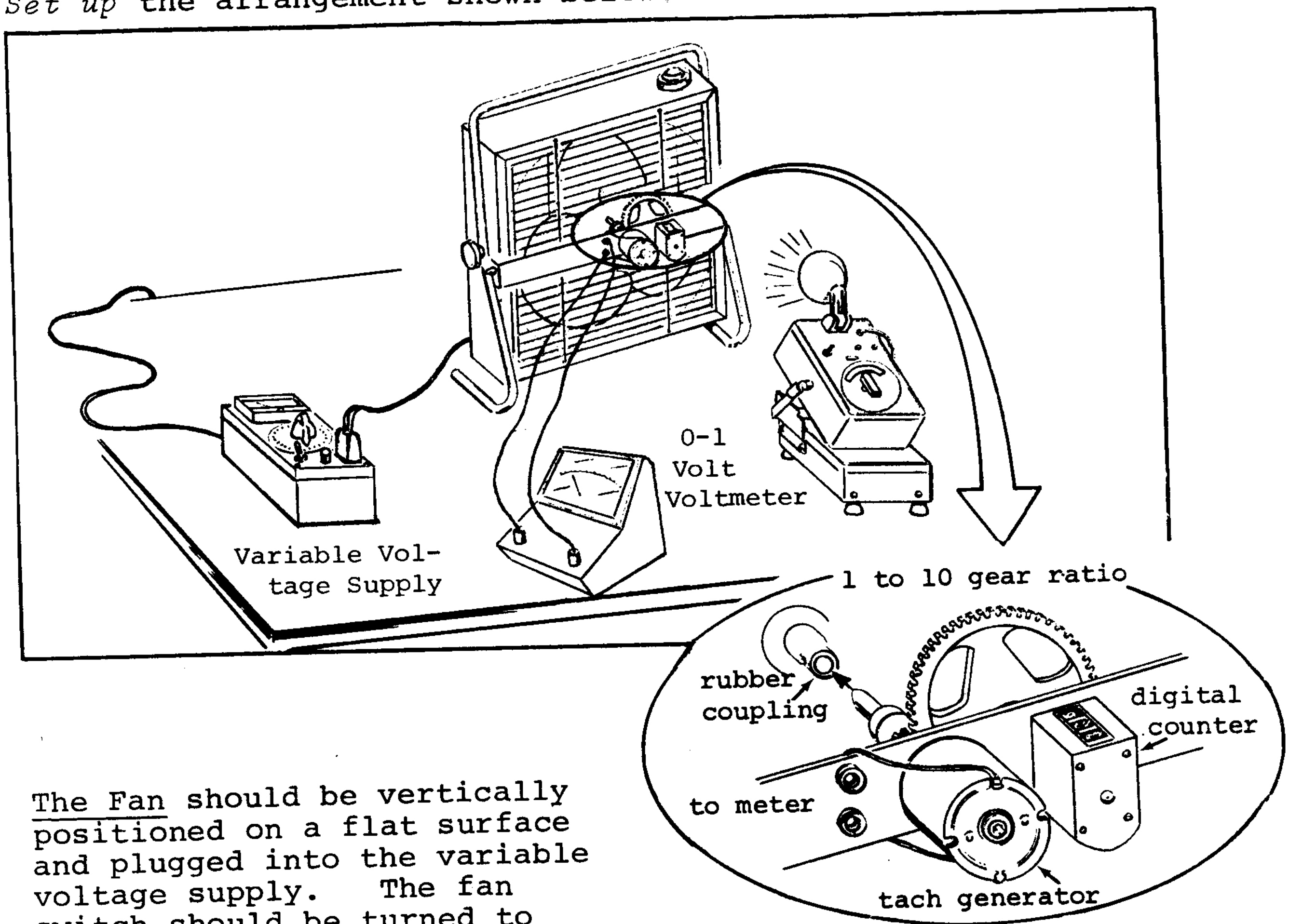
THE BASIC RULE: *The true rotational rate is the highest strobe rate that stops the motion of the reference mark and only the reference mark.*

The illustration below shows what you might see with a rotating disk when the motion is stopped.

OBSERVATION	STROBE RATE
DIM: *too low* 	The motion is stopped, but the strobe rate is not correctly matched to the angular speed. If the strobe rate is half the angular speed, then the rotator makes exactly two revs between flashes. You would still see the motion as stopped, but the reference spot would be dim.
CLEAR: *right on* 	There is just one revolution in the time between flashes and one image of the reference mark. Test your strobe rate by doubling it: if two images appear, you were right on; go back to the previous rate.
SEVERAL SPOTS: *too high* 	The time between flashes is not long enough for a complete revolution. For example, if the strobe rate is three times the angular speed, there is 1/3 revolution between flashes. Three images of the reference spot appear in different places.

THE BASIC SET UP:

Set up the arrangement shown below:



The Fan should be vertically positioned on a flat surface and plugged into the variable voltage supply. The fan switch should be turned to *HIGH*. The fan blade should be firmly mounted to the motor. Make an easily visible reference mark on the hub.

The Variable Voltage Supply should be plugged into a 110 volt outlet. Changing the voltage supplied to the fan will change its rotational speed.

The Strobe should be positioned so it will shine on your reference mark on the hub. The room light should be lowered so that the strobe flashes show up brightly.

The Tach Generator is mounted on the front screen and coupled to the fan shaft by a rubber sleeve. Be sure this coupling is firmly seated and will not slip.

The Digital Counter is geared to the tach generator and records directly the number of revolutions of the fan blade.

The Meter should be a dc voltage meter with a range from 0 to 1 volt. Connect the meter to the output terminals of the tach generator.

THREE SIMPLE EXPERIMENTS

LEARNING TO USE THE STROBE:

PROCEDURE:

1) *Turn on the voltage supply* and increase the voltage to a maximum so that the fan blades are turning at top speed. Note the voltage produced by the generator.

2) *Turn on the strobe* and set it to read about 1000 rpm.

Increase the strobe setting slowly while watching the hub, until your reference mark is stopped by the flashing strobe.

3) *Explore the behavior* of the reference spot when you pass through the exact strobe rate that stops the motion. Which way does the spot move when the rate is slightly too high?. .too low?

Vary the rate to double, one half, etc. of the true rate.

4) *Adjust the strobe rate* until the reference mark is exactly stopped.

DIRECT MEASUREMENT OF ROTATIONAL SPEED:

The most direct way to measure rotational speed is to count the number of revolutions in a given time period. To count the number of revolutions you can use the digital counter attached to the tach generator.

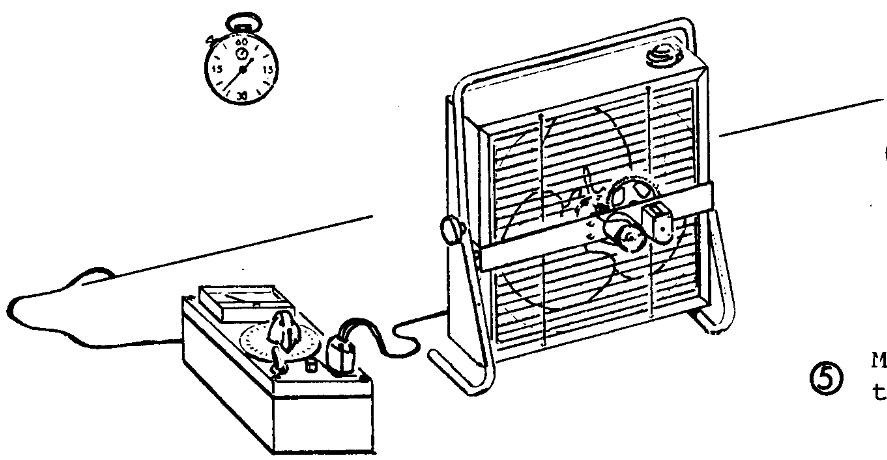
The counter itself registers every tenth of a revolution. However, the ratio of the gears was made ten to one so that it now reads directly the number, n, of revolutions of the fan. Follow the procedure in the illustration below.

SETUP AND PROCEDURE:

① Keep strobe matched to fan's rotational speed.

② Start stop watch when counter passes 000.

③ Stop watch at end of 1000 rev. Record time.



④ Calculate average angular speed.

⑤ Make several trials.

$$\text{Average Angular Speed} = \frac{\text{n turns}}{\text{time interval in minutes}} = \frac{1000}{\quad} = \quad \text{rpm}$$

How does this compare with the strobe reading?

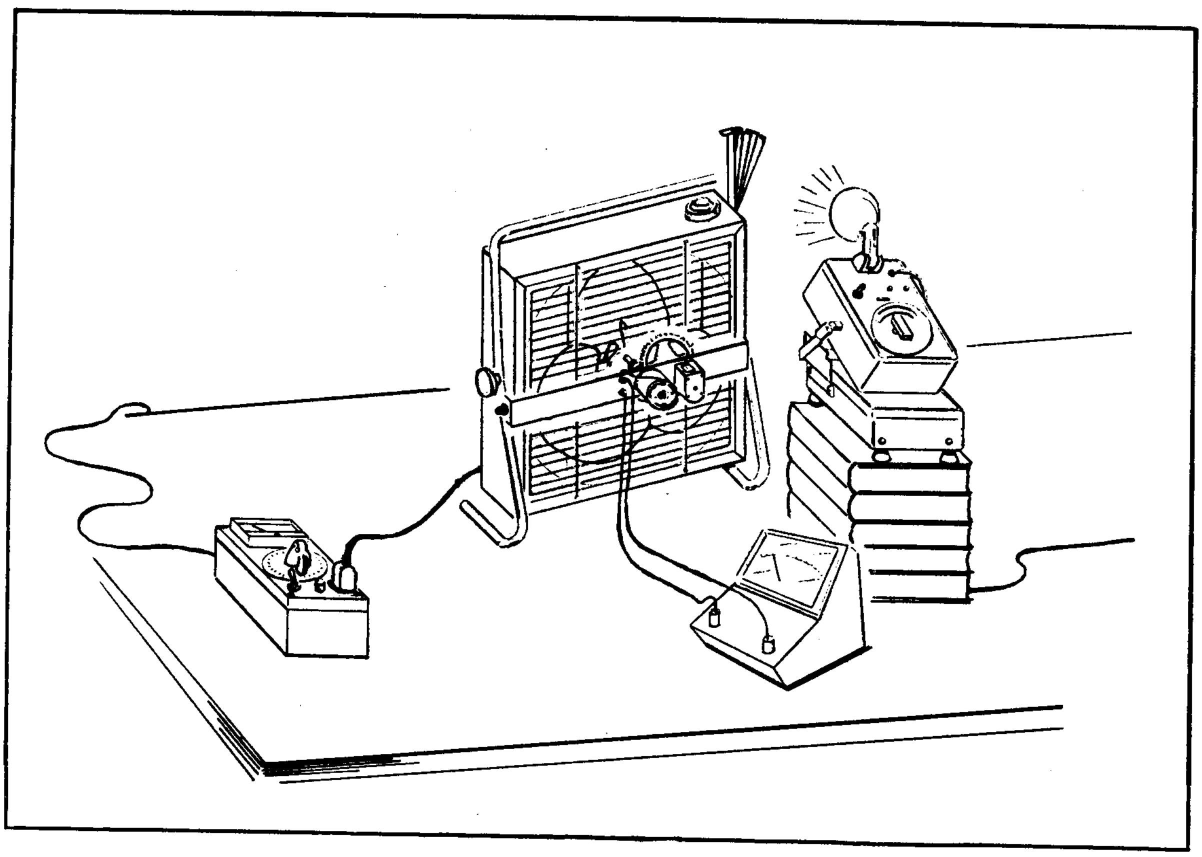
USING A VIBRATING REED TACHOMETER:

Earlier we discussed briefly the vibrating reed tachometer. In this experiment you can see how the vibrating reed tach works.

Bolted to the side of the frame is a thin strip of spring steel. Being springy it snaps back when pushed aside. Anything springy tends to vibrate at a certain fixed rate or frequency. This natural frequency depends on the mass and the stiffness of the spring. The combination of small mass and stiff spring means a high frequency. A large mass and weak spring means a low frequency. For the reed the natural frequency can be changed by simply changing its length. The longer the reed the lower the natural frequency.

When the reed is attached to the fan, the small vibrations of the fan will make it vibrate. If you watch closely, you will see that the reed vibrates a little at any fan rpm. However, it will vibrate most strongly when the fan rpm is equal to its own natural frequency. This strong response is called *resonance*. Follow the procedure at the top of the opposite page to see how the principle is used to make a vibrating reed tachometer.

SETUP:



PROCEDURE:

1) *Adjust the length* of the reed until it is about six inches long. Clamp it firmly at this length.

Record its length in the table following page 16.

Snap the reed so that it vibrates back and forth.

2) *Measure the natural frequency* of the reed using the stroboscope. The same general rules for using the strobe to measure the angular speed of rotating systems apply to using it to measure the vibration frequency of vibrating systems.

Record the natural frequency of the reed.

3) *Turn on the fan* and adjust its speed until the amplitude of the reed vibrations is a maximum.

Record the tachometer voltage at this resonant speed.

4) *Increase the length* of the reed by about an inch and repeat your measurements.

Record your results.

5) *Decrease the length* of the reed by two inches and repeat.

AN EXPERIMENT...

...TO CALIBRATE A TACH GENERATOR

In this experiment you will calibrate a dc motor as a tach generator. The motor is coupled to the rotating fan so that it is being driven as a dc generator. The voltage output of the motor-generator will be compared to the reading of the calibrated strobe which is set to just stop the motion of the rotating fan.

Several readings of tach voltage vs rpm should be made covering the full range of fan speeds. From these data a calibration graph can be made for later reference. The detailed steps of the experiment and important notes are given below.

PROCEDURE:

1) *Set the fan speed* with the variable voltage supply until the output of the tach generator is stable at the nearest tenth of a volt below the maximum output.

2) *Change the strobe rate* until you exactly match the rotational speed.

3) *Record your values* of tach voltage and strobe rate in the table following page 16.

4) *Repeat the measurement* for every tenth volt output of the tach generator. Carry this down as low as you can.

A FINAL MEASUREMENT...

...RELATING ANGULAR SPEED and NUMBER OF REVOLUTIONS

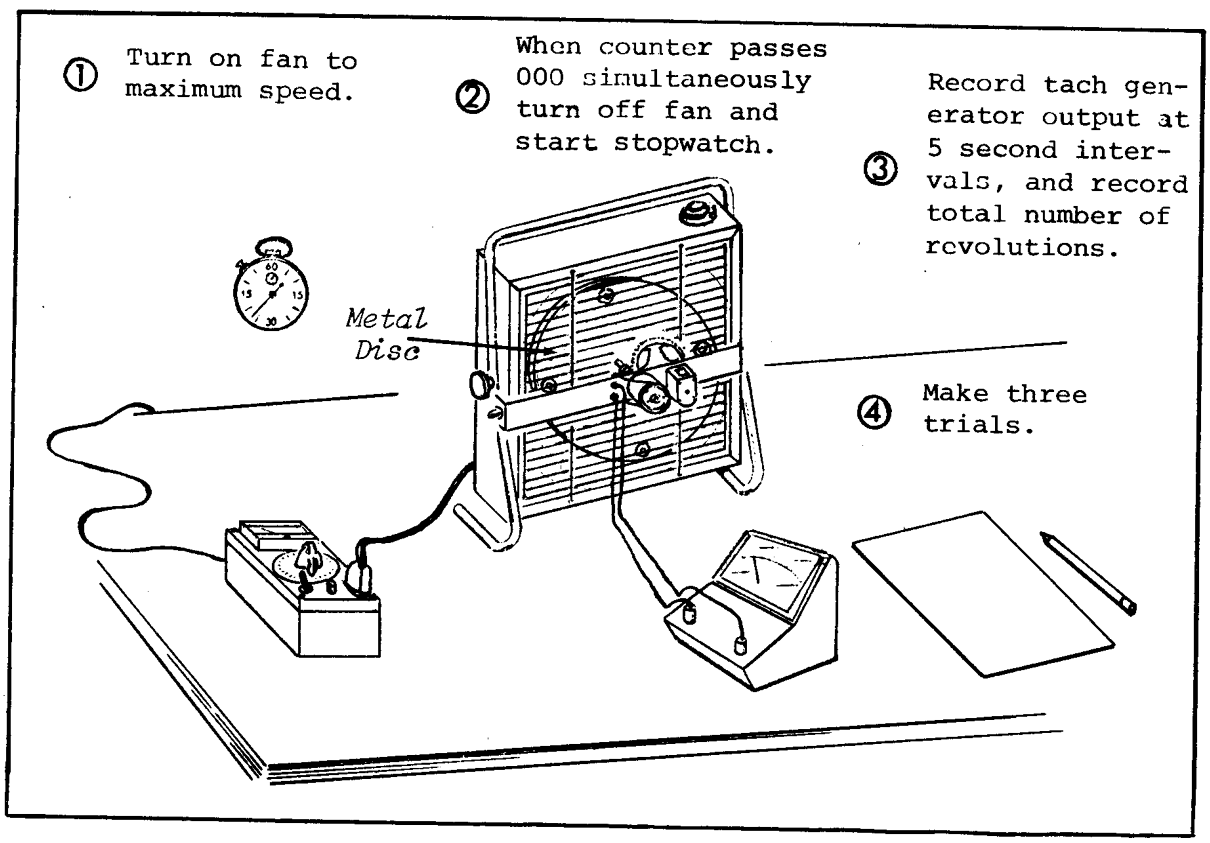
By this time you should be familiar with the various components of the apparatus of this module. As a final experiment you will examine the manner in which a rotating object slows down. Since the fan blade is light and its blades interact strongly with the air, it slows down too quickly for study. Thus you will replace it with a heavier metal disk that takes somewhat longer to slow down.

In this experiment you will record the angular speed of the disk at equally spaced time intervals as it slows down.

At the same time you will count, with the counter, the total number of revolutions that the disk makes from the time you shut off the power. Later you will see that total revolutions and angular speed are closely related.

Since the data must be taken quickly, it is best if two persons work together. One can call off the time intervals while the other records the tach generator output. Follow the procedure in the diagram below, and record your data in the table provided.

SETUP AND PROCEDURE:



DATA TABLES

	Vibrating Reed Frequency		
Trial	1	2	3
Length	______	______	______
Natural frequency (from strobe)	______	______	______
Tach Voltage at Resonance	______	______	______
Rotational Speed at Resonance (from calibration graph)	______	______	______

Tach Generator Calibration

Generator Voltage (volts)	Strobe rate (rpm)

- tear out page -

Slow down
Behavior

Time, Sec	Trial #1 Tach Volts	Trial #1 Rev./Min.	Trial #1 Rev./Sec.	Trial #2 Tach Volts	Trial #2 Rev./Min.	Trial #2 Rev./Sec.	Trial #3 Tach Volts	Trial #3 Rev./Min.	Trial #3 Rev./Sec.
0									
5									
10									
15									
20									
25									
30									
35									
40									
45									
50									

Total number of revolutions from:

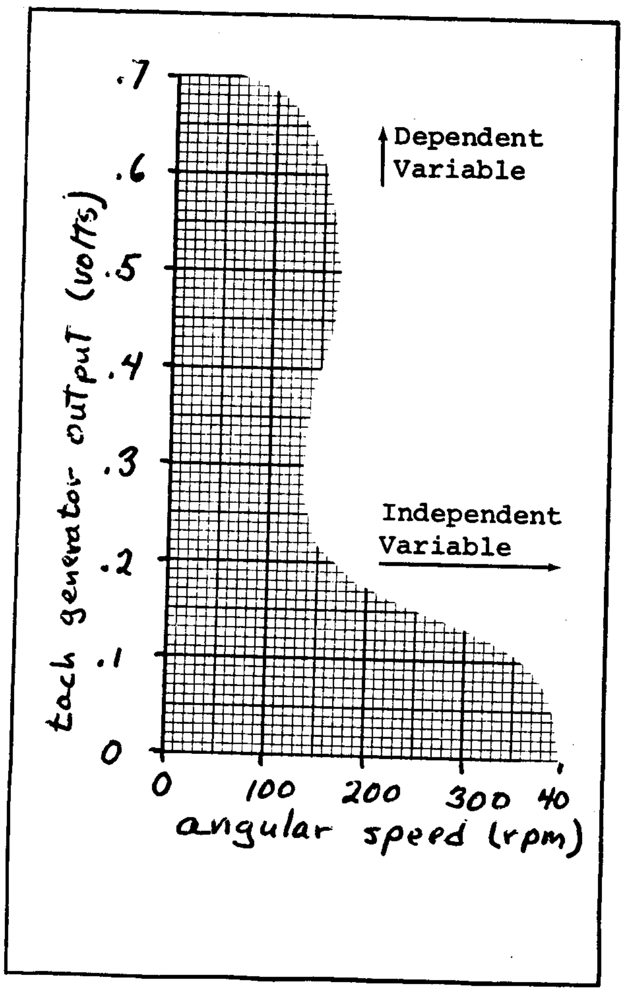
counter ________ ________ ________

graph ________ ________ ________

DRAWING THE CALIBRATION GRAPH

Writing your data in a table is generally the fastest and most accurate way to record them during the heat of an experiment. However, when you later want to get a picture of what was happening, or estimate values between the points you took, a graph of the data may be more convenient.

