The Bicycle

A Module on Force, Work, and Energy

ISU

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TEACHER'S GUIDE

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THE BICYCLE
TEACHER'S GUIDE

INTRODUCTION

The Bicycle, a Physics of Technology module, is intended to give students an understanding of some basic concepts of physics involving force, work, and energy. The teaching strategy is, using a familiar device, to allow the student to try things, raising questions in the process, then gradually finding the answers to some of the questions. In this module, new ideas are always introduced by way of experiment with the bicycle and the questions thereby raised are gradually explored. Many (most?) of the experiments with the bike are definitely not in the high precision category, but they are usually convincing (in the 10% to 20% ballpark) and have the tremendous advantage of obvious relevance. Also, students seem to have come to expect ultra-high precision from physics, and it is good for them to see that one can learn a great deal from relatively casual observations. Students usually enjoy doing the module.

Because of the nature of my teaching experiences these past few years, I think the best way for students to do The Bicycle is mostly on their own, but with a teacher always available to answer questions and help--when asked. Some teachers won't feel comfortable with this style, and they might get better results from their students by providing more direction and structure. I hope this Teacher's Guide will be of assistance to both kinds of teacher.

GOALS

You will notice that the goals for this module are listed and illustrated at the end of each section. When working with other PoT modules, so many students complained about having the goals at the beginning--even though they were clearly to be met after doing the module--that this seems an appropriate concession. The questions accompanying the goals are illustrative of questions students might expect to see in an exam.

PREREQUISITES

To be able to handle this module efficaciously, students will need some basic math skills and understanding of a few important physics concepts. The prerequisite math skills include graphing, simple trigonometry involving sines and cosines, and finding the circumference of a circle of known radius. They will also need a knowledge of metric (SI) units. The physics concepts needed include velocity, the difference between weight and mass, gravitational potential energy, kinetic energy, and work. These topics are expanded upon in the module but the students should have at least a rough knowledge of them before they begin. Many students will have learned enough about these topics in high school courses, prior work in your course, or other PoT modules. The prerequisites self-test will give them a good idea of where they stand. Often students will get help concerning their areas of weakness from peers; this is certainly to be encouraged.

At any rate, it will require some judgement on the part of the teacher and of the student to know when one is ready to start the module. Don't be too demanding; it is better to start a bit underprepared and make up the deficiencies as they are discovered than to be discouraged from doing the module by being forced into a great deal of preparation.

TABLE OF CONTENTS OF THE MODULE

OVERVIEW OF THE MODULE

Section A. In this section, students learn about the nature of force and work. They learn that the work input to the bicycle depends upon the force applied to a pedal and on the direction of the applied force. They also discover that, except for small frictional losses, the work put into the bicycle in a static (slowly moving) situation is the same as the work output, regardless of gear ratio employed.

Then the students calibrate the speedometer with a stroboscope. They learn how a strobe can "stop" repetitive motion and some of its implications and about linear relationships between variables. The calibrated speedometer is used later in the module.

Section B. This section develops the concept of rotational kinetic energy, starting with an understanding of translational kinetic energy. Then an exploration of energy losses is begun by examining the losses to friction in the wheel bearings and in the tires. Conservation of energy concepts are used implicitly and explicitly.

Section C. The very important losses to air resistance are now explored by the students. Students make a fairly direct measurement of the force of air resistance and find that is proportional to the square of the speed, indicating that the air flow is turbulent.

There is included an optional experiment using the bicycle generator to measure another energy transformation, and energy transformations are discussed in general.

DISCUSSION OF ACTIVITIES

Experiment A-1. Comparing Work Input and Work Output. (pg. 9)

The purposes of this experiment are to demonstrate that, except for small frictional losses, the work input is always the same as the output and that the work depends both on applied force and on the direction of the force. In the ensuing discussion, students are led to the concept that $W = \vec{F} \cdot \vec{r}$, although it is never stated in these terms, and to the first hint of a conservation law.

The experiment may be done in several different gears. In each gear, the students hang a known weight (a lead brick or a "student body") from one pedal and measure the resultant force impelling the bike forward. They find that the impelling force varies with the hanging weight, with pedal position, and with gear ratio. By measuring the distance the bike must move forward for one slow rotation of the pedals--and implicitly taking into account the vector nature of the applied force and the pedal displacement--they find that the work input is roughly equal to the work output in every gear.

There is no difficulty in doing this experiment, but the concepts are fairly sophisticated and students will need to be sure they understand before moving on. This is a good time to start to impress on the students that the 8 or 10 digit accuracy they have available on their electronic calculators frequently is meaningless physically and that good, approximate measurements and calculations can often yield the information required.

Questions

A. 1. The impelling force is maximum when the pedal crank is horizontal and zero when the pedal is at the top or bottom positions.

2. See the graph of Figure 7, page 15.

3. There is a linear relationship.

4. The lower the gear ratio, the greater the impelling force.

B. Questions 1, 2, 3, and 4 are straightforward and require measurement and/or calculation.

5. This question is hinting at the vector nature of force and displacement: to be pursued in the discussion of this experiment.

C. Again, the questions elucidate the nature of vectors and the answers indicate that the work done is not simply force times distance.

Experiment A-2. Calibrating the Speedometer. (pg. 12)

In this experiment students learn to use a stroboscope to "stop" periodic motion and to interpret the results. They get some insight into the meaning of a linear function and into calibration procedures, in general. It should be impressed upon the students that, if one can draw a smooth calibration curve, the graphed line is more likely to be representative of the quantity being measured than any single data point. In this experiment, for example the actual speed corresponding to a speedometer reading of, say, 12 mph should be taken from the straight line rather than from an actual data point.

If there is time, some of your students might enjoy exploring the symmetry properties of wheel spoking by using the strobe. If the construction paper mask is removed and the entire wheel is illuminated by the strobe, the "stopped" configurations as the wheel slows provide some insight into the symmetry of the spoking. Of course, this same insight can be arrived at simply by examining the spokes carefully, but with the stroboscope it is more fun and easier.

Please caution your students to keep their fingers away from the apparently "stopped" but actually turning wheel.

Questions

A. page 13

1. When rpm = fpm, 2 fpm, 3 fpm . . ., the wheel is "stopped" with a single image, so it would be easy to confuse these cases. When rpm = 1/2 fpm, the wheel is "stopped" with a double image. Thus, "stopping" the wheel at a given rate, then doubling the flash rate is a good way to tell when rpm = fpm.

2. Two images, three images, etc., equally spaced.

page 14

1. Half a revolution minus the distance between adjacent spokes.

2. For 36-spoke wheels, the speed is 17/18 of the initial speed.

B. 1. Speed = (rotation rate) x (circumference), with the necessary conversions to get it into miles per hour (sorry about that, but it is still very difficult to find bicycle speedometers calibrated in km/hr). A sample calculation is done on page 21 of the module.

2. For a 36-spoke wheel, it is 17/18 v_o, 16/18 v_o, 15/18 v_o, etc.

C. Figure 15, page 20, was made from actual student data, showing the speedometer they used to be quite accurate and linear.

Mini-Experiment (page 19)

The final speed is surprisingly close to the same for trials in different gears, and students can learn about conservation of work-energy from the experiment. However, in the lower gears, of course, the final speed is reached much more quickly, thus illustrating that the rate of doing work--the input power--depends dramatically upon the gear ratio used. The interested student who is familiar with Newton's Second Law will realize that this comes about because, in the lower gear and with the same weight on the pedal, the impelling force is greater, thus producing a greater acceleration of the bike and rider.

Experiment B-1. Rotational Kinetic Energy of the Rear Wheel (page 23)

Here the students are gradually eased from the concept of translational kinetic energy to the concept of rotational kinetic energy. They learn the important point that there is no difference in the two; the use of rotational kinetic energy is just a convenient device which makes it easy to express the kinetic energy of a rotating object.

If possible, it is probably better for the teacher to give the mass of the rear wheel to the students; removing and replacing the rear wheel is a bit tricky and time consuming. (Although, it seems that every group of students that works on the module has at least one expert who wants to remove the rear wheel.)

In this experiment the weight hung from the pedal should be quite large, at least a 10-15 kg mass. A lead brick works well, two work better and, if the bike is rideable mounted on its stand, a student suddenly standing on a pedal, as described in the module, works best.

Questions

A. 1. Straightforward.

2. The maximum speed can be found with the use of the stroboscope. Easier, if you have a speedometer which can be attached to the rear wheel, is to use the speedometer. See the section of this Teacher's Guide on apparatus for advice on this point.

3. Straightforward.

4. Of course there are frictional effects involved, but if the students have done the experiment carefully, the calculated kinetic energy will be more than the work input. This is because the mass is not all concentrated at the rim and the contribution of a unit mass near the hub to the kinetic energy is much less than the contribution of a unit mass near the rim. One hopes that the students will think of this reason for the discrepancy at this point but, if not, let them go on. They will learn about it in the discussion of this experiment.

B. 1. The maximum speed of the rear wheel is about the same in each gear, except in the lowest gears, where it is much smaller. This effect is most pronounced when the weight used is a student and it is due to the fact that the weight cannot fall fast enough to "keep up," as explained in the discussion section of the module.

2. The rate of doing work--the power--is drastically different in the different gear ratios. This is because in the lower gear ratio the force acting on the mass to be moved is greater and thus the acceleration is greater.

C. 1. Straightforward calculation of $1/2\ \underline{m}\ \underline{v}^2$ for the added weights. Note that $\underline{v} \neq \underline{v}_{rim}$.

2. Since the wheel has not changed its mass or maximum speed, the kinetic energy of the extra weights is just equal to the extra work that needs to be done to attain the maximum speed.

3. Since the speed of the weight depends on the rotational speed of the wheel and the distance of the weight from the axle, the kinetic energy will decrease if the weight is moved closer in and the rotational speed is kept the same. At this point, some students are able to deduce that, if the distance from the axle to the weights is halved, the kinetic energy is reduced by a factor of four.

Experiment B-2. Energy Losses to Friction (page 25)

In this experiment students use conservation of energy concepts to get a measure of the frictional forces resisting the motion of the bicycle. They learn that the retarding forces produced by the bearings are very small and that the forces produced by the rolling resistance of the tires depend very strongly on the inflation pressure. If they do the experiments carefully, they can get a fairly good measure of this dependence on pressure and deduce its functional form.

<u>Questions</u>

A. 1. Here conservation of energy is invoked.

2. A straightforward calculation; the students already know the circumference of the wheel.

B. 1. Since they have been instructed to start with the same kinetic energy as the rear wheel had in Part A and since only the rear wheel is to produce drag in the "<u>gedanken</u> experiment," the answers will be the same as for question A2 above.

2. $\text{drag} = \dfrac{\text{initial kinetic energy}}{\text{distance rolled}}$

C. 1. The students need to know the mass of the front wheel. The bearing friction turns out to be less than that of the rear wheel, but in the same ballpark.

2. The same calculation as before.

D. 1. (Weight of rider) x (distance pedal drops).

2. Measured distance.

3. $\text{drag} = \dfrac{\text{work put into bike}}{\text{distance rolled}}$

This will turn out to be much greater than the drag of the wheel bearings and is mainly due to the rolling resistance of the tires. At these low speeds, air resistance is negligible.

4. The distance turns out to be nearly the same in all gears because the work input is the same.

E. 1. The lower the tire pressure, the smaller the distance the bike rolls and the greater the retarding force.

2. Subtract out the bearing forces. Remember to include both wheels. If the experiments is done reasonably well, the graph will look like Figure 32, page 39.

3. Most students' graphs come out to be a reasonably convincing straight line, with quite a bit of experimental scatter. (This is definitely not a high-precision experiment, but it is instructive.)

4. The frictional force would become very small. Steel tires would be a very good approximation to this.

Experiment C-1. Air Resistance (page 42)

This experiment yields very nicely the $\underline{v}^2$ dependence of the air resistance (turbulent flow) and shows that, as the speed increases, air resistance soon becomes the dominant resisting force.

Questions

A. 1. Figure 36 is taken from actual student data. Note that only the ratio of the two areas is necessary, not an absolute measure of frontal area.

2. The effective (frontal) area of the plastic sheet decreases as the sheet is deflected. Students are shown how to handle this complication on page 47 of the module.

3. Here students must use the calibration of the wind-force measurer, the area correction for the deflection of the plastic sheet, and the ratio of frontal area of the bike and rider to the plastic sheet.

B. 1. The graph turns out to be definitely not linear.

2. Figure 41, page 49, was made from actual student data. The $\underline{v}^2$ dependence of the force is very obvious and convincing.

Experiment C-2. Generator Power (page 45)

In this experiment the students learn how much work is necessary to turn the generator and how much of that goes into electrical energy. They calculate an efficiency for the production of electrical energy, but are warned that the useable energy (light) is very much less.

Questions

A. 1. Straightforward. Done as before.

2. The "force" calculated here is really the force slowing the bike down. Since it is applied near the rim of the wheel it can be calculated approximately by the method used in the earlier experiments and subtracting out the previously measured retarding forces.