This is particularly true of the data you took of tach generator output vs rpm. Since you want to use the tach generator as a tool to measure rotational rates, you want to convert the voltage readings to rpm quickly.



If the relation turns out to be linear; that is, if tach voltage is directly proportional to rpm, then you can convert any voltage reading to rpm by multiplying by a constant. Follow the procedure below to get a calibration graph for your tach generator. Use the graph paper provided on the following page.

STEPS IN GRAPHING:

1. CHOOSE AND LABEL AXES

Normally you put the quantity that you can change (the independent variable) on the horizontal axis. Therefore we choose the horizontal for the rpm setting. Tachometer voltage is a reading that follows from your setting (dependent variable) so it is on the vertical axis. Both axes should be labeled with the names of the variables and their units.

2. CHOOSE AND LABEL SCALES

You should choose the number of units per division so the range of your data *nearly fills the paper.* Be sure to choose convenient divisions that are easy to plot. For example, 10 divisions to be 100 rpm or .1 volt.

Indicate which divisions you have chosen by putting a light hatch mark next to the line with the appropriate number.

3. PLOT THE POINTS

Each point on the graph represents a pair of values from the table. For example, suppose one pair is 0.17 volts and 250 rpm. The desired point on the graph is the intersection of two imaginary, perpendicular lines from those points. Put a small dot at the intersection. Since dots are hard to see, it helps to put some geometrical shape around or through it such as: ⊙, ⊡, ▲, X. to make it more visible. Be sure that the shape is centered on the dot.

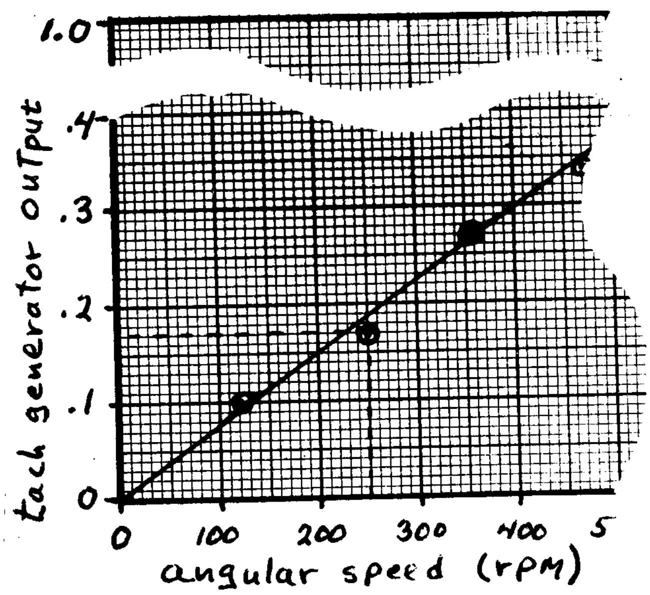
4. DRAW A SMOOTH CURVE

The measured points will probably not fall perfectly on any straight line or curve. If the points look close to being on a straight line, draw a line that follows their general trend. There should be the same number of points above and below the line. If the points seem to lie along a curve, try to judge its shape by eye, and draw a smooth curve. *Don't connect the points with a jagged line.* The angular speed and voltage changed smoothly, not erratically.

Be careful in drawing this graph. It is a calibration graph and all your data after this will depend on your work here. A straight edge or draftsman's French curve will help you draw your curve accurately.

5. TITLE YOUR GRAPH

The graph should be titled so that anyone, including you, will know what it represents. You should include what the two variables are, how they were measured, what specific apparatus it refers to, when it was measured and by whom.



6. CALCULATE THE SLOPE

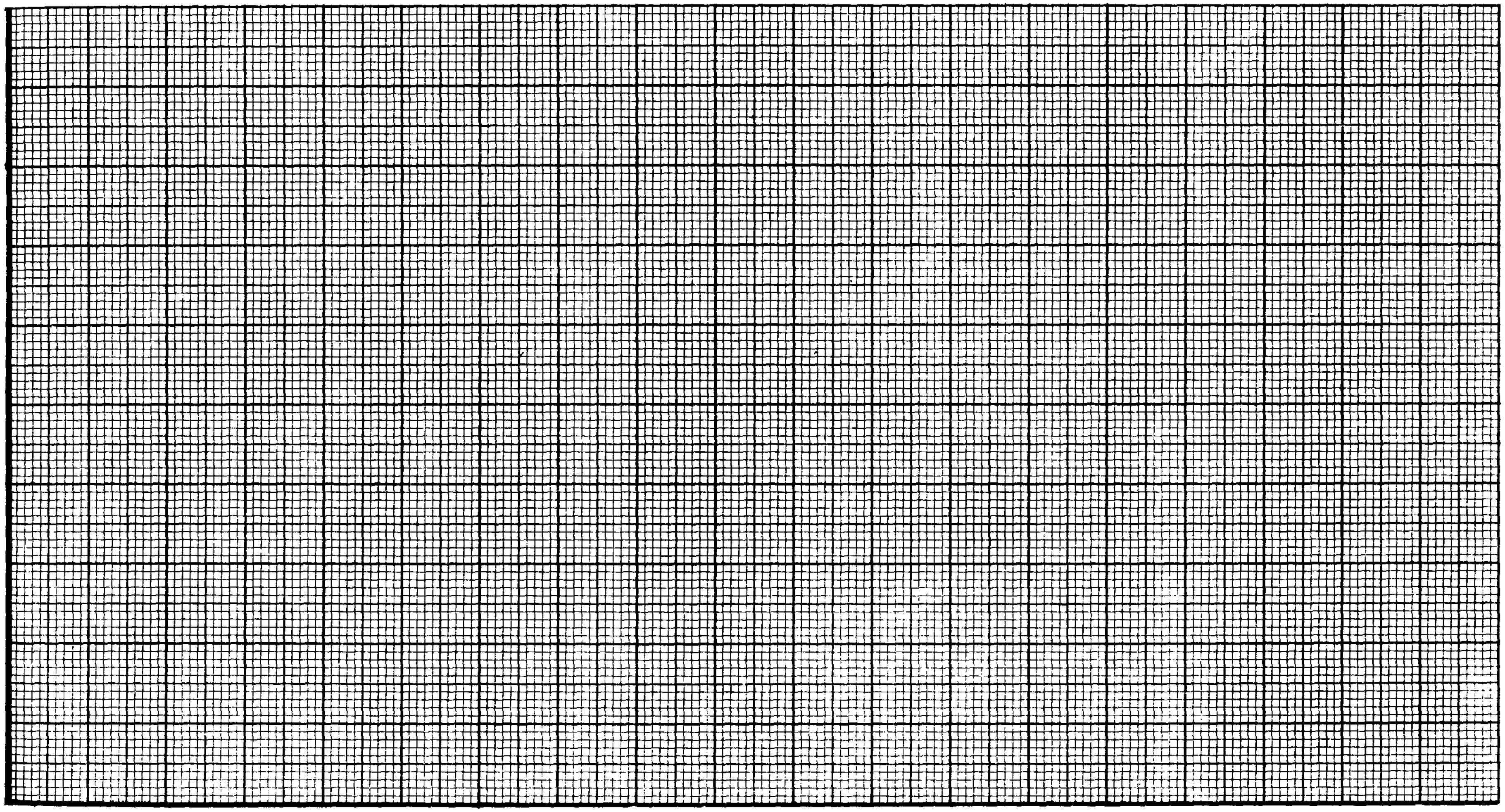
Your curve should be a straight line. This is convenient since a simple constant can be used to determine the rpm from any voltage reading. This constant is the *slope* of your line.

Since the line goes through 0, the slope can be calculated from any pair of points *that lie on the line*. For best accuracy choose the largest values that lie on your line. Simply divide the rpm reading by the corresponding tach voltage to give the number of rpm per volt. This number multiplied by any voltage reading will give the corresponding rpm. Try it.

Tach voltage vs rpm
for tach gen # 3
using strobe # 4 as
calibration standard

by R. P. Emm 5/23/73

CALIBRATION GRAPH



Angular Speed (rev/min) (from strobe)

Slope = ________ $\frac{\text{rpm}}{\text{volt}}$ = $\frac{\text{rpm}}{\text{volt}}$

- tear out page -

- tear out page -

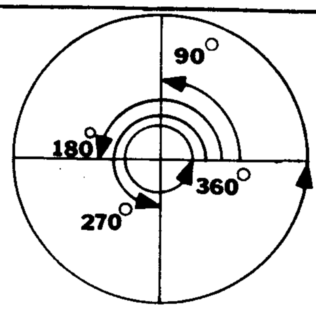
SPARE GRAPH PAPER

THE DESCRIPTION OF ROTATION

CHOOSING A SCALE FOR ANGLES:

Up to now we have been using the number of revolutions to describe the angle through which a rotating body has turned. Each revolution means that the body has turned completely around once.

From geometry you will recall that degrees are another measure of angular rotation; there are 360° per revolution, 90° in ¼ revolution and so on. In rotational kinematics there is a third angular measure that is particularly convenient, the *radian*.



The degree is the most common angular unit. In one revolution, all points turn through an angle of 360°.

THE RADIAN

The radian is useful when you want to relate the linear distance traveled by a point to the angle turned. As seen in the diagram, the distance traveled along an arc depends on the angle and the radius. In fact it is directly proportional to each of them; double the radius and you double the arc traveled or double the angle and you double the arc.

If we define:

$$\boxed{\text{one revolution} = 2\pi \text{ radians}}$$

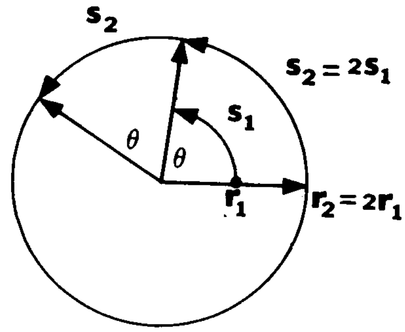
then we have the simple mathematical relation:

$$\boxed{s = \Theta r}$$

if Θ is measured in radians.

This makes sense when we note that the arc traveled in one complete rotation of $\Theta = 2\pi$ is simply the circumference C. Thus we get the familiar expression:

$C = 2\pi r$.



The linear distance, s, traveled by a point along an arc is proportional to the radius, r, and the angle Θ. If the angle is measured in radians then $s = \Theta r$.

SIZE OF THE RADIAN

While this is convenient mathematically, it doesn't give a feel for how much of an angle one radian is. Since we are most familiar with degrees, even though they have no logical basis, let us relate it to degrees.

We know that:

one revolution = 360°

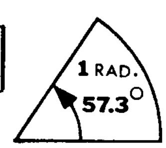
and

one revolution = 2πrad

= 6.28rad

Dividing gives:

$$\boxed{57.3° \text{ per radian.}}$$



MOTION OF A POINT ON A ROTATING BODY:

LINEAR DISTANCE AND ANGULAR DISTANCE:

Another way of looking at the relation $s = \Theta r$ is that it relates the linear distance, s, traveled by a point on a rotating body to the angular distance traveled, Θ. All points on a rigid body will move through the same angle. But the linear distance, or arc, will be greater with greater distance from the center of the rotation.

Thus $s = \Theta r$ expresses the relation between linear distance and angular distance. The next step is to relate linear speed to angular speed.

LINEAR SPEED AND ANGULAR SPEED:

First we must choose symbols for the two quantities. We will use the common convention of:

v = linear speed
ω = angular speed

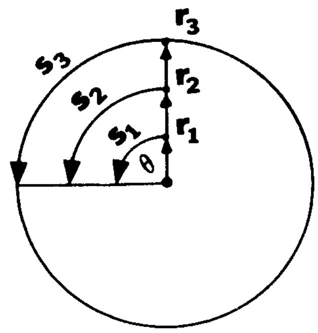
The linear speed, as you will recall, is the *linear* distance traveled, s, per unit time, t. That is:

$$v = \frac{s}{t}.$$

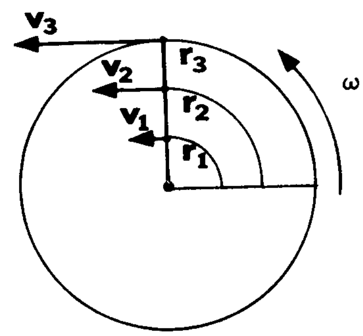
Similarly the angular speed, ω, (omega) is the *angular* distance traveled, Θ, per unit time. Or:

$$\omega = \frac{\Theta}{t}.$$

For the angular speed, we have been using the number of revolutions per minute (or second). If we instead used radian measure, that is ω in radians per minute, then again we have a simple *relation between linear speed and angular speed:*



For a given angular rotation the linear distance traveled by a point is proportional to its distance from the center of rotation.



For a given angular speed, the linear speed of a point is also proportional to its distance from the center. Its direction is tangent to the circumference.

$v = \omega r$ where ω is measured in *radians* per minute (or second).

A mathematical derivation of this relation is given on the opposite page.

Thus we see that although all points on a rigid rotator will be moving at the same angular speed ω, their linear speed is proportional to their distance from the center of rotation. The greater the distance, the greater the linear speed.

MATHEMATICAL DERIVATION OF $v = \omega r$

The relation $v = \omega r$ can be determined by a mathematical route. First we agree that the linear speed, v, is the change in arc length, Δs, in a time, Δt;

$$v = \frac{\Delta s}{\Delta t}$$

and that the angular speed is the change in angle, $\Delta\Theta$, in the same time, Δt:

$$\omega = \frac{\Delta\Theta}{\Delta t}$$

Then, starting with the distance - angle relation $s = \Theta r$, if the angle changes by $\Delta\Theta$ in the time Δt then the linear distance will change by Δs in the same time. r of course remains the same so that,

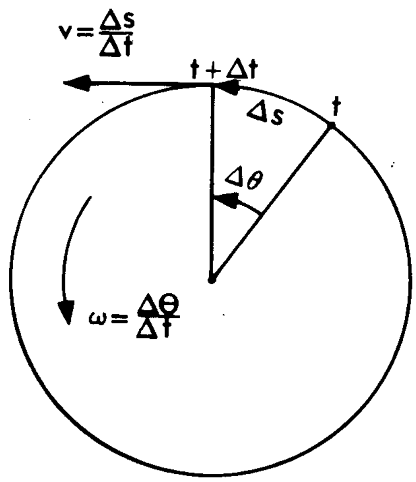
$$\Delta s = \Delta\Theta r$$

If we divide by the time interval Δt, we have:

$$\frac{\Delta s}{\Delta t} = \frac{\Delta\Theta}{\Delta t} r$$

But these are the values for v and ω above. Substituting, we have:

$$v = \omega r.$$

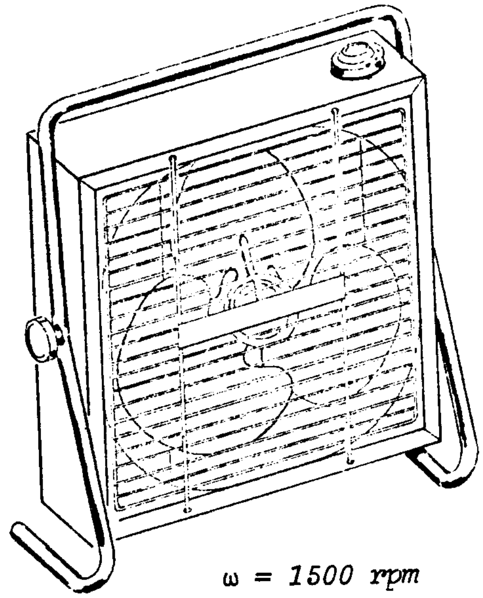


DIRECTION OF THE LINEAR SPEED:

A final note about the linear speed is that its *direction is tangent to the circumference* or at right angles to the radius. Thus if a body should suddenly be released from a rotator, it will go off at the linear speed in a direction tangent to the circumference at the point of release. As we will see, the linear speeds can become quite large for even moderate angular speeds. If things are not rigidly attached, they can go off as rather dangerous projectiles.

A SAMPLE CALCULATION:

As an example of how to use these relations in calculations, let us consider the fan. Assume that its maximum rotational speed is measured to be 1500 rpm. Then let us answer the following questions:



ω = 1500 rpm

1. *What is the angular speed in radians per second?*

We were given: $\omega = 1500\frac{rev}{min}$

Since there are $60\frac{sec}{min}$: $\omega = 150\cancel{0}\frac{rev}{\cancel{min}} \times \frac{1}{6\cancel{0}}\frac{\cancel{min}}{sec}$

$= 25\frac{rev}{sec}$

There are also $2\pi\frac{rad}{rev}$, so that: $\omega = 25\frac{\cancel{rev}}{sec} \times 2\pi\frac{rad}{\cancel{rev}}$

$$\boxed{\omega = 157\frac{rad}{sec}}$$

2. *What is the linear speed of a bolt attached to the fan blade a distance 6 inches from the axis?*

We have the ralation: $v = \omega r$

Substituting ω in $\frac{rad}{sec}$

and r in feet we get: $v = 157\frac{rad}{sec} \times .5\ ft$

rad can be dropped since it, like π, is dimenionless: $\boxed{v = 78.5\frac{ft}{sec}}$

Since $60\frac{mi}{hr}$ is $88\frac{ft}{sec}$, the bolt is traveling nearly 60mph.

3. *How far does the bolt travel in a minute?*

Since the linear speed is: $v = 78.5\frac{ft}{sec}$

In 60 seconds it travels a linear distance: $s = vt$

$= 78.5\frac{ft}{\cancel{sec}} \times 60\cancel{sec}$

$\boxed{s = 4710ft}$ nearly a mile

or in another way:

Since the angular speed is: $\omega = 157\frac{rad}{sec}$

In 60 seconds it travels through an angle of: $\theta = \omega t$

$= 157\frac{rad}{\cancel{sec}} \times 60\cancel{sec}$

$= 9420rad$

Since: $s = \theta r$

this is a linear distance: $s = 9420rad \times .5ft$

$= 4710ft$ as above.

TOTAL ANGLE WHEN ANGULAR SPEED CHANGES

In the preceding discussion the angular speed was assumed constant. In your last experiment, however, the angular speed changed as the fan slowed down. In the sample calculation we saw that for a constant angular speed, ω, the total angle rotated in a time, t, could be calculated from:

$$\Theta = \omega t$$

But if ω is not constant, this relation cannot be used. In the following analysis we will describe a way of determining the total angle turned (or number of revolutions) from your slow-down data. Since you also counted the number you will be able to double check the method.

DRAWING THE GRAPH

1. CONVERT TACH VOLTAGE READINGS TO RPS Using your calibration graph, or your conversion constant, determine the rpm values for each set of slow-down data and enter them in your table. Then divide each value by 60 sec/min to obtain the angular speed in rps.

2. GRAPH YOUR DATA Use the graph paper on the opposite page to plot your data of angular speed in rps against time in sec. It is customary to regard time as the independent variable so that angular speed should be put on the vertical axis.

   Follow the procedure described earlier for the calibration graph. Use different symbols for the three trials and indicate which is which. Draw smooth curves through the data with a French curve.

ANALYZING THE GRAPH

A graph like the one you have just drawn gives a useful picture of how the fan's angular motion changes with time. It tells you, for example, that the fan does not slow down at a uniform rate. The change in angular speed is large in the first 10 seconds, but less in the last 10 seconds.

Any graph of angular speed against time has another useful feature - you can get from it the total number of revolutions turned during any time interval.

Look at the illustration on the next page. Note the small "almost rectangle" shaded between t_1 and t_2. It is "almost" a rectangle because the top is formed by the sloping curve. However, if you draw a short horizontal line at the midpoint, then you do have a rectangle with about the same area as before. The area of the rec-

- tear out page -

SLOW DOWN BEHAVIOR

time (sec)

Angular Speed (rps)

- tear out page -

SPARE GRAPH PAPER

tangle is given by its base times its height. What does this area mean? The height is an angular speed, the average angular speed in the interval between t_1 and t_2. The base is the time interval t_2-t_1. Their product is:

$$\text{Area} = \text{base} \times \text{height} = \text{sec} \times \frac{\text{rev}}{\text{sec}} = \text{revolutions}.$$

Therefore the shaded area is:

$$A = 13\frac{\text{rev}}{\text{sec}} \times 2\text{sec} = 26 \text{ rev}.$$

This says that in the time interval, $t_2 - t_1$, the fan made 26 revolutions.