B. 1,2. Obviously, it will be a larger force.

3. This usually comes out about 80%.

APPARATUS NOTES

This module can be done largely using equipment which is readily available or easily put together. Exceptions will be noted in the following pages. Sizes, for example the amount of mass to hang from a pedal, are mostly arbitrary, and the numbers quoted here are just some convenient ones. Substitutions may be freely made.

Experiment A-1. Comparing Work Input and Work Output (page 9)

Apparatus:

1. Multi-speed bicycle
2. Hanging weights - 10 kg or more
3. Spring balance - 0-20 N
4. Spring balance - 0-100 N
5. Heavy string
6. Meter stick

Experiment A-2. Calibrating the Speedometer (page 12)

Apparatus:

1. Bicycle with speedometer
2. Stand for bike
3. Driving motor and wheel
4. Continuously variable stroboscopic light

Notes:

1. For this experiment, any speedometer may be used. However, I have found it very convenient, if a bit expensive, to use the Erisman Pacemeter manufactured by Erisman Industries, Inc., because it can easily be adapted to the rear wheel. It is an electronic speedometer used by racing bicyclists because it is very light weight and it introduces no extra friction. Three small magnets are mounted on the spokes of the wheel, equally spaced and equidistant from the axle. A tape recorder pick-up head (i.e. a small coil) is mounted on the frame so that the magnets pass very near it as the wheel turns. The electronic circuitry converts the frequency of electric pulses into a speed which is read directly from a meter. The device is battery operated. One reason for the high cost is that the speedometer also incorporate a tachometer to measure the rotational rate of the pedals, a boon to racers in training, but not of much use to your students. The retail price was about $75 in 1976, but the manufacturer has indicated a willingness to offer educational institutions a discount. It may help to point out that it is being used for The Bicycle module. Information may be obtained from:

Erisman Industries, Inc.
521 S. Maguire Street
Warrensburg, MO 64093

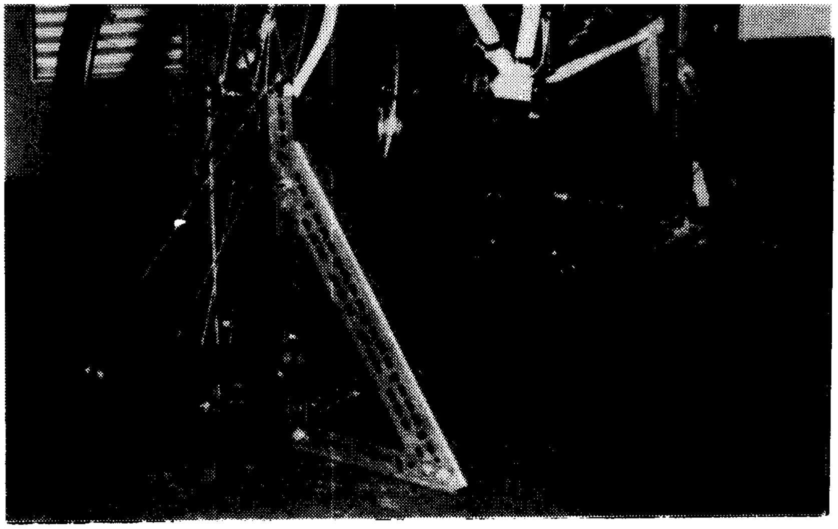


Figure 1: The bicycle stand

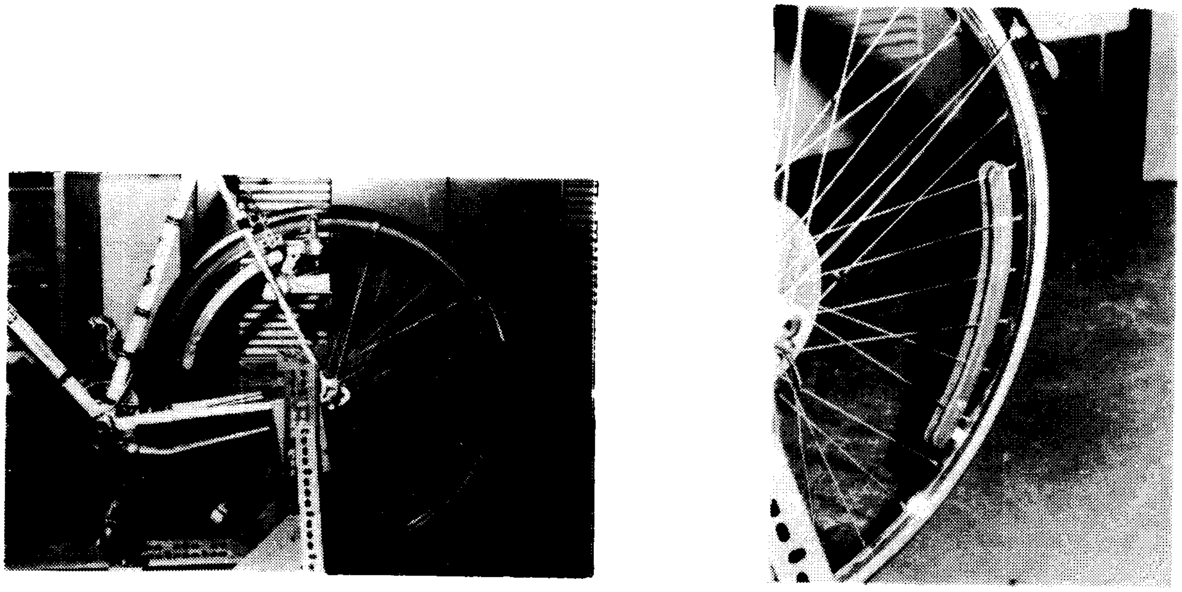


Figure 3: The extra weights attached to the rear wheel

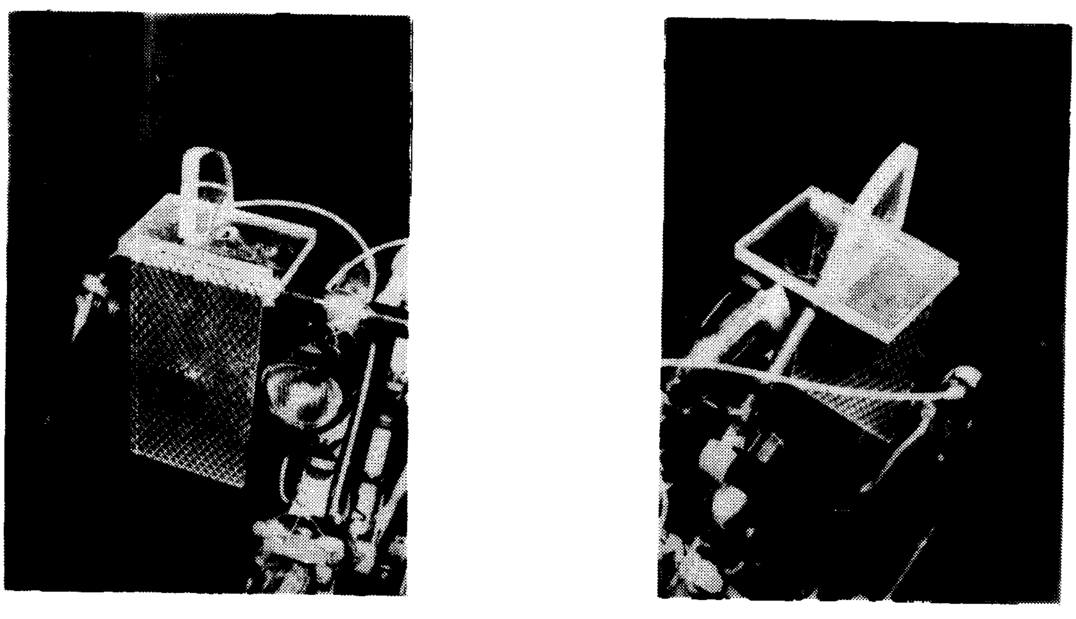


Figure 4: The "wind-force measurer"

Some electronic whiz may figure out a simple circuit that could be home-made by teachers. If so, please let me know. One alternative is to use the magnets and tape head with an oscilloscope. This works well with the bike on the stand, and can be used as an alternative way to calibrate a standard speedometer, but it has obvious problems when the bike is being ridden.

2. Any stand which holds one or both wheels off the floor will do. I used a home-made stand made of standard slotted angle iron (one trade name is "Dexion") which bolts together easily. The stand is shown in Figure 1, and the bike is bolted down to it by means of a turnbuckle, attaching the bike frame at the pedals to the stand. This stand is sturdy enough for students to "ride" the bike without any danger of tipping or collapsing. There are also available inexpensive repair stands which will support either wheel, but not a rider. They may be ordered at any bike shop.

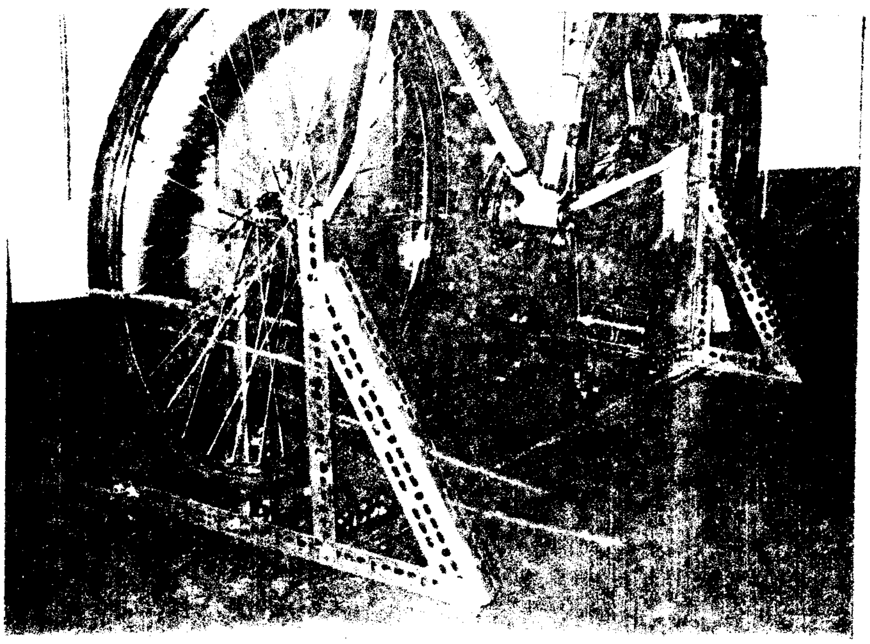


Figure 1: The bicycle stand

3. The driving motor and wheel can be any convenient motor and wheel. A standard (1725 rpm) motor (1/4 hp is plenty) with a 4-inch diameter drive wheel will drive a bicycle wheel at about 25 mph. I used a hard rubber wheel mounted on the motor shaft, but a V-pulley works well also. The motor should be on a stand which can be slid on the floor so that the drive wheel makes firm contact with the tread of the bicycle tire and then pulled back slightly to let the bike freewheel.

4. The strobe can be any which has a continuously variable flash rate.

Experiment B-1. Rotational Kinetic Energy of the Rear Wheel (page 23)

Apparatus:

1. Bicycle on stand
2. Balanced weights for bicycle wheel
3. Rear wheel speedometer or strobe light

Notes:

2. The weights used were home-made by pouring molten lead into a wooden mold. They are about 1/4 inch thick and have the dimensions shown in Figure 2. Each of the four weights has a mass of about .45 kg. The mold was made on a milling machine, but it can easily be made by cutting it out of 1/4 inch plywood and nailing that down on a board.

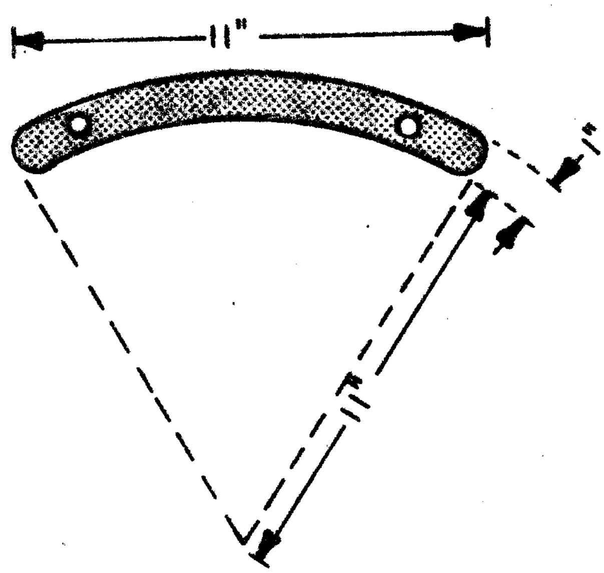


Figure 2: The dimensions of the added lead weights (thickness 1/4")

The weights are attached in pairs, one on each side of the wheel, and with the two pairs diametrically opposed. They are just bolted onto the wheel. This is shown in Figure 3.