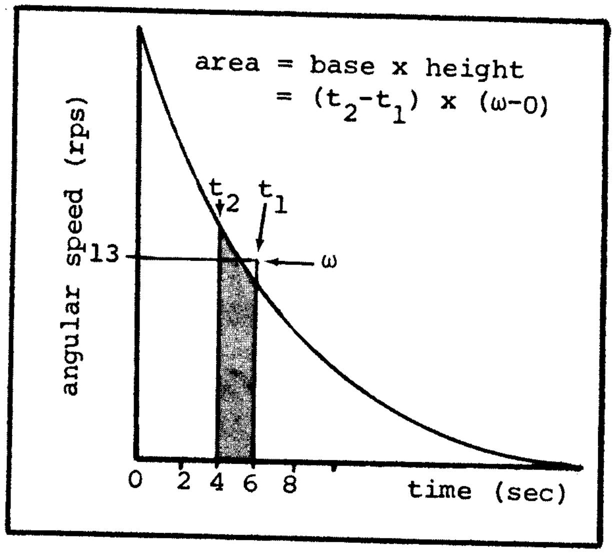
ESTIMATING THE AREA:

There are two ways to do this. The first is to break the curve up into rectangles as described above, and add up the areas of all the rectangles. A cruder, though surprisingly accurate, method is to *approximate* the curve by a *triangle of the same area*, and calculate the area of the triangle.

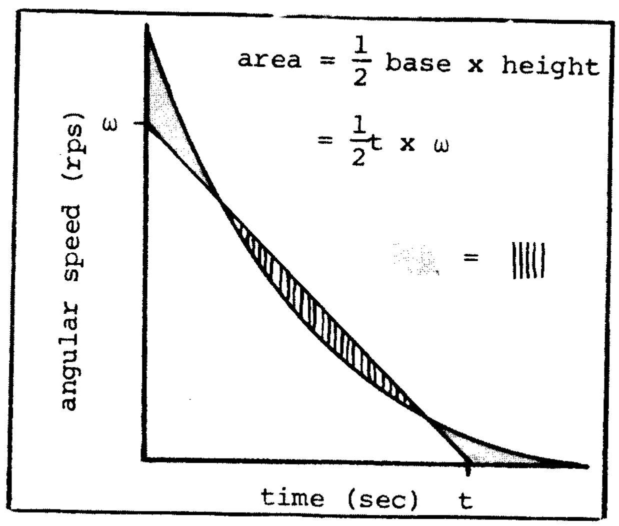
Draw a diagonal through the graph such that the area of the curve not included equals the extra area that was added. This is only done by eye so your judgement is important.

The area of the triangle is:

$$\text{Area} = \tfrac{1}{2}(\text{base}) \times (\text{height}) = \tfrac{1}{2}\ \text{time intercept} \times \text{speed intercept}$$



If one can determine the total area under the curve, then one would have the total number of revolutions turned after the power was shut off.



3. CALCULATE THE NUMBER OF REVOLUTIONS Use this method to calculate the total number of revolutions from your three graphs. A clear plastic ruler will help you estimate the proper diagonal. Enter your values in the space provided in the data table. How well does this value compare with the number you counted?

REVIEW

SUMMARY:

In Part I of the module, the quantities used to describe rotational motion were introduced, *angle turned* and *angular speed*. Angular speed, the rate at which a body is rotating, is commonly expressed in either rpm or rad/sec.

There are several kinds of *tachometers* for measuring angular speed, electronic tachometers, vibrating reed tachometers, tach generators and stroboscopes. Stroboscopes can also stop a repetitive motion.

Angular speed can also be obtained using a revolution counter and a watch. The total number of revolutions divided by the time interval gives the average angular speed.

The *vibrating reed tachometer* utilizes resonant behavior. A reed, or any body with mass and springiness, has a certain natural frequency at which it vibrates. When the reed is attached to a vibrating piece of machinery, it vibrates most strongly when driven at its natural rate.

The following relations can be used to calculate the various rotational parameters.

Linear distance (arc),

$$s = \Theta r$$

Linear speed,

$$v = \omega r$$

On a graph of angular speed against time the area under the curve gives the total number of revolutions in a given time interval. The area can be estimated by calculating the area of a triangle of equivalent size.

CONVERSION TABLES:

ANGLES	To go from	revolutions	to	radians	multiply by 6.28
		radians		revolutions	.159
		revolutions		degrees	360.
		degrees		revolutions	.00278
		radians		degrees	57.3
		degrees		radians	.0174
ANGULAR SPEED	To go from	$\frac{rev}{sec}$	to	$\frac{rev}{min}$	multiply by 60.
		$\frac{rev}{min}$		$\frac{rev}{sec}$	.0167
		$\frac{rev}{min}$		$\frac{rad}{sec}$	.014
		$\frac{rad}{sec}$		$\frac{rev}{min}$	9.55
		$\frac{rev}{sec}$		$\frac{rad}{sec}$	6.28
		$\frac{rad}{sec}$		$\frac{rev}{sec}$	.159

QUESTIONS:

1. Suppose you have used a stroboscope to stop the motion of a rotating wheel that has one bright mark on it. Only one image is visible. How can you be sure that the strobe rate equals the angular speed?

2. Explain briefly the operation of a tachometer generator.

3. What is the angle turned in radians when a point on a wheel of radius 1.7 feet moves a linear distance of 3½ ft?

4. A technician turns a control knob through an angle of 1°. How far does a pointer on the rim (radius 3/4 inch) move?

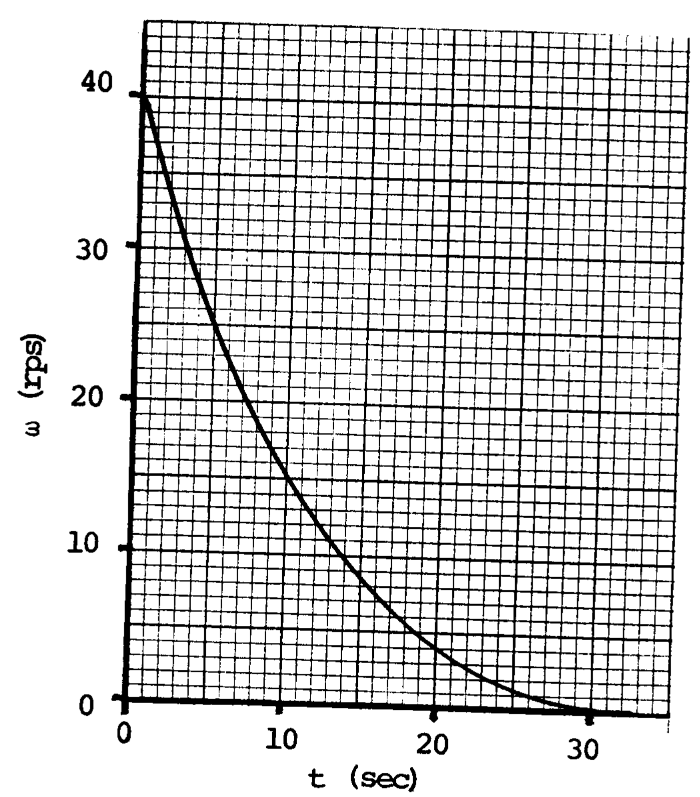
PROBLEMS:

1. You have stopped a rotating motion with a strobe rate of 1000 flashes/min. A single image of the mark is visible. List several possible angular speeds.

2. You are going to draw a graph of voltage against rpm over the range of 0-1500 rpm. You can use 8 inches for the horizontal axis. What should the horizontal scale be? That is, how many rpm should be represented by one inch?

3. Refer to the graph. What is the average angular speed in the time interval 0 to 5 sec? What is the total number of revs turned?

4. Calculate the linear distance moved by a point on a wheel 2 feet in diameter when it turns through 3½ revolutions.



5. An engine flywheel is turning at 4000 rpm. The radius of the wheel is 9 inches. What is the linear speed of a point on the rim in ft/sec? ft/min? mi/hr?

6. The period of revolution of a light airplane propeller is 0.03 sec. The radius of the blade is 3 ft. Find the angular speed and the linear speed of a point on the tip of the blade.

7. A point on the tread of an automobile tire has to move at the same linear speed as the car. What is the angular speed (rad/sec) of a tire of 18 in, radius when the car is moving 150 mi/hr? Use the relation 88 ft/sec = 60 mi/hr.

8. The manufacturer states the maximum safe linear speed of a point on the outer surface of a grinding wheel is 4000 ft/min. What is the maximum safe angular speed if the wheel is of 6-in diameter? Calculate first in rad/sec, then rpm.

PART II

ROTATIONAL DYNAMICS

WHAT IS ROTATIONAL DYNAMICS?

In Part I you saw how angular motion was described using such ideas as revolutions, angles turned, and angular speed.

Now in Part II you will investigate changes in angular motion, and the causes of these changes. Motors speed up and slow down. Airplane propellors come to rest when the engine is turned off. What brings about these different changes? The answer lies in the study of rotational dynamics.

ANGULAR ACCELERATION:

One simple way to describe a change of motion is by change of angular speed, ω. We will represent a change in angular speed by $\Delta\omega$, where Δ (delta) means "change in" or, "small interval of:.

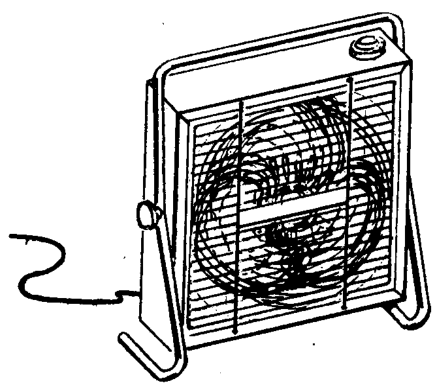
The rate at which the angular speed changes with time is called the *angular acceleration*. Its symbol is α (alpha).

Mathematically, the definition is:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

where, Δt means a small time interval.

Rotational dynamics is the study of the causes and effects of angular accelerations.



Rotational dynamics is the description of angular acceleration. We will consider only the rotational dynamics of rigid bodies on a fixed axis, such as the fan blade.

RIGID BODIES ON FIXED ROTATION AXES:

Our attention will be directed only to *rigid bodies*, solid pieces of matter that keep a fixed shape as they turn.

And further, we shall only consider rigid bodies that *rotate about a fixed axis*. Such things as automobile wheels, crankshafts, propellor blades, motor armatures are all rigid bodies that rotate about a fixed axis.

The behavior of rigid bodies whose rotation axis can change, for example, baseballs, bicycle wheels and gyroscopes, is somewhat more complicated. And the behavior of non-rigid, rotating bodies, such as a can of water, can be extremely complicated.

WHAT DETERMINES SPEED CHANGES?

FORCE AND MASS IN LINEAR MOTIONS:

You know from previous study and experience that forces cause changes in linear motion. That is, a force F produces a change in linear speed Δv. The amount of this change depends on the force, the length of time it acts, and the mass of the object. In symbols:

$$F = m\frac{\Delta v}{\Delta t}$$

where $\frac{\Delta v}{\Delta t}$ is, of course, the linear acceleration, a.

This expression is called the *linear dynamic equation* since it expresses the relation between forces and changes in linear speed.

If we write it as:

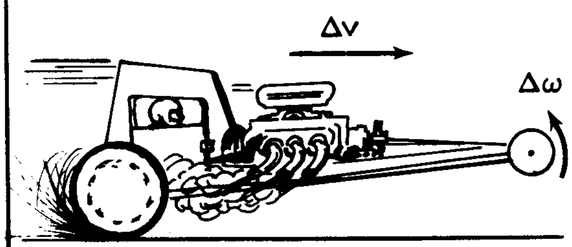
$$\Delta v = \frac{F}{m}\Delta t,$$

this says that the change in linear speed, Δv, of a body of mass m is greater, the greater the force and the longer it acts; but is less if its mass is bigger.

TORQUE AND MOMENT OF INERTIA IN ANGULAR MOTIONS:

A change in angular speed, $\Delta\omega$, does not depend on force or mass alone. It also depends on how the force is applied and where the mass is located.

A change $\Delta\omega$ depends on three features of a force: its *magnitude* or strength, the *direction* in which it acts and the *position* where the force acts with respect to the axis. In linear speed changes only magnitude and direction are important, but in rotational motion, position is another essential factor.



Forces on a dragster cause important changes of linear speed. It may also cause torques that produce unwanted changes in angular speed.

The term that expresses this information is the *torque*. We will use the Greek letter τ (tau) to indicate torque. When forces are applied to rigid bodies on a fixed axis, it is the resulting torque that will determine how much angular acceleration will occur.

A change $\Delta\omega$ also does not depend only on the *magnitude* of the mass. The *position* of the mass, or its distribution, with respect to the axis is also important. That is expressed by the term *moment of inertia*, I.

THE ROTATIONAL DYNAMIC EQUATION:

The expression which relates torque and moment of inertia to changes in angular speed is called the *rotational dynamic equation*. It is:

$$\tau = I \frac{\Delta\omega}{\Delta t}$$

where $\frac{\Delta\omega}{\Delta t}$ is the angular celeration α.

This is the rotational equivalent to the *linear dynamic equation:*

$$F = m \frac{\Delta v}{\Delta t}$$

on the opposite page. Torque and mement of inertia replace force and mass in rotational motion, and angular speed and angular acceleration replace their linear counterparts.

THE THREE EXPERIMENTS:

Thus the behavior and characteristics of rotating bodies are properly expressed by torque and moment of inertia. (See for example the description of the tach generator on page 9.)

Your experiments in this part of the module will be to explore these parameters and see how they describe the changes in angular speed of the fan. The experiments will be to:

• MEASURE STALLING TORQUE

The purpose of the first experiment is to measure the stalling torque of the fan motor and therby become familiar with the idea of torque. Stalling torque is the torque required to just prevent a motor from turning, that is to stall it.

• MEASURE DYNAMIC TORQUE

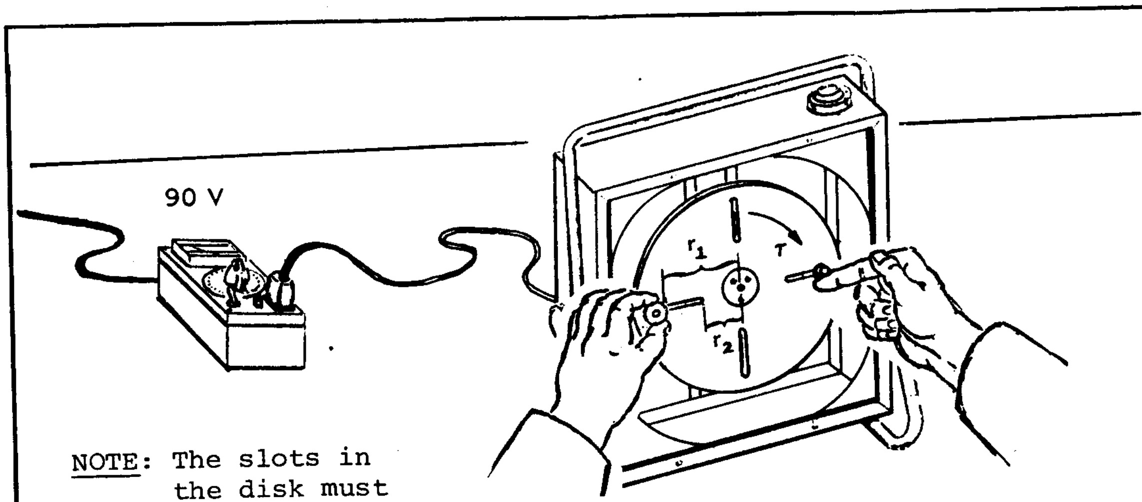
In the second experiment you will determine the torque when the motor is rotating. Will the torque be the same? That you will determine.

• DETERMINE MOMENT OF INERTIA

Finally you will change the size of what the motor has to rotate. Here you will see that it is not only *how much* mass you add but *where* you put it as well. This involves the notion of *moment of inertia*.

A SIMPLE EXPERIMENT... ...TO MEASURE STALLING TORQUE

The purpose of this experiment is to measure the stalling torque of the motor. This is a common specification for motors. Here you will use heavy washers and the force of gravity to produce the necessary torque. You will also see how important the mounting radius is in determining the number of washers required to stall the motor.



NOTE: The slots in the disk must be horizontal for the measurement.

CAUTION: *Do Not Let The Motor Stall For More Than A Few Seconds Since It Will Overheat!*

PROCEDURE:

1) *Set up the fan* as shown in the diagram.

Romove the protective grill and make sure that the set screw of the disk is tight.

Turn the variable voltage supply to 90 volts but be sure it is OFF.

2) *Hang a number of washers at r_1* on the disk but hold the disk lightly as shown, so that the *slot is horizontal.*

3) *Turn on the voltage* while holding the fan.

Remove the washers one at a time until there are just enough left to stall the motor. This may require some judgement. Be sure that the measurement is made with the slot horizontal.

Turn off the voltage.

Record the number of washers that just stalls the motor and the radius r_1 in the table after page 34.

4) *Repeat the measurement* for the radius r_2 where r_2 is equal to $\frac{1}{2}r_1$.

5) *Weigh a number of washers* and record the mass per washer in the table.

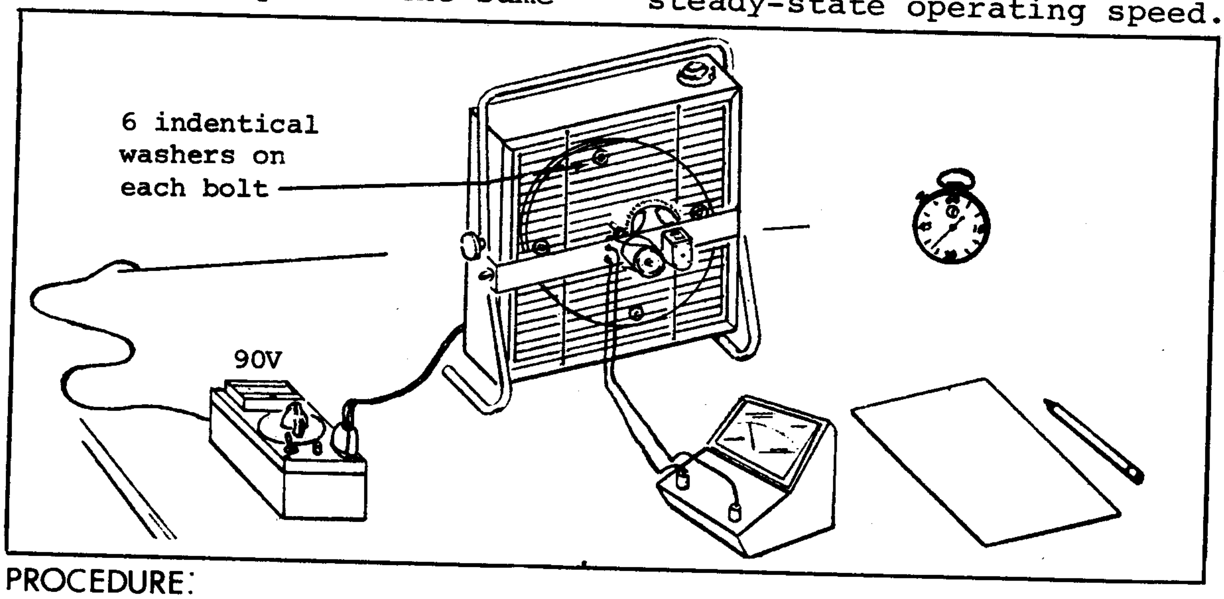
AN EXPERIMENT...

...TO MEASURE DYNAMIC TORQUE

You have seen that the motor exerts a definite torque and that it can quite easily be stalled.

You will now investigate the effect of that torque when the rotor is allowed to turn. It is reasonable to assume that the motor torque is constant for a given voltage, though perhaps not the same as the stalling value.

You will measure the angular speed with the tach generator at equal time intervals after turning on the motor. From a graph of this data you will see that the motor torque is constant at low rpm, but falls off at higher angular speeds as the fan reaches its steady-state operating speed.



PROCEDURE:

1) *Keep the variable voltage supply at 90 V,* the same as in the last experiment.

Securely mount 6 identical washers to each of the 4 bolts at maximum radius. The washers will increase the time to reach top speed for easier measurement.