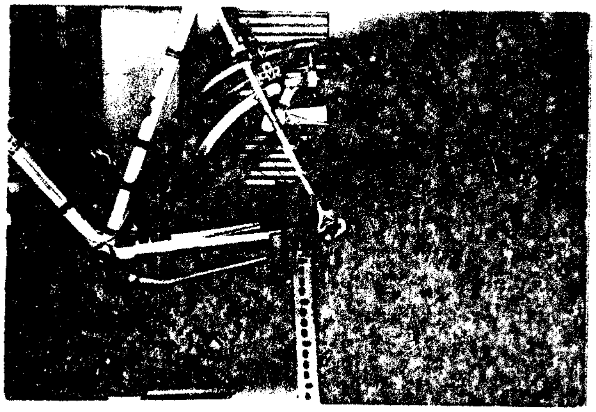
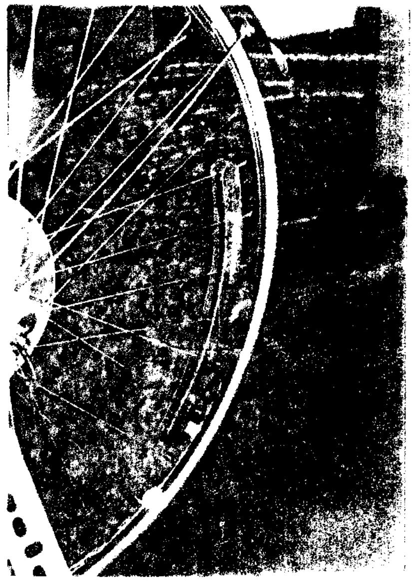



Figure 3: The extra weights attached to the rear wheel

3. The rear-wheel speedometer may be the electronic speedometer described earlier. I don't know of any other speedometers that can be attached to the rear wheel. Alternatively, the strobe light can be used to measure the maximum speed of the wheel, but this is more difficult since the wheel starts slowing down immediately.

Experiment B-2. Energy Losses to Friction (page 25)

Apparatus:

1. Bicycle and stand
2. Tire pump capable of 70 lb/in^2
3. Pressure gauge capable of 70 lb/in^2

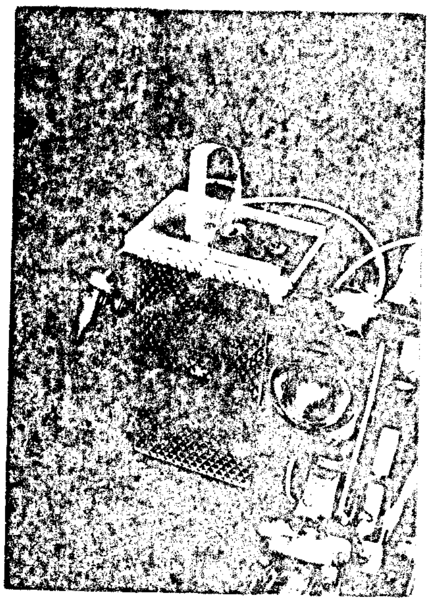
Experiment C-1. Air Resistance (page 42)

Apparatus:

1. Bicycle equipped with speedometer
2. "Wind-force measurer"

Notes:

2. Figures 4 and 5 show the "wind-force measurer", and Figure 6 is a shop drawing of it, as I built it.



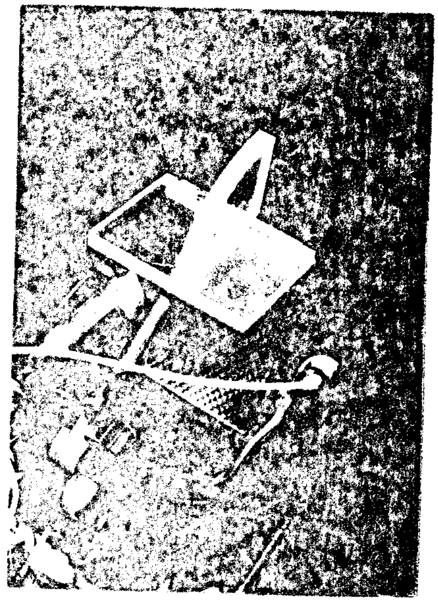


Figure 4: The "wind-force measurer"

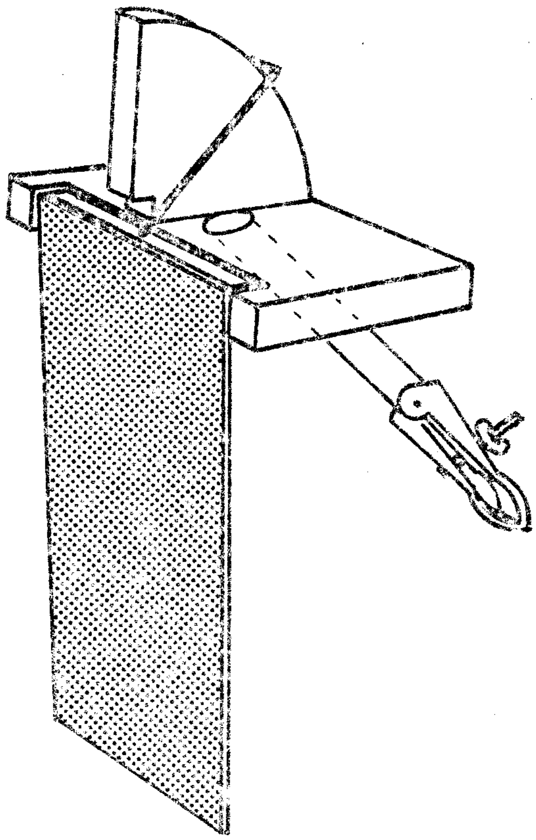


Figure 5: The "wind-force measurer"