Replace the protective grill and be sure that the tach generator is operating properly.

2) *Make a table* in the space provided on the data page to record your data. The table should look similar to the one used when taking the slow-down data of Part I. Have enough columns for 3 trials. Be sure you have the calibration graph for the tach generator.

3) *Turn on the fan* and record tach generator voltage readings at 5 second intervals until the speed stops changing. At least two persons should work together and you might make a couple of practice runs.

4) *Repeat the measurement* at least three times.

AN EXPERIMENT...
...TO DETERMINE MOMENT OF INERTIA

The moment of inertia, I, of a body is related to both the mass of the body and to how that mass is distributed. It is analogous to mass in linear motion.

The quantity I determines how a body will respond to an applied torque. Since I depends on mass distribution, it depends upon the location of the axis of rotation, and therefore you must speak of I about a certain axis. Here you will investigate rotation about the geometrical center of a disc.

How do you find I for an object? You cannot just use a scale, as you do to measure mass. You can calculate I for some simple shapes, but your disk and rotor is not simple. You could find I by applying a *known* torque and observing the change of angular speed. However, you do not yet know the torque of the motor.

Here you will use a method that works in a wide range of cases. It is not necessary to know the torque, only to know that it is constant. By adding additional mass at known positions, you can increase the moment of inertia in a known way. By then timing how long it takes for the torque to bring the object up to the *same* angular speed you can determine the original moment of inertia as well as the applied torque.

PROCEDURE:

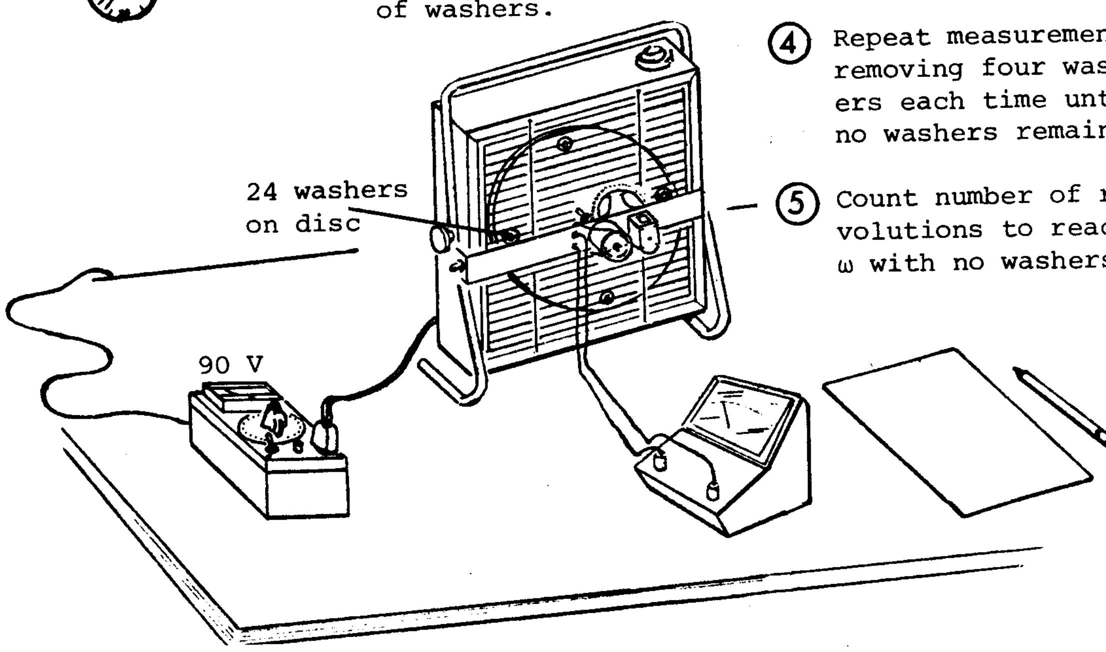
1. Turn on fan and start watch.

2. Stop watch when tach indicates some voltage within constant torque range, about 1300rpm. Record time and number of washers.

3. Remove 1 washer from each bolt. Repeat time measurement to reach *identical* tach reading.

4. Repeat measurement removing four washers each time until no washers remain.

5. Count number of revolutions to reach ω with no washers.



- tear out page -

DATA TABLES

STALLING TORQUE

Fan Voltage_______________V r_1 = $r_2 = \frac{1}{2}r_1$

Mass per washer____________g   No. washers at r_1___at r_2___

DYNAMIC TORQUE

(make table here)

MOMENT OF INERTIA

Fan Voltage   V = ____________V

Washer Radius   r = ____________cm

Mass per washer   m = ____________g

Tach reading when watch stopped = ____________volts

angular speed ω = ____________rpm

= ____________rps

Number of revs to reach ω with no washers: = ____________revs

θ = ____________radians

(make table here)

- tear out page -

CALCULATIONS

STALLING TORQUE

Force = No. Washers X Mass Per Washer X Acceleration Of Gravity

F_1 = ________ X ________g X 980 $\frac{cm}{sec^2}$ = ____________dynes

F_2 = ________ X ________g X 980 $\frac{cm}{sec^2}$ = ____________dynes

Stall Forque = Force X Lever Arm

$\tau_1 = F_1 \times r_1$ = ________dyne cm

$\tau_2 = F_2 \times r_2$ = ________dyne cm

Are these the same?

KINETIC ENERGY....AND WORK

$KE = \frac{1}{2} I_o \omega^2$

$W = \tau\Theta$

ANGULAR ACCELERATION

RATE OF CHANGE OF ω:

The rate at which the angular speed is changing is called the angular acceleration α (alpha). In symbols:

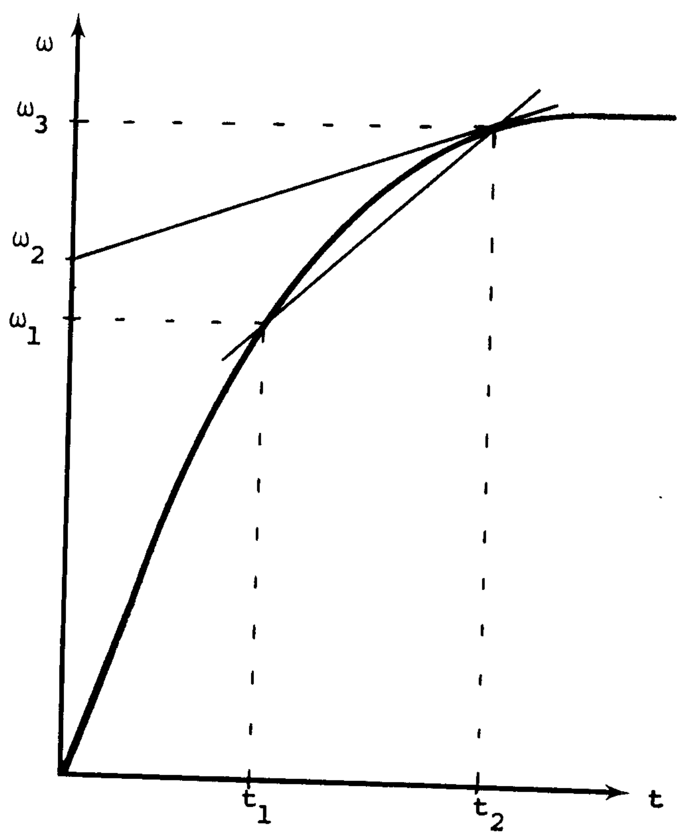
$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$$

The angular acceleration α can be found from graphs of ω against t. For any two points on the curve, measure the $\Delta\omega$ and the Δt. The ratio $\Delta\omega/\Delta t$ is the *average angular acceleration* during that time interval.

If the two points are very close together, so they are at essentially the same time, then $\Delta\omega/\Delta t$ is the *slope* of the curve at that point. This slope of the curve at any time gives the *instantaneous angular acceleration* at that time.

If the slope of the curve is the same over the time interval, then the angular acceleration is constant and the angular speed changes linearly with time. If the slope is changing, then the angular acceleration is changing as well as the angular speed.

An upward slope means a positive acceleration and an increasing angular speed. A downward slope means a negative acceleration (or deceleration) and a decreasing angular speed.



from t_1 *to* t_2:

$$\textit{average } \alpha = \frac{\omega_2-\omega_1}{t_2-t_1}$$

at t_2:

$$\textit{instantaneous } \alpha = \frac{\omega_3-\omega_2}{t_2-0}$$

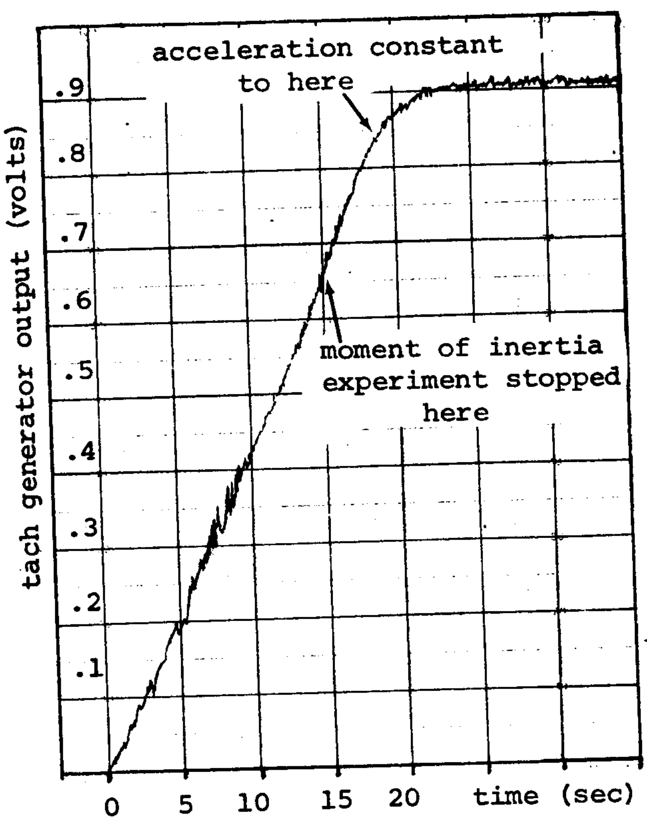
ANGULAR ACCELERATION FROM YOUR DATA

THE FIRST STEP. . .

. . . in determining the dynamic torque of the fan motor will be to determine the angular acceleration from the ω-t graph of start up behavior.

1. DRAW THE ω-t GRAPH using your data and the piece of graph paper on the opposite page. Follow the proper labeling and graphing procedure described in the previous section.

   Your graph should be quite linear over most of its range. A typical behavior is shown below. Only when the disk is almost up to its final operating speed does it depart from linearity. This says that the angular acceleration is constant up to some rpm.



Typical start up behavior of a fan. Continuous curve was made by connecting the tach generator output to a servo-recorder.

2. CALCULATE THE ANGULAR ACCELERATION over the constant range and record it on your graph.

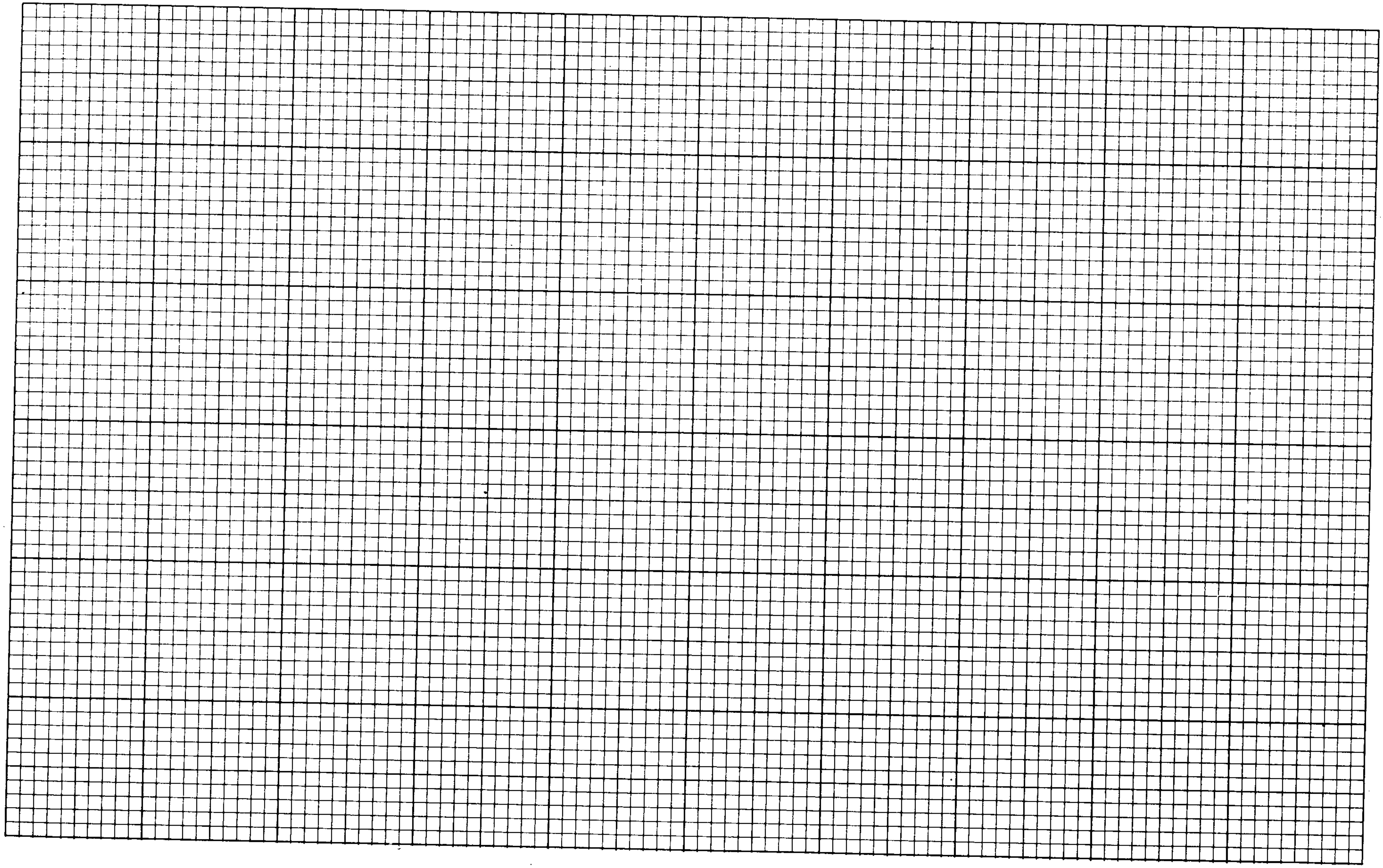
   The fact that the angular acceleration is constant indicates that the motor torque is constant. This is an important conclusion for your other experiment, the determination of moment of inertia. As you may recall, one of the criteria for that experiment was that the torque be constant, at least up to the rpm value you chose for your time measurement. Was it? If not, your later analysis may be in error.

THE NEXT STEP. . .

. . . is to relate the angular acceleration to the torque. In order to do that we must gain a better, and more mathematical, understanding of what is meant by torque.

MEASUREMENT OF DYNAMIC TORQUE

Angular Speed (rps)



time (sec)

angular acceleration =

- tear out page -

- tear out page -

SPARE GRAPH PAPER

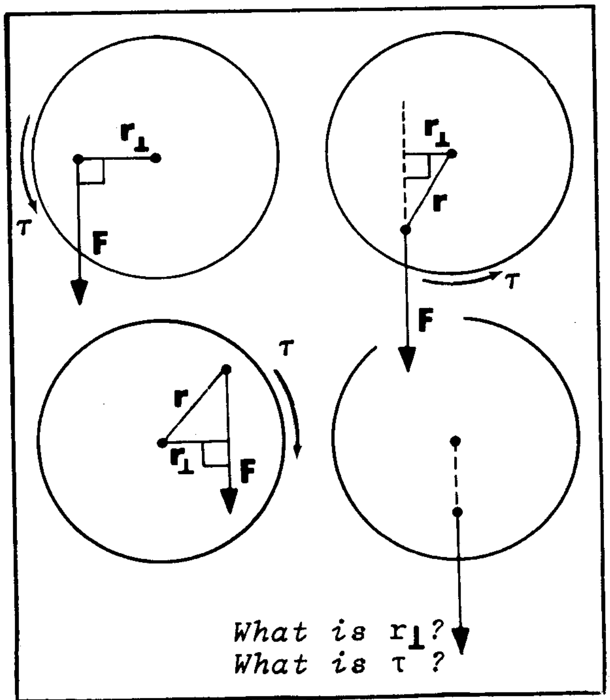
TORQUE

FORCE TIMES LEVER ARM:

A definition of torque, τ, is:

$$\tau = \text{force} \times \text{lever arm}$$
$$= Fr_{\perp}$$

It is the force times the perpendicular distance from the axis. The perpendicular distance, or lever arm, is the distance measured from the axis of rotation along a line at right angles to the direction of the force. (See the illustration.)

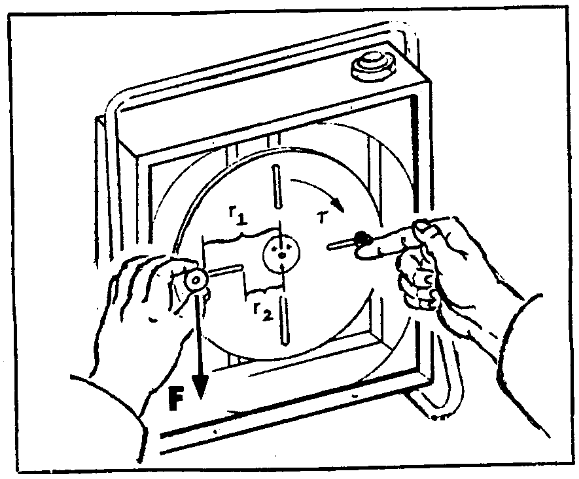


If the force is applied at right angles to the radius r, then $\tau = Fr$. If it is at some other angle then r must be calculated from the geometry.

Torque, then, is a measure of the extent to which an applied force will produce an angular acceleration. As can be seen from the diagram it depends on the *magnitude* of the force, the *distance* from the axis of rotation and the *direction* relative to the radius.

STALLING TORQUE FROM YOUR DATA:

In your measurement of stall torque, the motor produced an internal electromagnetic torque to the rotor of the motor. You, in turn, produced an external counter torque by hanging washers on the disk. The importance of having the slot horizontal was so that $r = r_{\perp}$ and the torque was simply $\tau = Fr$.

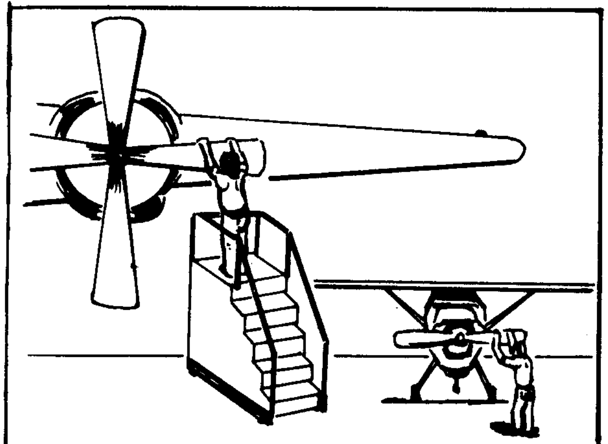


3. CALCULATE THE STALL TORQUE of your fan motor. Use the space provided on the back of the data table. Recall from the linear dynamic equation that F = ma. Thus the force is the mass of the washer times the acceleration due to gravity, g. The force will be in dynes if m is in grams and g is 980 cm/sec^2. If the radius is expressed in centimeters then the units of *torque will be dyne cm.*

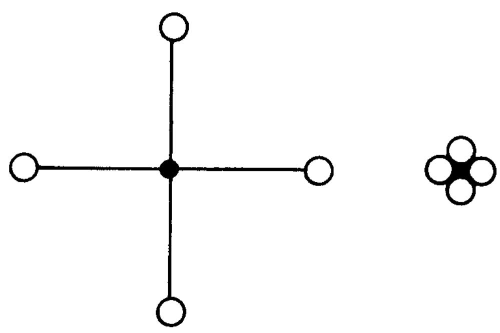
MOMENT OF INERTIA

WHAT MAKES A BODY HARD TO TURN?

A change in the angular speed of a rotating body does not depend only on the torque. Common sense tells us that the rotator itself must have an effect on the motion. Some bodies are harder to turn than others. We will now investigate what features of a rigid body affect its rotation.



In the illustration above the larger propellor is clearly much harder to turn since its mass is bigger. But in the illustration below, the masses are identical. Is one harder to turn (accelerate) than the other?