WIND FORCE MEASURER

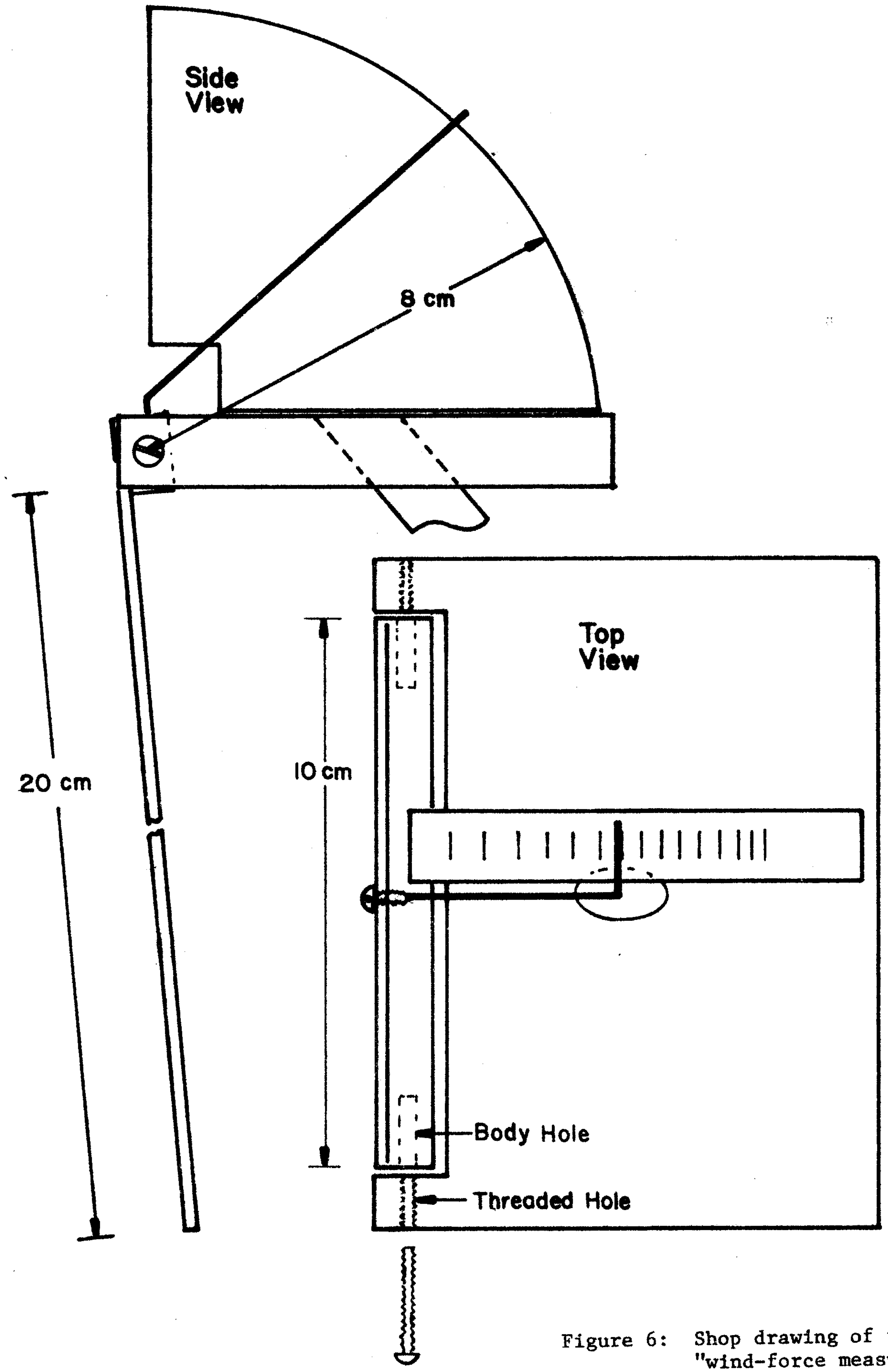


Figure 6: Shop drawing of the "wind-force measurer"

(Full Scale)

The gadget is made of plastic, and the hanging piece is a sheet of thin (about 3/32") plastic. This sheet is attached to a plastic bar, about 3/8" x 1/2" x 10 cm, which pivots on two screws coming in from the side.

None of the dimensions are critical! Use whatever sizes are convenient. Note that, when hanging at rest, the plastic sheet is not quite vertical. This could be remedied with a bit of fanciness in construction, but that is entirely unnecessary.

The device is attached to the bicycle by means of a standard laboratory clamp which fits the hole in the support piece. Keep the clamp as short as possible to avoid a long lever arm which shakes the device excessively when the handle bars jitter a bit.

Another helpful hint is to lay out the scale on a piece of paper, using $\underline{r}\ \theta$, where $\underline{r}$ is the radius of the curved piece on which the scale is pasted and θ is in radians. For $\underline{r} = 8$ cm, as in Figure 6, this comes out to about 0.7 cm on the scale for each 5° of deflection. To minimize parallax--the rider sees the scale from above--the zero point should be about where the pointer is shown in Figures 4, 5, and 6.

POST TESTS

Here are some questions you may--or may not--want to use for testing students' knowledge of the subject matter in the context of the module. I believe that multiple-choice questions are antithetical to the spirit and approach of this module, and so have included none. Perhaps, for expedient grading, you will wish to write some.

I must confess that I generally find other people's test questions inappropriate for my students; thus I usually write my own. But I offer these for whatever they are worth. I suggest that the student be allowed an open book and notes. Both tests are a bit long, and students will probably need about two hours to do each.

POST-TEST I

1. A bicycle with 27-inch diameter wheels is in a gear where the number of teeth on the crank sprocket is 52 and the number on the wheel sprocket is 26. The diameter of the circle the pedals move in is 12 inches.

   a. For each turn of the crank, with the chain always tight, how far will the bike move? (Don't count coasting.)

   b. If you stand with your full weight on one pedal, what will be the maximum impelling force given to the bike? (Use 60 kg for your mass.)

   c. If you stand with your full weight on one pedal, what will be the minimum impelling force given to the bike? Explain.

2. When the front wheel of the bike is "revved" up by an electric motor and running at a constant speed, it is found that a strobe light flashing at a rate of 800 rpm "stops" the wheel so that four equally spaced images of the valve stem appear stationary.

   a. At what rate is the wheel turning?

   b. What should the speedometer read? (27" wheel)

   c. If there are 36 spokes, with the nipples equally spaced, at what speeds should the strobe "stop" the nipples?

   d. How would you determine if the speedometer is linear?

3. In Experiment B-1, suppose that you (mass = 60 kg) suddenly stand on a pedal whose crank is initially horizontal, and that in so doing you cause the rear wheel to turn at a rate of 250 rpm.

   a. You now add an extra, balanced mass at a distance of 25 cm from the axle of the wheel. If the extra mass is 1 kg, how much higher must the pedal start in order to produce the same maximum speed as before?

   b. Suppose that the extra mass of 1 kg is now placed on the spokes a distance of 15 cm from the axle. How will the result be different from part a?

   c. If you did this experiment in the lowest gear ratio, you probably found that adding the weights to the wheel didn't make much difference in the maximum speed of the wheel when the pedal started from the same height. How do you account for this?

4. Suppose that you pedal the bike you worked with in the lab a distance of a kilometer (0.62 miles) in two minutes.

   a. How much work do you do against the friction of the wheel bearings? (Remember, there are two wheels.)

b. If the tires are fully inflated, how much work do you do against tire resistance?

c. If you have a following wind of 16 km/h (10 mph), about how much work do you do in overcoming air resistance?

d. If the end point is about 10 m higher in altitude than the beginning point, about how much work do you do against gravity?