Mass certainly has an effect, but it is not the whole story. The two bodies at the bottom left have the same mass, but it is located very differently with respect to the axes. One is harder to turn than the other.

Just as the lever arm of a force affects its torque, so the distribution of mass in a body affects its rotational properties. The quantity that expresses both the mass of a body and how it is distributed is called the *moment of inertia, I.*

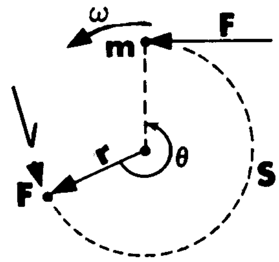
In the following pages we will derive an expression for the moment of inertia for the simplest possible case, a rotating point mass. While a point mass rarely occurs in practice, it will serve two purposes.

First, any rotating body can be made up of a collection of point masses, so it will suggest the general relation between mass and its distribution.

And second, when you added washers to the disk, this was essentially the same as putting a point mass at a fixed rotational radius. Thus we will be able to calculate the *increase* in moment of inertia from our expression.

DERIVATION OF I FOR A POINT MASS

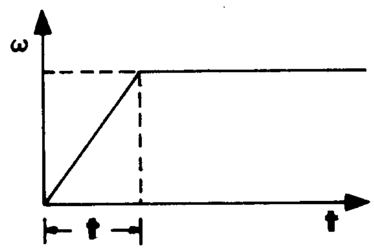
To begin, let us review the case under consideration and the procedure we will follow. We are calculating the moment of inertia of a point mass m, rotating at constant angular speed ω at a fixed radius $\bar{r}$. In order to reach the speed ω, a torque τ must have acted on the mass for some time Δt. This torque is the equivalent of a perpendicular force F acting on the mass at $\bar{r}$.



The derivation strategy will be to equate the work done by the force to the gain in kinetic energy. Comparing this to the dynamic equation:

$$\tau = I\alpha$$

will give us the expression for the moment of inertia of a point mass.



THE ROTATIONAL KINETIC ENERGY

For any mass point moving with *linear* speed $\bar{v}$, the kinetic energy is given by:

$$KE = \tfrac{1}{2}mv^2$$

The linear speed of the rotating mass point is given by $v = r\omega$, so that its *rotational kinetic energy* is:

$$\boxed{KE = \tfrac{1}{2}mr^2\omega^2}$$

THE WORK DONE

The work done when a force F pushes a mass for a distance S is:

$$W = FS$$

For the rotating mass point the distance S is an arc of $S = r\Theta$ so that the work done by a force perpendicular to r on m through an angle Θ is:

$$W = FR\Theta$$

Since Fr is simply the torque τ, we have:

$$\boxed{W = \tau\Theta}$$

CONSERVATION OF ENERGY

By conservation of energy we know that:

$$\text{Work done} = \text{Energy Gained}$$

For the mass point, assuming no other interactions, this is:

$$\tau\Theta = \tfrac{1}{2}mr^2\omega^2$$

From Part I we remember that Θ, the angular rotation, can be simply calculated as the area under the ω-t curve. For the example at the left this is a triangle of height ω, and base t. Thus $\Theta = \tfrac{1}{2}\omega t$.

Substituting this into the conservation of energy equation gives:

$$\tfrac{1}{2}\tau\omega t = \tfrac{1}{2}mr^2\omega^2$$

Dividing by $\tfrac{1}{2}\omega$ gives:

$$\tau t = mr^2\omega$$

or:

$$\tau = mr^2\left(\frac{\omega}{t}\right)$$

(ω/t) is the slope of the ω - t curve or the constant angular acceleration, α. Thus:

$$\tau = mr^2\alpha$$

Earlier we stated that the rotational dynamic equation was:

$$\tau = I\alpha$$

where I is the moment of inertia of the rotating object. Thus for a mass point the moment of inertia must be:

$$\boxed{I = mr^2}$$

Moment of inertia of a mass point

UNITS OF MOMENT OF INERTIA

The quantity mr^2 expresses both the mass and its location. Since it depends on the *square* of the radius, we see that the location of mass in rotational motion is even more important than its size.

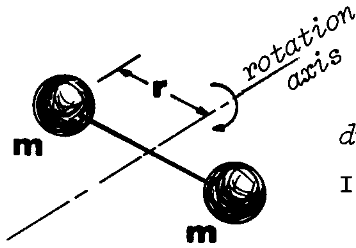
Further, moment of inertia has units of mass times distance^2. Thus the typical moment of inertia units are: gm cm^2, kg m^2 and slug in^2. For more on units and conversion factors see the Review.

Refer back to the illustration of the tach generator on page 9 and note the listing of the moment of inertia of its rotor.

I FOR RIGID BODIES

POINT MASSES AND RINGS

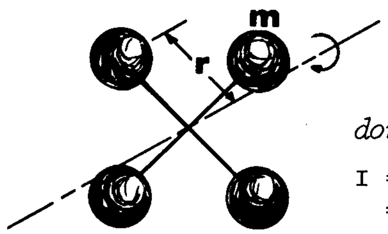
The expression $I = mr^2$ does not hold for all rigid bodies. For a body other than a point mass, one can consider it to be made up of large numbers of point masses. Then one adds up all the mr^2 for the point masses at different positions.



dumbbell

$I = 2mr^2$
$= Mr^2$

This adding up process can get quite complicated for odd shapes, but is easy for some simple ones. For a dumbbell, a double dumbbell, or a thin ring, all the mass points are at the same radius. Let us call each point mass m, and the total mass M. Then the sum of all the mr^2 is just Mr^2.

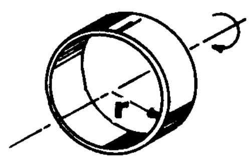


double dumbbell

$I = 4mr^2$
$= Mr^2$

This is the case when you added the washers. The increase in moment of inertia was:

$$I = \text{No. washers} \times \frac{\text{mass}}{\text{washer}} \times r^2.$$



thin ring

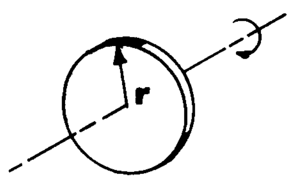
$I = Mr^2$

A DISK

When you add up the mr^2 for all the points in a disk of uniform thickness, you get:

$$I = \tfrac{1}{2}Mr^2$$

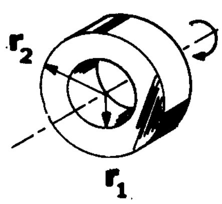
regardless of how thick it is.



disk
(any thickness)

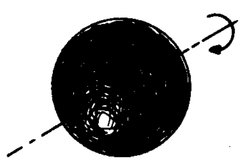
$I = \frac{1}{2}Mr^2$

OTHER SHAPES



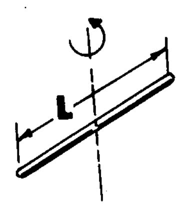
hollow cylinder

$I = \frac{1}{2}M(r_1{}^2 + r_2{}^2)$



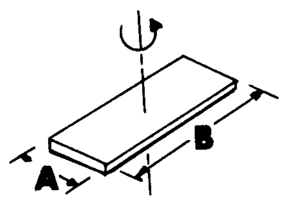
sphere

$I = \frac{2}{5}Mr^2$



slender rod
(axis through center)

$I = \frac{1}{12}ML^2$



Rectangular Plate
(axis through center)

$I = \frac{1}{12}M(A^2 + B^2)$

I FROM YOUR DATA

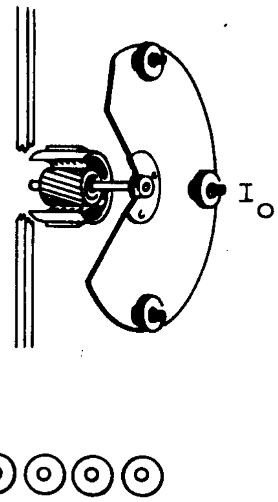
THE AIM OF THE ANALYSIS..

. . is to determine the moment of inertia, I_o, of the rotating disk without the washers. Actually I_o includes more than the disk because the hub, some bolts, and the rotor of the motor were all rotating. While one can calculate the moment of inertia of the disk alone, it is not possible to calculate, for example, that of the rotor.

The washers were added as essentially point masses at a fixed radius. Thus the *increase* in moment of inertia can be calculated as:

$$\Delta I = \text{No. washers} \times \frac{\text{mass}}{\text{washer}} \times r^2$$

$$\Delta I = nmr^2.$$



$I_o = ?$

but $\Delta I = nmr^2$

where n = no. washers

m = mass/washer

r = radius to washers

WHY IT WORKS:

According to your last graph, the motor torque τ was constant for the time, t_o, during which it accelerated the disk from 0 to your final rpm, which we will call ω. If it was constant for a load of 24 washers it was almost certainly constant for a lesser number.

From the dynamic equation, we know that with no washers:

$$\tau = I_o \frac{\omega}{t_o}$$

or:

$$\tau t_o = I_o \omega$$

When the washers were added, the moment of inertia increased by ΔI and the time it took to reach ω increased by Δt. Thus the dynamic equation becomes:

$$\tau(t_o + \Delta t) = (I_o + \Delta I)\omega$$

Dividing the second equation by the first gives:

$$\frac{t_o + \Delta t}{t_o} = \frac{I_o + \Delta I}{I_o}$$

or:

$$1 + \frac{\Delta t}{t_o} = 1 + \frac{\Delta I}{I_o}$$

or:

$$\frac{\Delta I}{I_o} = \frac{\Delta t}{t_o}$$

or:

$$I_o = t_o \frac{\Delta I}{\Delta t}$$

Since $\Delta I = nmr^2$ we have:

$$\boxed{I_o = t_o mr^2 \frac{n}{\Delta t}}$$

Another way of writing this is:

$$n = \left(\frac{I_o}{mr^2 t_o}\right) \Delta t$$

This says that if you make a graph of the number of washers n, against the increase in time, Δt, that it took to reach ω; the curve would be a straight line and the slope of that line would be equal to:

$$\text{slope} = \frac{I_o}{mr^2 t_o}$$

Everything in the expression is known but I_o so that:

$$I_o = mr^2 t_o \text{ X slope}$$

Once I_o is known, it can be used to calculate the constant dynamic torque τ from the dynamic equation:

$$\tau = \frac{I_o \omega}{t_o}$$

The dynamic torque can then be compared to the static or stall torque calculated earlier.

DETERMING I_o and τ:

1. GRAPH n AGAINST Δt using the graph paper on the opposite page. Follow the proper graphing procedure described in Part I. Draw the best straight line you can through the points.

2. DETERMINE THE SLOPE of the straight line and record it in the space provided.

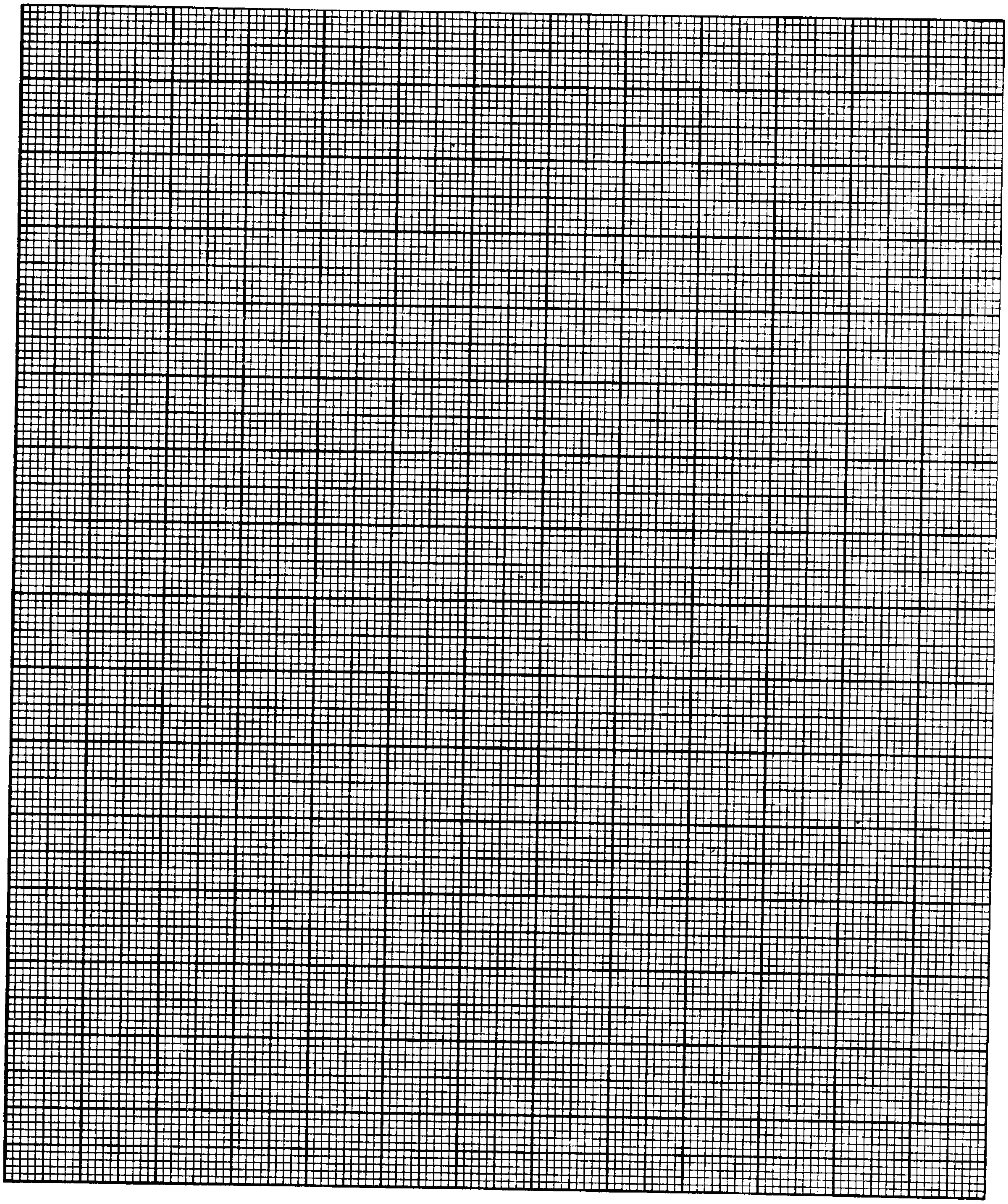
3. CALCULATE I_o from the expression above. You know the slope, and you have measured the values of m (mass per washer), r (radius to the washers) and t_o (the time it took for the fan to reach ω with no washers). Record I_o on your graph.

4. CONVERT YOUR VALUE into lb in^2. You may find it convenient to use the conversion table at the end of this section. Compare your value with that of the tach generator illustration on page 9.

5. CALCULATE THE DYNAMIC TORQUE from your value of I_o. How does it compare to the stalling torque at the *same supply voltage?* Be sure to use ω in rad/sec.

- tear out page -

GRAPH OF n VS Δt



Slope =

Moment of inertia
I_o =

Dynamic Torque
τ =

- tear out page -

SPARE GRAPH PAPER

ROTATIONAL KINETIC ENERGY

INTRODUCTION:

Kinetic energy is an important feature of a body's motion for several reasons.

First, it gives more information than speed alone, since it includes the body's moment of inertia.

Second kinetic energy is related to work. It tells you directly how much work is required to bring a body up to a certain speed.

And finally, energy is a conserved quantity. It never disappears, but simply goes into other forms and the total amount can always be accounted for. For example, when a rotating wheel slows down, its KE decreases. In this case it converts into heat to the bearing and to the air in which it rotates.

CALCULATING KE AND WORK:

On page 40 we derived the kinetic energy of a point mass on a rotating body to be:

$$KE = \frac{1}{2}mr^2\omega^2$$

Later we found the moment of inertia of the mass point to be:

$$I = mr^2$$

Thus the *rotational kinetic energy* can be written:

$$KE = \frac{1}{2}I\omega^2$$

In the same way that the moment of inertia of a rigid body can be considered as the sum of the moments of a collection of mass points; so too is the kinetic energy of a rigid body the sum of the kinetic energies of all the mass points.

Thus we may write the general kinetic energy expression as:

$$KE = \frac{1}{2}I_O\omega^2$$

Rotational Kinetic Energy of a Rigid Body.

where I_O is the moment of inertia of the body.

We also derived the work done in getting a mass point up to speed as:

$$W = \tau\Theta$$

where Θ is the total angle turned to get the disk up to the angular speed ω. This expression, too, can be shown to apply to any rigid body.

Further we argued that, by conservation of energy, we must have:

$$W = KE$$

if there are no other interactions such as frictional forces.

You are now in a position to test our assertion.

PROCEDURE:

1. CALCULATE THE ROTATIONAL KINETIC ENERGY of the rotating disk with no washers on it from your value of I_O and ω. Record it in the space provided on the calculations page.

2. CALCULATE THE WORK done in getting the disk up to the angular speed ω. Use the value of torque that you calculated and the angle Θ that you counted with the counter.

3. COMPARE YOUR VALUES. How well do your values agree?

USING ROTATIONAL KINETIC ENERGY

FLYWHEEL. . .

. . .operation depends on the fact that, when turning with angular speed ω, it has

$$KE = \tfrac{1}{2}I\omega^2.$$

A flywheel keeps this KE (and, therefore, its angular speed) unless the energy is transferred to some other form, for example heat due to bearing friction.

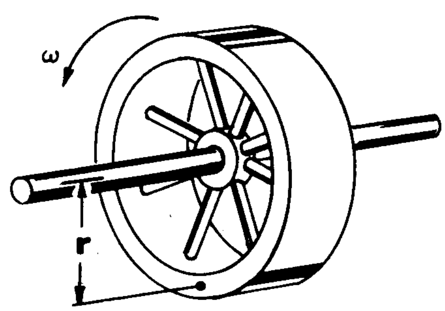
In a gasoline engine work is done on the flywheel, storing KE. In a one-cylinder engine, there is only one power impulse in every two revolutions and the KE of the flywheel is necessary to keep all the parts in motion until the next power stroke. Without a flywheel, the engine might stop completely, or move in a rough and jerky manner.

GOOD FLYWHEEL DESIGN. . .

. . .means getting the maximum energy storage in the required space with the least possible weight. Energy storage is proportional to $I\omega^2$. Thus there is an advantage in running a flywheel as fast as possible, since energy storage increases with the square of ω. But speeding up the flywheel requires expensive gearing, and excessive speed can cause a flywheel to fly apart.

Thus the designer's job is to get the largest possible I without making the wheel too heavy or too big. Look at the expressions for I for various rigid body shapes on page 42. Note that all have the expression mr^2 but many are preceded by some reducing fraction. The ring shape, however, has no such fraction. Thus flywheels are frequently shaped like a ring with a very light support structure.

$$I = Mr^2$$



$$KE = \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}Mr^2\omega^2$$

REVIEW

SUMMARY:

Rigid body dynamics is concerned with the factors causing changes in rotational motions. These include their causes, and the nature of the bodies that rotate.

Angular acceleration is the rate of change of angular speed:

$$\alpha = \frac{\Delta\omega}{\Delta t} .$$

Angular acceleration results from the application of a *torque*. Torque, τ, is defined as force times lever arm.

$$\tau = Fr_{\perp} .$$

Stalling torque is the torque that just prevents a motor from rotating. Dynamic torque is the torque exerted by the motor when it is rotating. The two motor torques are not necessarily the same.

The relation describing changes in angular speed is the *rotational dynamic equation*.

$$\tau = I\alpha$$

Moment of inertia is the basic dynamical property of a rotating body. It describes the body's mass and how the mass is distributed. Moment of inertia of a single point mass m at distance r from axis is:

$$I = mr^2 .$$

For dumbbell shapes and rings, where all the mass is at the same radius, the moment of inertia is also:

$$I = Mr^2 .$$

For other simple shapes there are other formulas.

For more complicated rigid bodies, I can be measured. This is done by observing quantitatively the effect of a constant torque as the moment of inertia is changed by a known ΔI. The formula used is:

$$I_o = \Delta I \left(\frac{t_o}{\Delta t}\right)$$

The rotational kinetic energy of a *mass point* on a rotator is given by:

$$KE = \tfrac{1}{2}mr^2\omega^2 .$$

The kinetic energy of any rotating rigid body (of moment of inertia I) is:

$$KE = \tfrac{1}{2}I\omega^2 .$$

CONVERSION TABLES

A NOTE ABOUT UNITS:

On the next page are given various units and conversion factors for torque and moment of inertia. These require some explanation, since the units commonly used in the technical literature (which we call *engineering units*) are not always the proper *absolute units. Absolute units must be used in calculations using the rotational dynamic equation.*

The confusion results from the difference between force and mass. The table below gives the proper force and mass units for the various unit systems.

Torque has units of force times length. Thus the proper absolute units are dyne cm, newton meter and pound feet (or inch ounce). However, commonly used engineering units are gm cm and kg m since gm and kg are often used as units of weight.

Moment of inertia has units of mass times length squared. Thus the proper absolute units are: gm cm^2, kg m^2 and slug ft^2. However, commonly used engineering units are lb ft^2 (or ounce in^2) since we rarely use the English unit for mass, the slug.

The conversion tables list both engineering and absolute units. Any engineering unit must be converted to the absolute unit before making calculations using the rotational dynamic equation.

Force =	Mass x	Acceleration	Unit System
Weight =	**Mass x**	**Gravitational Acceleration**	
dyne	gram (g)	$980 \frac{cm}{sec^2}$	cgs
newton (nt)	kilogram (kg)	$9.8 \frac{m}{sec^2}$	MKS
pound (lb)	slug	$32.2 \frac{ft}{sec^2}$	English

HOW TO USE THE TABLES:

If you have a unit in column A and you want a unit in row B multiply by the factor shown.

FOR EXAMPLE: *If you have* gm cm^2 *and you want* lb in^2 *multiply by:*

$$3.42 \times 10^{-4} \frac{\text{lb in}^2}{\text{gm cm}^2}$$

$$A(\text{gm cm}^2) \times 3.42 \times 10^{-4} \frac{\text{lb in}^2}{\text{gm cm}^2} = B(\text{lb in}^2)$$

TORQUE

A \ B	ABSOLUTE UNITS					ENGINEERING UNITS	
	dyne cm	nt m *	lb in	lb ft	oz in	gm cm	kg m
dyne cm	1	10^{-7}	8.85×10^{-7}	7.38×10^{-8}	1.42×10^{-5}	1.02×10^{-3}	1.02[illegible]
nt m *	10^{7}	1	8.85	.738	142	1.02×10^{4}	.102
lb in	1.13×10^{6}	.113	1	.0833	16	1150	.0115
lb ft	1.36×10^{7}	1.36	12	1	192	1.38×10^{4}	.138
oz in	7.06×10^{4}	7.06×10^{-3}	.0625	5.21×10^{-3}	1	71.9	7.19×10[illegible]
gm cm	980	9.80×10^{-5}	8.67×10^{-4}	7.23×10^{-5}	.0139	1	10^{-5}
kg m	9.80×10^{7}	9.80	86.7	7.23	1390	10^{5}	1

MOMENT OF INERTIA

A \ B	ABSOLUTE UNITS				ENGINEERING UNITS		
	gm cm^2	kg m^{2*}	slug in^2	slug ft^2	lb in^2	lb ft^2	oz in^2
gm cm^2	1	10^{-7}	1.06×10^{-5}	7.38×10^{-6}	3.42×10^{-4}	2.37×10^{-6}	5.46×10^{-3}
kg m^{2*}	10^{7}	1	106	.738	3420	23.7	5.46×10[illegible]
slug in^2	9.43×10^{4}	9.43×10^{-3}	1	6.94×10^{-3}	32.2	.224	515
slug ft^2	1.36×10^{7}	1.36	144	1	4640	32.2	7.41×10^{4}
lb in^2	2930	2.93×10^{-4}	.0311	2.16×10^{-4}	1	.00695	16
lb ft^2	4.21×10^{5}	.0421	4.47	.0311	144	1	2304
oz in^2	183	1.83×10^{-5}	1.93×10^{-3}	1.35×10^{-5}	.0625	4.34×10^{-4}	1

* indicates the SI (Standard International) Unit

QUESTIONS:

1. Name three torque units.

2. In "Soap Box Derby" races unpowered cars roll down an incline. In coming down the hill, gravity does work on the cars and they receive a definite amount of KE depending on the change of height. This KE is shared between the car's linear motion and the wheel's rotation. What kind of wheels would give the highest possible final speed?

3. In terms of work and energy, what must be done to stop a moving car?

4. Explain briefly the meaning of each of the four quantities in the basic dynamic equation.

$$\tau = I \frac{\Delta\omega}{\Delta t}$$

5. Why are record player turntables made of heavy metal rather than, say, light, plastic?

PROBLEMS:

1. The stalling torque of a small motor (Hurst model CA) is given by the manufacturer as 100 inch-ounces.

   a) Calculate the torque in lb ft.

   b) The motor is connected to a pulley of one inch radius, and a string is wrapped tightly around this pulley. How much weight could be lifted or supported by this string?

2. The Sigma model 9AK4J2 stepping motor is advertized as having a hold torque of 950g cm. This is not a proper unit of torque, since grams are units of mass, not force, What is meant is the *weight* of one gram. Convert this spec to dyne cm.

3. An engine flywheel has almost all of its mass concentrated in a ring at an average distance of 15cm from the axis of rotation. Its mass is 9000g. Calculate I.

4. A child's playground toy consists of a thin, narrow plank, 12 feet long pivoted at its center. The mass of the plank is 18,000g. A child sits at each end of the plank. Assume each

child has a mass of 45,000g.

a) Calculate I for the plank about the axis at the center.

b) Calculate I for each child about this axis.

c) Calculate the total I for the same system by adding the three values obtained above.

d) State briefly how the children could reduce I.

5. A steady torque of 10 lb ft acts on a wheel (free to turn) for 40 sec. The wheel starts at rest, and is turning at 1800rpm after 40sec. Calculate the KE of the wheel after 40sec.

6. A certain "stepping-motor" is advertised as being able to produce a "step" of precisely 18° rotation in 2.0 millisec. It is further stated that the maximum torque produced by the motor is 3.5×10^5 dyne cm. Calculate the maximum I that the motor can "step" at this rate:

Try the problem in small steps:

a) Calculate the rotation in rad.

b) Calculate the average ω during a step.

c) Calculate $\Delta\omega$, assuming that torque is constant.

d) Calculate I.

7. A grinding wheel consists of a flat disc 15 cm in diameter, with a mass of 500 gram. It is driven by a motor that exerts a constant torque of 1.3×10^6 dyne cm. How much time is required to bring the wheel up to 1725 rpm?

8. A certain flywheel has KE = 15,000ft lb at full speed. The torque resulting from friction is 75 lb ft. How many revolutions will the wheel turn before coming to a stop?

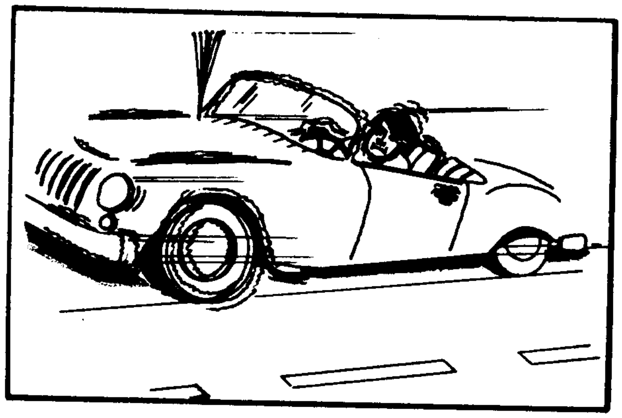
PART III

STATIC AND DYNAMIC BALANCE

THE EFFECTS OF UNBALANCE:

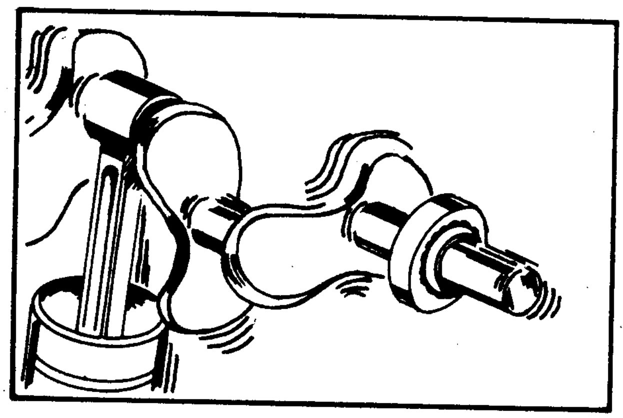
VIBRATION:

Unbalance not only is extremely annoying but can cause serious destructive effects. The motion of an unbalanced rotor sets up forces that make the rotor and its supports vibrate, sometimes violently.



BEARING WEAR:

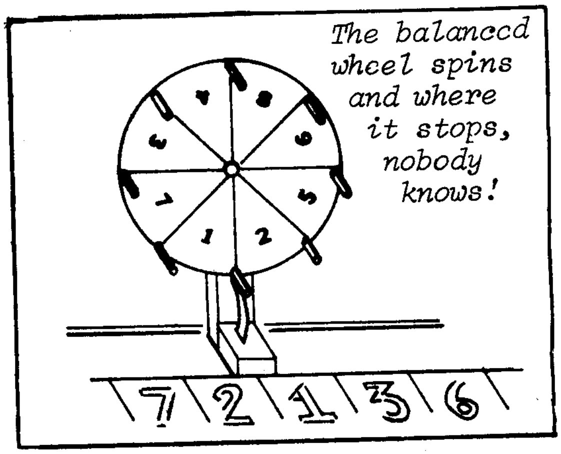
Rotating shafts are supported in structures called bearings. These are designed to have low friction, to keep the shaft properly aligned, and to carry the weight of the rotating parts. If bearings are well lubricated and not overloaded with excess weight, they last a long time, that is, the metal wears away slowly. However, unbalance forces cause rapid wearing away of metal and destroy bearings quickly.



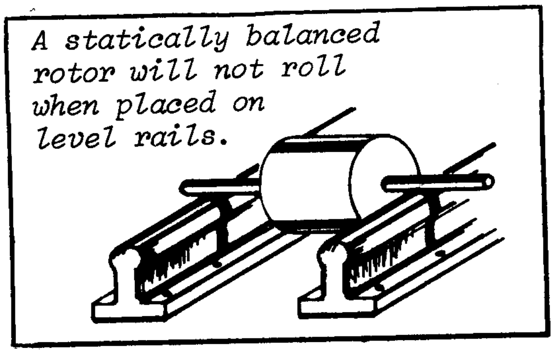
NOISE - HUMAN EFFECTS:

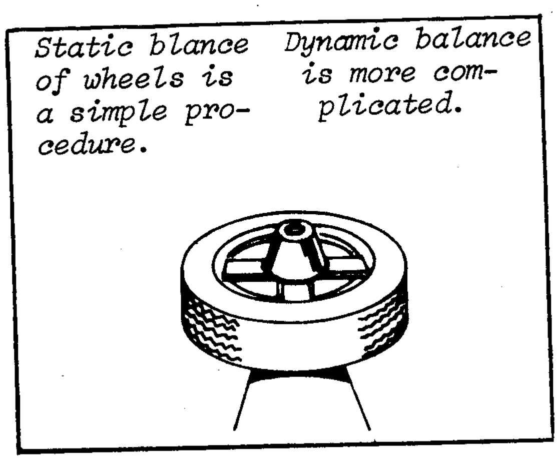
Noise is an obvious result of vibration; this is because the effect we call sound is directly produced by vibration. We now know that excessive noise can be harmful to human beings; our hearing can be permanently demaged and we do not work efficiently. Even if these did not happen, a noisy environment is simply an unpleasant place to be.

WHAT IS STATIC BALANCE?



The balanced wheel spins and where it stops, nobody knows!


A statically balanced rotor will not roll when placed on level rails.


Static blance of wheels is a simple procedure. Dynamic balance is more complicated.

It is easy to see when a wheel mounted on a shaft is in balance. Turn the wheel gently several times. Does it always stop with the same point down? or is there no "heavy spot"? A statically balanced rotor shows no preferred heavy spot; it stops anywhere. This test is useful for getting a rough idea of balance, but it is not very sensitive.

A better method for a large, heavy rotor is to place it on a carefully leveled set of rails. If it shows no tendency to roll in any position, it is in static balance.

Most common wheel balancing machines support the wheel by a needle at the center of its rotation axis. If a bubble then indicates that the wheel is level, it is statically balanced.

This is the method commonly used with auto wheels. The front wheels, especially, should be statically balanced because the reaction forces can set up an unpleasant, and sometimes dangerous pounding.

CENTER OF MASS:

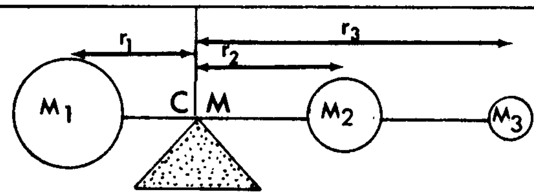
The *center of mass* (CM) is a very useful idea in the study of dynamics and balance. It is a sort of average position of a body. It is the point where (for many purposes) the mass of a body may be assumed to be concentrated.

How do you find the CM of a body? First you need to understand the idea of *moment*, which is a way of describing a mass and its position. The moment of a mass is the product of the mass and its distance from a point.

For example, suppose you have a body made up of several small masses, where is its CM? The CM is located at the point where the moments are the same on either side of it.

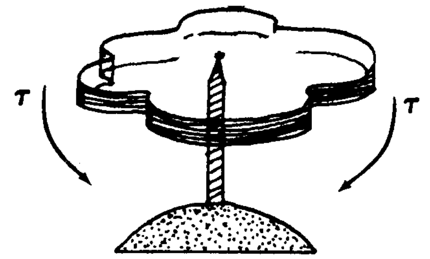
The CM is important in balance because, when a body is supported at its CM (by a needle, say) you know that the moments are equal on either side of the needle. Because of gravity, there are weight forces on the masses. Those weight forces in turn produce moments of force. A moment of force is what we have been calling torque.

When weight torques are equal and opposite, the body does not tend to turn. It is in balance. Because of this gravitational effect, the CM is more commonly referred to as the *center of gravity* (CG).



The CM is where:

$$M_1r_1 = M_2r_2 + M_3r_3$$



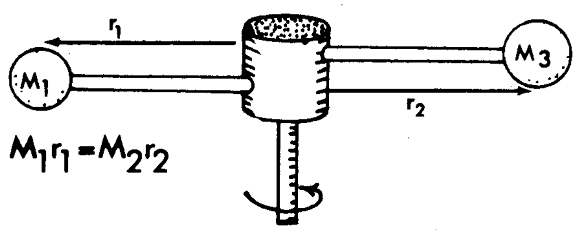
When supported at its CM in a gravitational field, all the gravitational torques at work on a body are equal and it remains level. Hence CM = CG.

WHAT IS DYNAMIC BALANCE?

A wheel that is in perfect static balance may well vibrate wildly when rotating. Something is still wrong with it. That something very often is dynamic unbalance. The explanation of dynamic unbalance is not simple and the methods for curing it are not easy.

The problem arises in rotating objects that have some thickness. As in static unbalance, it is due to an improper distribution of mass. The mass may be perfectly distributed with respect to radius but unequally distributed along the axis. Thus for objects that have both diameter *and* thickness, both static and dynamic balance must be considered.

To dynamically balance a rotating object is largely trial and error. You put on some additional weights to balance the distribution and see whether or not you have decreased the vibration.



This dynamic duo is in static balance, but try to spin it and you're in for trouble.

YOUR EXPERIMENTS:

In the final part of this module you will experimently observe the effect of an unbalanced rotating object, if you haven't already. Then you will learn some of the techniques used to bring such systems into static and dynamic balance.

Later in the module, we will describe the reasons why these techniques worked.

AN EXPERIMENT....

....TO OBSERVE THE EFFECTS OF UNBALANCE...........

The aim of this first experiment is to acquaint you with the practical effects of static unbalance. First you put the rotor out of balance by adding one washer and observe its behavior. Then you balance the rotor approximately by removing the washer, and again observe the behavior. You will carry out a more precise balancing operation in the next experiment.

You cannot measure the vibration quantitiatively at this point, but the differences should be obvious and easy to describe in words.

This experiment is primarily exploratory. The procedure suggests some things to do but you should investigate the cause and effect of unbalance to your own satisfaction.

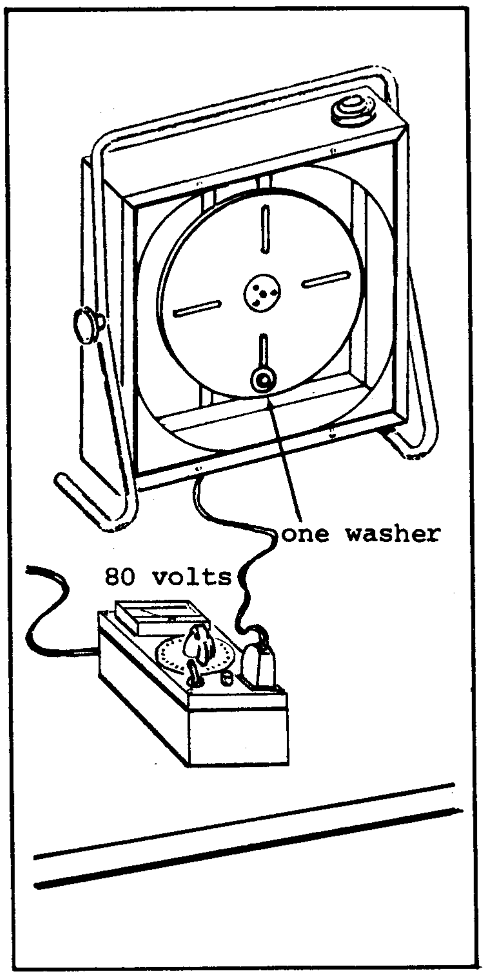
PROCEDURE:

1) *Set the voltage supply at 80 volts.* This will produce a low rpm so things don't get out of hand at the start.

2) *Mount the rotor with one washer on one bolt.* This will produce an *unbalance* as seen by the fact that the rotor's *natural* rest position is with this bolt down.

3) *Turn on the power supply* and observe the behavior of the fan and other objects on the table as the rotor comes up to speed.

4) *Vary the rotational speed* with the variable supply and observe the different vibrations that occur. You should find that different speeds produce large amplitudes for various *modes* of vibration of the fan and of other objects on the table.



5) *Describe with a sketch* at least three modes of fan vibration, in the data table at the end of these experiments. At what voltage does the fan start to "walk"?

6) *Explore the effects* of adding various washers at various positions.

...........AND.......TO ACHIEVE STATIC BALANCE.

In the previous part you observed the effect of roughly balancing a rotor that had been deliberately unbalanced.

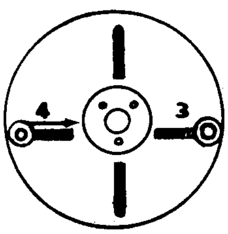
Now you will balance the same rotor carefully using a simple stationary balancing machine. This particular balancer is designed for balancing rotary lawn mower blades.

The rotor should now run with even less vibration. You will see that balance is achieved when you have an equal distribution of *moments* about the axis. This is *not* the same as an equal distribution of masses.

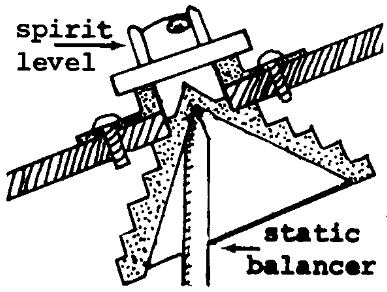
Follow the general guidlines below.

PROCEDURE:

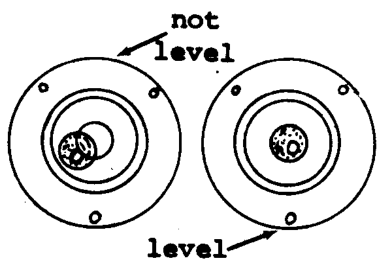
1) *Put 4 washers on one bolt, 3 on another* diametrically opposite. Initially locate them at maximum radius.



2) *Place the rotator on the static balancer* as shown. The disk will tilt because it is unbalanced.



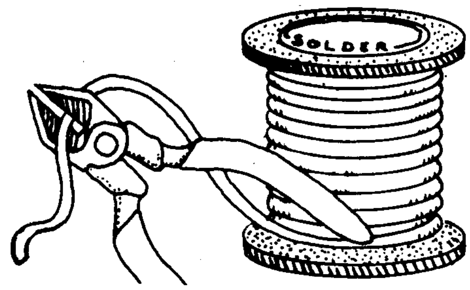
3) *Place the spirit level on the center of the disk.* When the level is level, the bubble will be within the black ring as shown.