5. a. For speeds at which one normally rides a bike, the force of air resistance is given approximately by:

$$\underline{F} = \underline{B}\,\underline{v}^2.$$

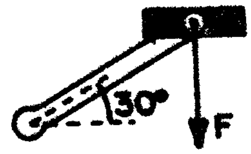
From your data, what is the value of $\underline{B}$? (Hint: Use your <u>linear</u> graph of $\underline{F}$ vs. $\underline{v}^2$.)

b. At approximately what speed is the force of air resistance equal to the other retarding forces acting on bike and rider?

c. The world's speed record on a bicycle is well over 100 mph. About what would be the force of air resistance at such speeds? (Thus, are such speeds possible? Obviously, yes, since there are these world records. How is it possible?)

POST-TEST II

1. You (mass 60 kg) stand on one pedal of the bike with the pedal in the position shown in the sketch. You have found by experiment that, when you produce the maximum impelling force in this gear, that force is 15 N.



   a. What is the impelling force produced?

   b. At what other position of the pedal will the impelling force be the same?

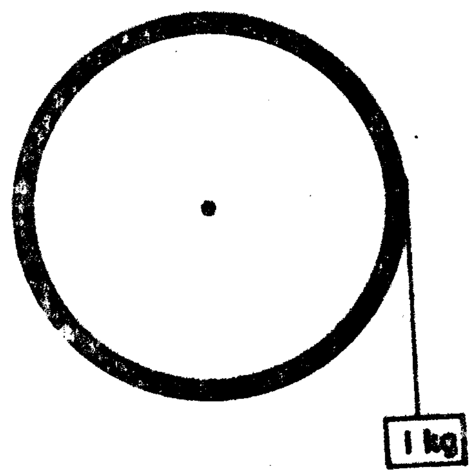
2. A bicycle speedometer is linear but not accurate. It is found experimentally that, when the bike is actually travelling at 20 mph, the speedometer reads 28 mph.

   a. Make a calibration graph for the speedometer.

   b. When the speedometer reads 10 mph, what is the approximate actual speed?

   c. If this bike, with 27-inch wheels, is put on a stand and the front wheel "revved up" until the speedometer reads about 32 mph, it is found that a strobe flashing at 900 fpm "stops" the wheel so that three equally spaced images of the valve stem appear stationary. At what rate is the wheel turning? (This is a bit tricky; there are two answers possible from the strobe information, but only one matches the speedometer information.)

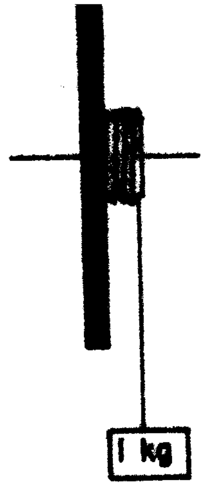
3. A 27" bicycle wheel is suspended high above the floor and a cord wrapped several times around its circumference. A 1 kg mass is tied to the cord and allowed to drop, starting from the position shown in the figure and accelerating the wheel in the process. Make the simplifying assumptions that the mass of the wheel is 2 kg, all of it concentrated at the rim, and that there are no frictional losses.



   a. When the 1 kg mass has fallen for enough that the wheel has made exactly two turns, what is the speed of a point on the rim of the wheel? (Hint: Use conservation of energy.)

   b. What is the rate of rotation of the wheel at that same moment?

c. Suppose a pulley is attached to the bicycle wheel and the same mass is suspended from a string wrapped many times around the pulley, as indicated in the sketch. When the hanging mass has dropped the same distance as it did in part (a), what is the speed of a point on the rim of the wheel?



d. How does the time required for the weight to drop in part (c) compare to the time in part (a)? Explain.

4. Suppose that you pedal the bike you used in the lab a distance of 5 km (3.1 miles) in 12 minutes and there is a headwind of 5 mph.

   a. About what is the net frictional force resisting your motion?

   b. About what is the force of air resistance?

   c. What is the net work you do on this trip?

5. a. Assuming that the speed is constant and the ground level throughout the trip of question 4, calculate the average power expended.

   b. With the same assumptions, calculate the power being expended at any instant.

   c. Suppose that the trip of question 4 is accomplished over the same distance in the same time, but that there are hills between the two end points and you do a lot of speeding up and slowing down. Discuss the work done and power expended on this trip, as compared to the level, constant-speed trip.

ANSWERS TO POST TESTS

POST TEST I

1. a. For each turn of the crank the rear wheel makes two turns. Thus

$$\underline{D} = 2\pi \text{ (diameter)}$$
$$\simeq 2\pi \times 27 \text{ in.}$$
$$\simeq 170 \text{ in.} \simeq 4.3 \text{ m}$$

b. $$\underline{F}_i \times \Delta\underline{D} = \underline{F}_t \times \Delta\underline{d}$$

For the maximum impelling force, the tangential force at the pedal is just the total force, $\underline{mg}$:

$$\underline{F}_i = \frac{\underline{m}\ \underline{g}\ \Delta\underline{d}}{\Delta\underline{D}}$$

$$= \frac{60 \text{ kg} \times 9.8 \text{ m/s}^2 \times \pi \times 12''}{2\pi \times 27''}$$

$$\simeq 131 \text{ N}$$

(Note: I used a full turn of the crank for convenience, using the same force throughout. If it makes you feel better, you may divide both numerator and denominator by 360 to get a 1^o turn of the crank.)

c. Zero.
At top dead center or bottom dead center of the pedal's travel, the force is perpendicular to the direction of pedal travel.

2. a. 200 rpm

b. $$\underline{v}_o = \left(200 \frac{\text{rev}}{\text{min}}\right) \times \left(27\pi \frac{\text{in}}{\text{rev}}\right) \times \left(\frac{1 \text{ mile}}{12 \times 5280 \text{ in.}}\right) \times \left(60 \frac{\text{min}}{\text{hr}}\right) \simeq 16 \text{ mph.}$$

c. at $8/9\ \underline{v}_o$, $7/9\ \underline{v}_o$, $6/9\ \underline{v}_o$ . . .

d. Plot a graph of actual speeds, as determined in parts (b) and (c) versus corresponding speedometer readings, and see if the curve is a straight line.