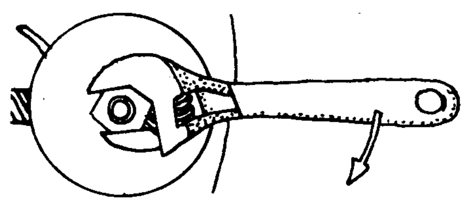


---

4) *Coarsely balance the disk* by adjusting the radius of the 4 washers. You probably cannot do it perfectly. This shows, however, that it is not mass alone that produces static balance.

5) *Finely balance the disk* by adding small pieces of solder. Cut short lengths and wrap them around bolts. Adjust their length until you get as perfectly balanced, or level, a disk as you can. You will probably have to put pairs of weights on two adjacent bolts. Clamp the solder under a nut or washer so it does not fly off when rotated.





---

6) *Rotate the disk on the fan* and see how well you did. Vary the speed and see if any resonances occur.

7) *Sketch the disk on the data page.* Record the radius of the two sets of washers from the axis.

AN EXPERIMENT....

....TO OBSERVE DYNAMIC UNBALANCE...........

Dynamic balance will be discussed fully in the remainder of the text, but now the aim is for you to have practical experience with it. Static unbalance is easy to detect and easy to cure. Both operations can be done on a simple balancing machine.

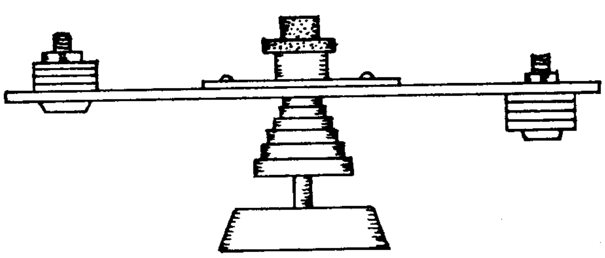
But, dynamic unbalance is a different story. Dynamic unbalance can be detected only by a dynamic test, that is by turning the rotor at high speed, and looking for certain effects.

Dynamic unbalance can indeed cause severe *vibration*, but the effect you will look for here is a slightly different one which we shall call *wobble*. The experiment should help clarify the meaning of wobble, but in brief it means a motion where the plane of the rotor no longer stays perpendicular to the axis of rotation.

PROCEDURE:

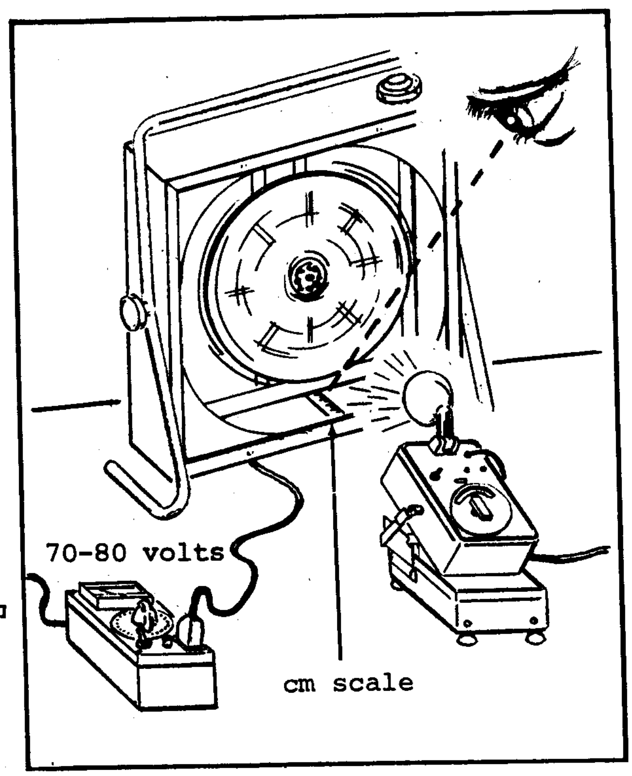
1) *Place two identical sets of 4 washers on opposite sides of the disk.* This change in relative positions of the masses is a key feature of dynamic unbalance. The masses and their moments are no longer in the same plane perpendicular to the rotation axis.

2) *Statically balance the disk.* Adjust the position of one (or both) bolts for good static balance. Note that the reversal of position does not affect your ability to get static balance.



3) *Mount rotor on fan shaft.* Dynamic unbalance can be detected and measured only by a dynamic test.

4) *Turn on the fan* at a low voltage (about 70-80 volts).



...........AND.......TO ACHIEVE DYNAMIC BALANCE.

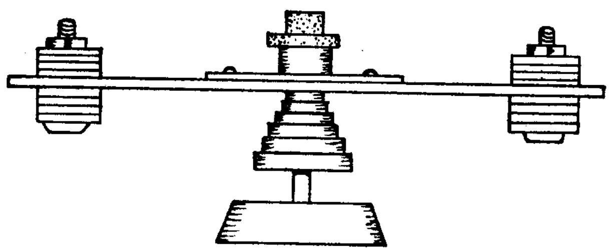
You have just seen the results of dynamic unbalance: forces that produce a wobbling motion. The rotor no longer stays aligned with respect to the shaft. If the rotor were rigidly attached to the shaft, these forces would act on the shaft itself. Then there would be vibration of the whole fan, excessive bearing loads, and other problems.

Our aim here is to see how the unbalance may be removed in this simple special case. The key word is *counterweights*: masses added so as to keep the moments evenly distributed in planes perpendicular to the axis of rotation.

By adding counterweights these moments will be in two planes rather than the original one. Of course, when mass is added, the static balance must be checked and preserved.

PROCEDURE:

1) *Add 4 washer counterweights* to each of the bolts as shown. Be careful that the two sets of washers in each plane ("upper" and "lower") are arranged in exactly the same way with respect to nuts, washers, etc.



2) *Statically balance the disk* in the usual way using the static balancer. Keep the bolts near their maximum radius.

3) *Dynamically measure and record the wobble amplitude* with the strobe as on the preceding page.

When an object is out of dynamic balance this method of adding counterweights is the procedure for bringing it back into balance. Unfortunately the decision of *how much* to add and *where* to put them is not clear and generally must be done by trial and error. In the final experiment you will see a standard method that is complicated but works.

5) *Stop the rotation with a strobe.* First stop the motion at the *highest* flash rate that gives a *single* image of the mark on the rotor. Then change the flash rate slightly so that the rotor appears to rotate slowly, and you can see the wobble in "slow motion".

6) *Measure wobble amplitude* by doubling the flash rate of the strobe. At twice the rpm, you will see the two extremes of the wobble. Read the amplitude along the cm scale on the frame by sighting over the edge of the disc.

7) *Sketch the orientation of the disk* as it rotates in the space provided in the data page. Show the position of the washers.

USING THE FLEXIBLE TABLE DYNAMIC BALANCER

Dynamic unbalance is a common effect, particularly with thick or long rotors, such as electric motor armatures, or crank-shafts. It is almost never obvious where mass should be added or how much. Modern automobile plants use sophisticated automatic machinery to balance crankshafts. But this lab is concerned with a simple device and a simple procedure for measuring and correcting dynamic unbalance on almost any rotor.

A symmetric, but *dynamically unbalanced* disk is provided. Your task is to add weights at the proper places to bring it into dynamic as well as static balance.

PROCEDURE:

1) *Mount the dynamically unbalanced disk* securely on the fan shaft. There should be no washers or other weights attached to the disk.

2) *Mount the fan on the flexible table.* Be sure the fan is seated in the slot in the table. Wing nuts should be tightened securely. Check that the table bounces freely without binding or hitting stops.

3) *Turn on the fan and adjust the speed* to the natural resonant bouncing frequency of the table. It is *important* that you achieve maximum amplitude of vibration since this amplitude will be your reference for later improvement.

Record the amplitude.

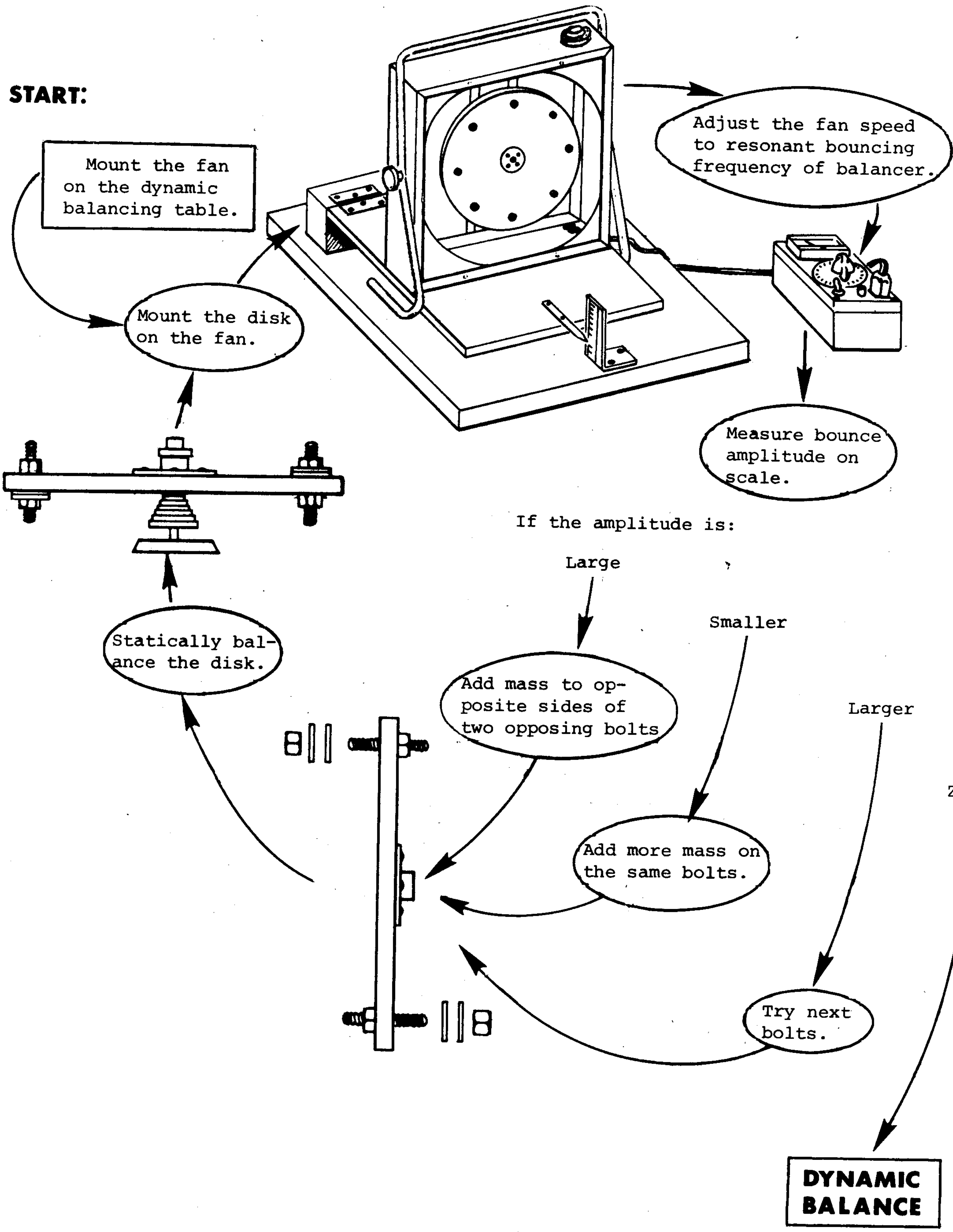
4) *Add a washer to each of two opposing bolts.* As in the previous experiment you should put one on one side and one on the other side of the disk to retain static balance.

5) *Turn on the fan and measure the bouncing amplitude.* The voltage for maximum amplitude should not have changed much but you might have to adjust it slightly.

Has the amplitude decreased?

6) *Move and trim the mass until the bounce amplitude is zero.* If the amplitude is greater try the next bolt. If the amplitude is smaller, try more washers on the same bolt. Keep trying various combinations until the amplitude is zero. The secret is to avoid bolts that make the vibration worse, stay with those that improve it. Be sure the disk is in static balance.

7) *Sketch disk and record masses added* on the data sheet.


START:
Mount the fan on the dynamic balancing table.
Adjust the fan speed to resonant bouncing frequency of balancer.
Mount the disk on the fan.
Measure bounce amplitude on scale.
If the amplitude is:
Large
Smaller
Larger
ZERO
Statically balance the disk.
Add mass to opposite sides of two opposing bolts
Add more mass on the same bolts.
Try next bolts.
DYNAMIC BALANCE

CALCULATIONS

-tear out page-

DATA PAGE

THE EFFECTS OF UNBALANCE (Description and sketch of 3 modes of fan vibration)

STATIC BALANCE (Sketch of disk showing washer position)

-tear out page-

DYNAMIC UNBALANCE (Sketch of orientation of unbalanced disk in rotation showing washers and positions)

DYNAMIC BALANCE

Wobble Amplitude before balance____________

after balance____________

FLEXIBLE TABLE BALANCER (Sketch of disk showing masses added and their position relative to reference marks)

STATIC UNBALANCE

UNBALANCED FORCES IN ROTATION:

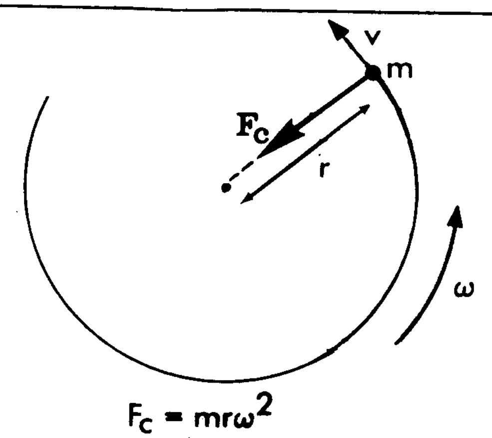
Our purpose is to describe the cause of static unbalance, and how it leads to vibrations. From this description it will then be clear what steps can be taken to cure the problem. Vibrations are caused by forces. Thus the first step is to understand what forces are presented.

THE CENTRIPETAL FORCE. . .

Suppose a point mass m on a rotor is moving with rotational speed ω. You know from Newton's first law that the mass wants to keep moving in a straight line at speed $v = r\omega$. To make the mass rotate as part of the rotor requires an inward force. This force is called a *centripetal force* and always acts radially inward. The magnitude of this force can be derived (see optional section on the next page), and is found to be:

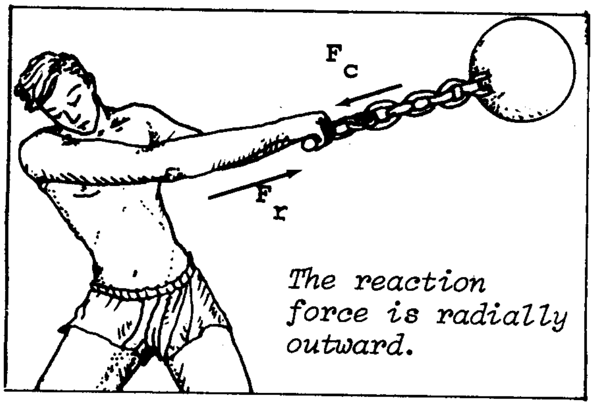
$$F_c = mr\omega^2$$

The Centripetal Force on a Mass Point.



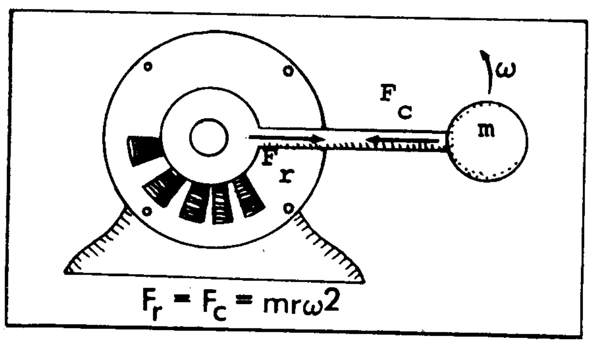
The centripetal force is radially inward.

. . .AND THE REACTION FORCE:



The reaction force is radially outward.

This inward centripetal force, of course, must be exerted by something. In the simple case of a weight on the end of a chain, it is exerted by your arm. For a mass rotating on bearings, it is the bearings that must exert the force.



According to Newton's third law, if the bearing exerts a force on the shaft, it feels an equal and opposite force. This force is called the *reaction force*, F_r. Its magnitude is equal to the centripetal force $F_r=F_c=mr\omega^2$ and its direction is radially outward on the bearing.

MATHEMATICAL DERIVATION OF $F_c = mr\omega^2$

The relation for the centripetal force on a rotating mass, m, can be mathematically derived. As in the derivation in Part I we will consider what happens to the mass during a small time interval, Δt. Referring to the diagram, we assume that during Δt, the mass moves from a to b through a small angle $\Delta\Theta$.

The linear velocity, v, at a and b is of the same magnitude, $r\omega$, but in a different direction. Since velocity is a *vector* quantity a small additional velocity, Δv, must have been added to produce the direction change. From Newton's laws we know that velocity changes are produced by forces. From the linear dynamic equation we have:

$$F = m\,\frac{\Delta v}{\Delta t}.$$

The value of v can be determined by referring to the vector addition geometry. The angle between v_a and v_b is also $\Delta\Theta$ so that:

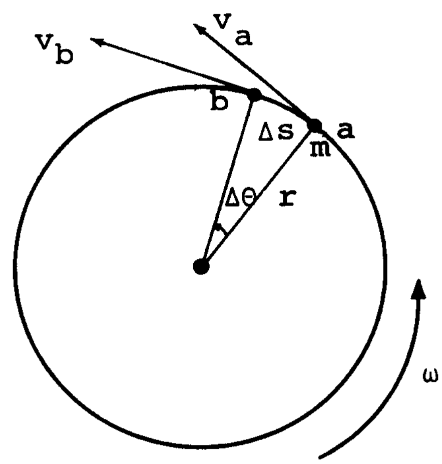
$$\Delta v \approx v\Delta\Theta,$$

where we assume that if $\Delta\Theta$ is small, Δv is nearly equal to the arc. From the top diagram we see that:

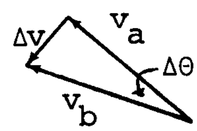
$$\Delta\Theta = \frac{\Delta s}{r}\,.$$

Putting this into the previous expression we get:

$$\Delta v = \frac{v}{r}\,\Delta s.$$



Δv = *additional velocity required to produce direction change.*



Putting this into the force equation gives:

$$F = m\frac{v}{r}\,\frac{\Delta s}{\Delta t}\,.$$

But $\frac{\Delta s}{\Delta t} = v$ so that:

$$F = m\frac{v^2}{r}\,.$$

And since $v = r\omega$ we have:

$$F_c = mr\omega^2.$$

The direction of F_c is the same as Δv, which is radially inward.

CENTER OF MASS...

The *center of mass* of a body is defined as *that point about which all the moments are equally distributed.* A rotational moment is simply the product mr of a mass m at radius r.

CENTER OF GRAVITY...

Begins to have meaning if one assumes the body to be in a gravitational field. Then there is a force on each mass point of mg. And if the point can only rotate about a fixed axis then the force becomes a torque of mrg (provided $r = r_\perp$).

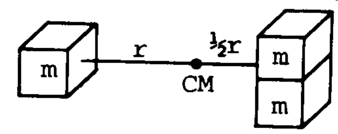
Thus the rotational center of mass is *that point about which the net gravitational torque is zero;* and the center of mass is identical to the *center of gravity*.

REACTION FORCES...

On the previous page we saw that the reaction force on a rotating point mass was $mr\omega^2$. This is simply the moment (mr) times the square of the angular speed.

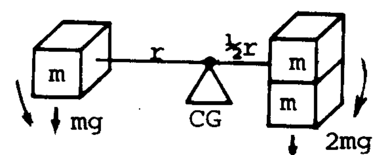
If a body rotates about its center of mass, these moments are equally distributed. Thus the reaction force of each mass point has an identical opposing reaction force on the opposite side of the rotation axis. The result is *no net reaction force on the bearings provided that the body rotates about its center of mass.*

CENTER OF MASS all rotational moments balance.



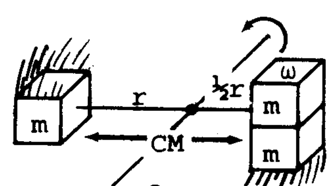
$mr = 2m\ (\tfrac{1}{2}r)$

CENTER OF GRAVITY all gravitational torques balance.



$\tau = mgr = 2mg\ (\tfrac{1}{2}r)$

ROTATION ABOUT CENTER OF MASS all reaction forces balance.



$F_r = mr\omega^2 = 2m\ (\tfrac{1}{2}r)\,\omega^2$

...AND STATIC BALANCE.

Thus the key to static balance is to be sure that the body rotates about the center of mass....or vice versa....be sure that the center of mass is at the axis of rotation.

THE CAUSE OF STATIC UNBALANCE...

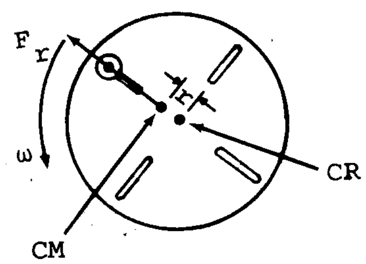
When a body rotates about an axis that is not through the center of mass, the reaction forces on the bearings do not all balance out. There are greater rotational moments on one side than the other. And there is a *net reaction force* on the bearing that rotates with the angular speed, pulling on the bearing as it goes.

The size of this net reaction force is simple to calculate. The body *acts* as if all of its mass, M, were concentrated at the center of mass at rotational radius, r, the distance between the CM and the axis of rotation. Thus *the unbalanced reaction force on the bearing* is:

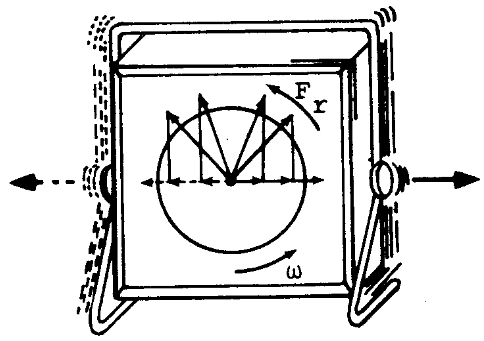
$$F_r = Mr\omega^2.$$

While r is generally quite small, sometimes only thousandths of an inch, M is the whole mass of the body and F_r can get quite large, particularly at large ω.

When the center of mass is not at the center of rotation, the net reaction force on the bearings is: $F_r = Mr\omega^2$



As the body rotates, the net reaction force on the bearings rotates with it.



The component of F_r in any direction oscillates at ω and will excite any natural modes at that frequency.

...AND VIBRATION:

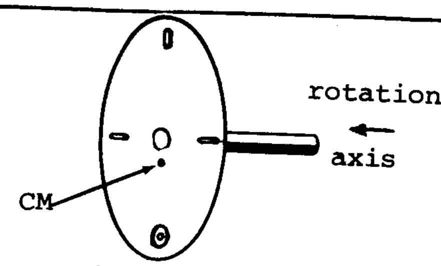
A real machine, like the fan, has several preferred directions in which it likes to vibrate. These modes of vibration have certain natural frequencies much like the vibrating reed. The direction of these natural modes can be at any angle.

When the disk rotates, the unbalanced force rotates around with it at the rotation frequency. The *component* of F_r in any direction oscillates back and forth at this frequency. Thus if ω exactly matches the natural frequency of a particular mode, it will act like a driving force in that direction and cause that mode to oscillate strongly.

This condition is called *resonance*. Different angular speeds will, of course, excite different vibrational modes of the fan, just as you observed in your experiment.

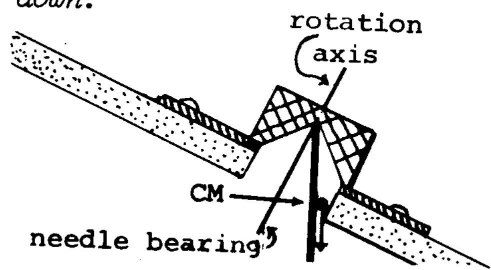
THE CURE FOR STATIC UNBALANCE...

As is obvious by now, the cure for static unbalance is to be sure that the rotation axis of a body coincides with the center of mass. While it is often difficult to move the rotation axis, it is generally not difficult to move the center of mass.



When an unbalanced disk is suspended vertically, the force of gravity pulls the center of mass down.

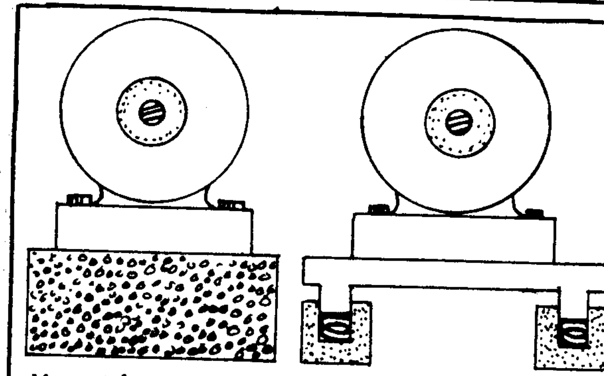
A rotor on its shaft can give a rough idea in which direction the center of mass is shifted since it pulls that side down. It is only a rough indication however since most bearings have significant static friction.



A more sensitive method is to suspend it horizontally at the center of rotation on a delicate needle bearing. An off axis CM will show up as a tilt.

A considerably more sensitive method is to suspend the body *horizontally* at its axis by a needle bearing, as you did with the static balancing machine. Do you see why this is equivalent to a vertical suspension?

...AND VIBRATION



Mounting on a heavy base shifts resonant frequency.

Friction mount absorbs energy of vibration.

Sometimes it happens that you cannot get at the rotating system to balance it. For example the rotor of the fan motor may be out of balance even though the disk is perfectly balanced. At some angular speed a strong, unwanted vibration resonance may occur. What can you do? There are three choices:

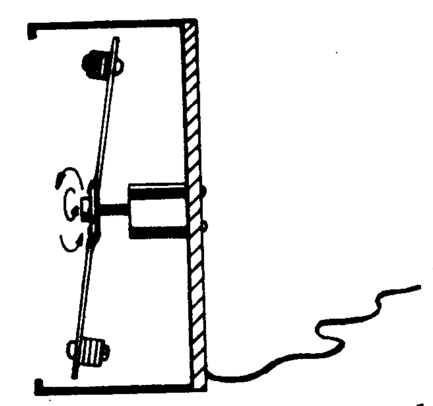
1. *Change the angular speed* so that the unbalanced reaction force no longer drives the vibration at its resonant frequency. Often this is not possible however.
2. *Change the resonant frequency.* This can often be done by mounting the system on a heavier base, that is change its mass.
3. *Mount the machine on some kind of damping mount* that absorbs the energy of vibration by friction. This prevents the amplitude of vibration from building up.

DYNAMIC UNBALANCE

UNBALANCED TORQUE IN ROTATION:

To explain static unbalance we looked at the forces present and how they acted on the bearings. To explain dynamic unbalance it is necessary to determine if these forces produce any torques.

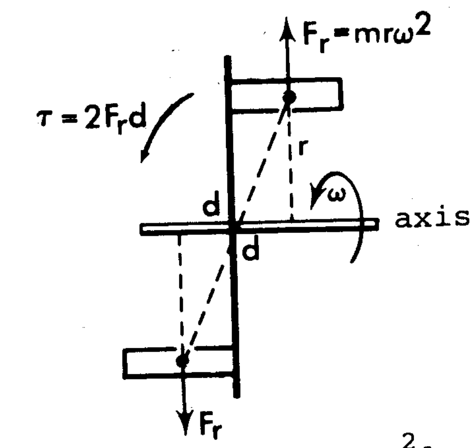
The rotor on the fan was a thin disk, almost a single plane. When it rotated, this plane was perpendicular to the rotation axis. All the reaction force lay in the plane of the disk and were also perpendicular to the axis. When you added washers to the disk, particularly when you added them to opposite sides, you began to give the disk thickness. With thickness, the analysis changes.



The dynamically unbalanced disk was seen to flex and wobble. If the disk were rigid, the wobble torque would all act on the motor bearings.

REACTION FORCES. . .

Consider the thin disk with two equal masses m added at equal radii r, but on opposite sides of the disk. At angular speed ω the reaction forces on these masses are both $mr\omega^2$. But, the reaction forces do *not* act along the same line, that is at the same point on the axis.



Wobble torque: $\tau = 2mr\omega^2 d$.

. . .PRODUCE WOBBLE TORQUES:

They are separated by a distance 2d. Two forces like these are sometimes called a *couple*. This couple clearly produces a torque whose magnitude is:

$$\tau = 2mr\omega^2 d.$$

This torque tries to twist the disc out of its normal perpendicular position, causing it to wobble as it turns.

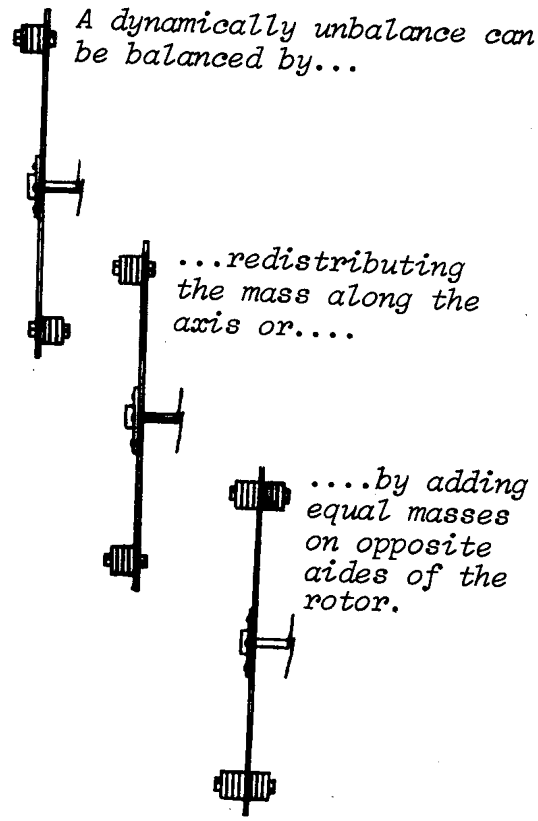
The disc in your experiment was flexible so it bent to adopt a tilted position as it turned. (Check your sketch of the dynamic unbalance experiment.) If the body were more rigid, then it would not bend. Instead the wobble torque would be transmitted to the rotational shaft and bearings to produce strain, wear, and vibration.

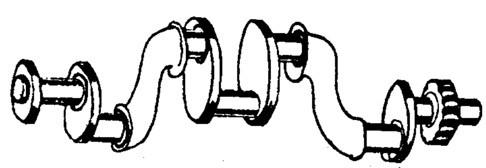
THE CURE FOR DYNAMIC UNBALANCE:

To eliminate dynamic unbalance one must either redistribute the mass of the rotor so that no couples exist, *or* add couples that are equal and opposite to the ones that are present.

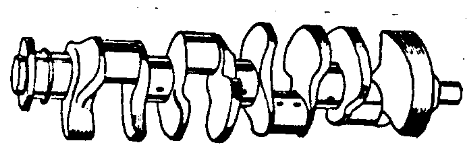
Adding a couple means to add a pair of weights (to preserve static balance) on opposite ends of a diameter and on opposite *sides* of the rotor.

In your first experiments on dynamic unbalance, either solution was clearcut since the unbalanced masses were large, you could *see* where they were, and you could easily move them. In your last experiment, as with most rotating bodies, the unequal distribution of mass was neither known or could it be moved. Thus a trial and error procedure had to be adopted to attempt to improve the problem.





Simple, dynamically unbalanced crankshaft of a 1930 Model A Ford, a slow turning engines...



Complex, dynamically balanced crankshaft of a modern high speed V-8.

SOME EXAMPLES:

Dynamic unbalance occurs most often in thick rotors, rotors that extend along the axis for some distance (d large), for example, tires, motor armatures and engine crankshafts. Unbalance effects increase with the square of the angular speed, so that crankshaft balance is of great importance in racing and high performance engines that turn at high speed.

There are many different techniques that are used to dynamically balance rotating bodies. In general each specific balancing problem leads to a specific technique to help reduce the guesswork. On the next page are a couple of examples of ways to reduce the number of trials and minimize the errors.

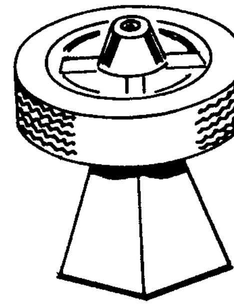
AUTOMOTIVE WHEELS:

Static balancers are essentially the same as the device you used to statically balance the disk. They are easy to use and are effective in most cases.

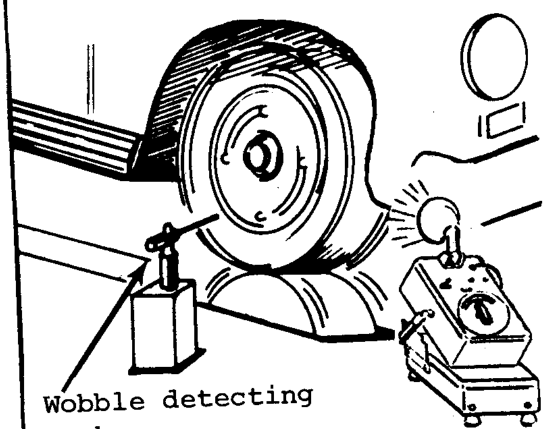
Auto wheels, especially front wheels, should be statically balanced. But wide tires, as any thick rotor, can be dynamically unbalanced, even though statically balanced. In fact, careless placement of static balancing weights may actually cause dynamic unbalance.

Automobile dynamic wheel balancers are essentially wobble detectors. The wheel is driven at high speed by an electric motor and the wobble amplitude is observed. One technique is to mount a small probe near the rim. As the wheel wobbles, the position of the probe, when the tire just strikes it, measures the wobble amplitude. The probe can be used to trigger a strobe and thus indicate where masses should be added on the wheel.

Static wheel balancing is simple. . . .



. . . .but dynamic wheel balancing requires complicated equipment and good judgement.

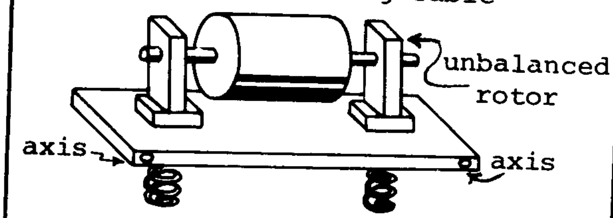


Wobble detecting probe

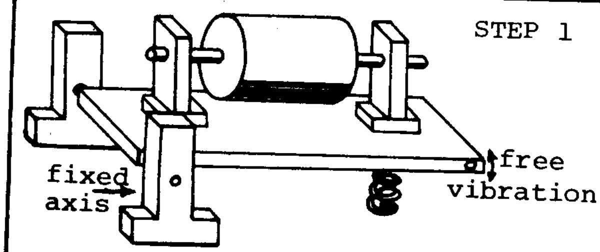
LARGE ROTORS:

The dynamic balancing of large rotors, such as large generator armatures, is essential if they are to run at high speeds. There is a fairly standard procedure that is followed for such rotors, provided you can get at both ends to add or remove mass. It utilizes a flexible table much like the one you used but the procedure is somewhat different. The procedure results in both static *and* dynamic balance of the rotor. On the next page this balancing procedure is illustrated.

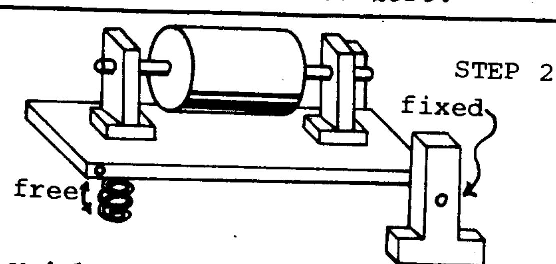
DYNAMIC BALANCING PROCEDURE:



The motor used to spin the rotor at the resonant bouncing frequency of the table is not shown.



Weights are added by trial and error to the right end of the rotor until the vibration amplitude is reduced to zero.



Weights are then added to the left end of the rotor. When the amplitude is zero, the rotor is both statically and dynamically balanced.

The rotating machine is mounted on a flexible table as shown. The table is mounted on two sets of springs. It can be pivoted about either of two axes. This arrangement has a resonant frequency of vibration and the amplitude can become large when the angular speed equals this frequency.

Step 1 consists in putting a shaft through 1 so that the table can pivot about that axis. The right end is then free to vibrate on the spring. The rotor is run and its speed is adjusted for maximum vibration amplitude - resonance. The rotor is stopped and mass is added anywhere on the right end plane. It is then run again, and the new vibration amplitude observed. Again it is stopped, the mass is moved one quarter turn around the rotor, and the vibration amplitude observed. This trial and error procedure is repeated until the vibration amplitude has been reduced to zero.

Adding only one mass has obviously statically unbalanced the rotor. Clearly this must be a two-step process.

In step 2 a shaft is put through the right hand table axis. The table now pivots about this, and vibrates at the left end. Note that this step is necessary in order to achieve *both* static and dynamic balance. The same trial and error procedure is repeated as before, masses being added to the left end plane. The object is again to reduce the vibration amplitude to zero.

Although the details are complicated, the basic idea of this procedure is simple. The table is a vibration amplifier that increases the vibration amplitude. By eliminating all vibrations you remove the reaction forces and the wobble torques.

REVIEW

SUMMARY:

Static and dynamic balance are important because unbalance can cause vibration and unnecessary large forces on bearings and moving parts. These lead to noise, rapid wear and breakdown.

Mass points on rotating bodies are acted on by centripetal forces. These act radially inward and are necessary to deflect the masses into circular paths. The forces are transmitted to the mass points by the rotor. But they must come from the rotor shaft and bearings. By Newton's third law there is an *outward* reaction force on the shaft and bearings. For each mass point this outward reaction force has magnitude.

$$F_r = mr\omega^2.$$

With a statically balanced rotor (one spinning about its CM) the net reaction force is zero. When a rotor is not statically balanced, the net reaction force is not zero. This force continually changes direction as the rotor turns. It puts strain on the rotor and bearings and can cause strong vibrations, particularly if the angular speed is the same as a frequency of natural vibration of the system.

Static balance exists when an object, supported at its center of mass, has no tendency to turn. That is, the net gravitational torque is zero. When rotating on the CM as axis, the rotor turns smoothly, without vibration.

Static unblance is corrected by adding masses at a position diametrically opposite to the CM. Static balancing machines use a level to indicate when the CM is at the point of support.

Degree of unbalance depends on the position of the CM with respect to the axis. The reaction force on the bearings is given by:

$$F_f = Mr\omega^2,$$

where M is the total mass of the rotor and r is the separation of the rotational axis and the CM.

Dynamic unbalance occurs when two masses on a rotor are located at two different points along the axis. The reaction forces on the two masses form a couple that results in a torque. This torque tends to make the rotor and the shaft wobble about an axis perpendicular to the rotation axis.

Dynamic unblance is corrected by adding pairs of masses in two planes separated along the axis. Pairs of mass are needed to maintain static balance.

QUESTIONS:

1. Suppose you cannot add masses to a statically unbalanced rotor, but were permitted to drill holes in it. What would you do to achieve balance?

2. State several important properties of CM.

3. The angular speed of an unbalanced rotor increased from 1000rpm to 2000rpm. How much are unbalance effects increased?

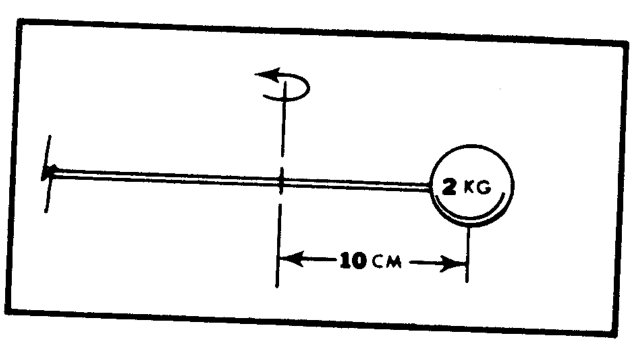
4. Could you have dynamic unbalance in a rotor made in the shape of a thin flat disk? Explain.

5. Can you suggest a reason why V-8 engines are now the most common, having completely replaced straight eights and even most sixes?

PROBLEMS:

1. A certian rotor has a mass of 10kg; its CM is 1cm from the axis. Where should you place a 0.1kg weight?

2. A certain rotor consists of a very light rod 10cm long with a 2kg mass on the end of it. The light rod also extends in the diametrically opposite direction. You have a 200g mass. Could you balance this rotor? How?



3. A rotor has a total mass of 5kg and its CM is 1cm from the axis. What is the reaction force on the bearings at 3000 rev/min?

4. The bearings on a certain rotor can withstand safely a reaction force of 100nt, about 20lb. The rotor has a mass of 10kg and the CM is 1mm from the axis. How fast can the rotor turn?

5. Dynamic unbalance results from two 100g masses at a radius of 30cm. They are separated along the axis by 2cm. Calculate the wobble torque at 300 rad/sec.