3. a. The kinetic energy imparted to the extra mass is

$$\underline{K.E.} = 1/2\ \underline{m}\ \underline{v}^2 = 1/2\ \underline{m}\ (2\pi \underline{r}\omega)^2$$

$$= 1/2\ (1\ \text{kg}) \left(2\pi \times 0.25\ \text{m} \times \frac{250}{60}\ \frac{\text{rev}}{\text{s}}\right)^2$$

$$\simeq 21.4\ \text{J}$$

$$= \underline{m}\ \underline{g}\ \Delta\underline{h}$$

$$\Delta\underline{h} = \frac{\underline{K}\ \underline{E}}{\underline{m}\ \underline{g}} = \frac{21.4\ \text{J}}{60\ \text{kg} \times 9.8\ \text{m/s}^2}$$

$$\simeq 3.6\ \text{cm}$$

b. $\underline{K.E.}' = \left(\frac{15}{25}\right)^2\ \underline{K.E.}$ for the same rotation rate

Thus $\Delta\underline{h}' = .36\ \Delta\underline{h} \simeq 1.3$ cm

c. The full explanation starts on page 33 of the module.

4. a. From data produced by my students, the retarding force produced by each wheel bearing was about 0.4 N. Thus:

$$\underline{W} = 2 \times (0.4\ \text{N}) \times 10^3\ \text{m} = 800\ \text{J}$$

b. Here the retarding force was about 1.7 N:

$$\underline{W} \simeq 1700\ \text{J}.$$

c. $\underline{v} \simeq 18.6$ mph, so the relative wind speed is 8.6 mph. From the data of page 48 of the module, the force of wind resistance is about 3.0 N and:

$$\underline{W} \simeq 3000\ \text{J}.$$

d. Assuming a mass of about 70 kg for bike and rider:

$$\Delta\underline{P.E.} = \underline{mg}\ \Delta\underline{h} = 70\ \text{kg} \times 9.8\ \text{m/s}^2 \times 10\ \text{m}$$

$$= 6860\ \text{J}.$$

5. a. For the data of page 48 of the module,

$$\underline{B} \simeq 0.2\ \frac{\text{N}}{(\text{m/s})^2}$$

b. From question 4, the other forces add to about 2.5 N. Then:

$$\underline{F} = [0.2\ \text{N}/(\text{m/s})^2]\ \underline{v}^2 = 2.5\ \text{N}$$

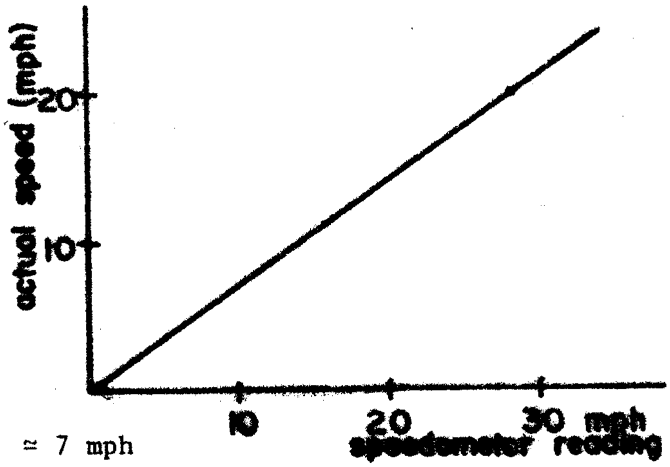
$$\underline{v} \simeq 3.5\ \text{m/s} \simeq 7.9\ \text{mph}.$$

c. $\underline{F} = \underline{B}\ \underline{v}^2 \simeq \frac{0.2\ \text{N}}{(\text{m/s})^2}\ (44.7\ \text{m/s})^2 \simeq 400\ \text{N}$ (One needs a wind shield.)

1. a. $\underline{F}_i = 15 \text{ N} \cos 30^\circ \simeq 13 \text{ N}$

   b. With the pedal crank 30° below the horizontal.

2. a.



Speedometer	Actual
0	0
7	5
14	10
21	15
28	20

   b. $\simeq$ 7 mph

   c. The two possibilities are rotation rates of 300 rpm and 600 rpm. That is, the wheel makes 1/3 and 2/3 revolutions between flashes, respectively. The speedometer reading corresponds to a speed of about 23 mph. At 300 rpm:

$$\underline{v} = \left(300 \frac{\text{rev}}{\text{min}}\right) \times \left(27\pi \frac{\text{inch}}{\text{rev}}\right) \times \left(\frac{1 \text{ mile}}{12 \times 5280 \text{ inch}}\right) \times \left(60 \frac{\text{min}}{\text{hr}}\right)$$

$$\simeq 24 \text{ mph}$$

   So 300 rpm is the correct answer.

3. a. $\Delta \underline{K.E.} = \Delta \underline{PE}$

$$1/2 \; \underline{m} \; \underline{v}^2 = \underline{mg} \; \Delta \underline{h}$$

$$\underline{v} = \sqrt{2\underline{g} \; \Delta \underline{h}} = (2 \times 9.8 \text{ m/s}^2 \times 2 \times \pi \times 0.69 \text{ m})^{1/2}$$

$$\simeq 9.2 \text{ m/s}$$

   b. $$\omega = \frac{\underline{v}}{2\pi \underline{r}} = \frac{9.2 \text{ m/s}}{2\pi \times .69\text{m}} \times 60 \frac{\text{s}}{\text{min}}$$

$$\simeq 127 \text{ rpm}$$

   c. 9.2 m/s. Conservation of energy demands that it stay the same.

   d. Part (c) will take longer. This is like changing gears to use a smaller sprocket on the rear wheel; the wheel makes more turns as the weight drops, but it accelerates at a lesser rate.

For the teacher:

$$t_c = \frac{t_a}{r}\left[\frac{m\,r^2 - M\,R^2}{m \;+\; m}\right]^{1/2}$$, where R and r are the radii of the

wheel and the pulley, respectively, and M and m are the masses of the wheel and the hanging weight, respectively. (A good, old massless pulley!) But I see no reason to inflict this on your students, as long as they understand the concept.

4. a. From the student data of page 41 of the module, $F \approx 2.5$ N for friction.

   b. From page 48, $F \approx 17$ N (20.5 mph) for wind.

   c. $W = F_{net}$ x distance $\approx$ 19.5 N x 5,000 m

      $\approx$ 97,500 J

5. a. $P_{ave} = \frac{W}{t} = \frac{97{,}500 \text{ J}}{720 \text{ s}}$

      $\approx$ 135 watts

   b. $P = F\,v = 19.5 \text{ N} \times \frac{5000 \text{ m}}{720 \text{ s}}$

      $\approx$ 135 watts

   c. The fact that the wind resistance changes with speed means that the total work done and the average power expended will be different from the preceeding calculations. If it were not for this, there would be no difference; the extra work done going up a hill is regained coasting back down and the extra work used to speed up the bike is regained as the bike is allowed to coast to its initial speed. However, even if the changing wind resistance is ignored, the instantaneous power needed will change as you go up and down hills or speed up and slow down.