The Bicycle

A Module on Force, Work, and Energy

ISU

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To Evelyn

Without whose constant encouragement and understanding nothing would be possible.

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PREREQUISITES

In order to be able to work through this module, you will need to start with some basic math skills and an understanding of a few physics topics. The skills include graphing, the definition of the sine and cosine of an angle, and finding the circumference of a circle. The topics in physics include metric (SI) units, velocity, the difference between weight and mass, gravitational potential energy, kinetic energy, and work. You may have learned these topics in a previous course or PoT module, or you may learn them now with the help of your teacher. One good way to learn those topics connected with work and energy is to study the PoT module entitled The Pile Driver. You may also find the module The Electric Fan helpful.

Whatever method you use, the following self-test will tell you whether you have the necessary prerequisite knowledge. If you can answer all of the questions, you are ready to proceed. If you have trouble with some of them, get help from your teacher or fellow students before starting the module.

Prerequisites Self-Test

1. The table gives the position of a bike and rider at various measured times.

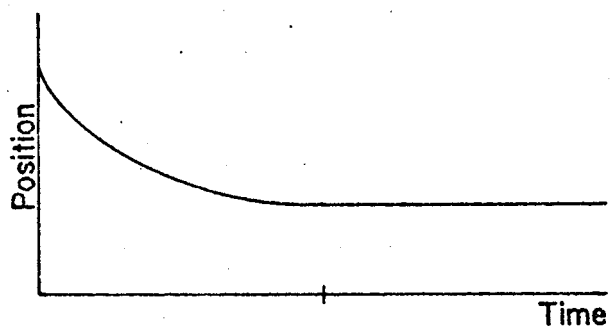
Position (m)	Time (s)
0	0
2.8	1
6.3	2
9.0	3
11.6	4
14.8	5
18.5	6
21.2	7

a. Graph the motion, that is, draw a graph of position versus time.

b. Is the graph linear (a straight line)?

c. What is the velocity of the bike?

2. Here is a different motion, given as a graph.

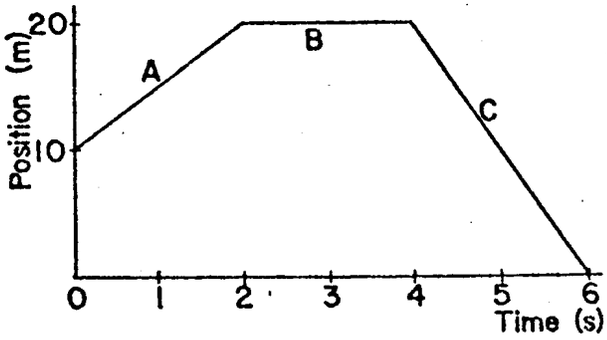


a. Is the graph linear?

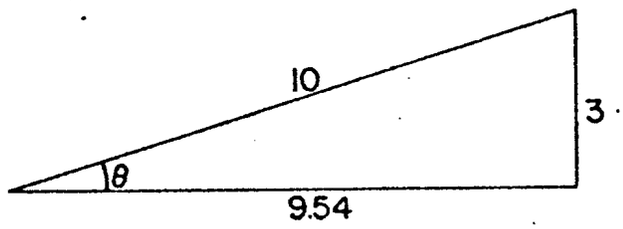
b. Is the velocity constant or changing?

c. What is the velocity for the latter half of the graph?

3. Here is yet another motion, shown graphically. What is the velocity (slope) in each of the three regions, A, B, and C?



4. The figure shows a right triangle, with the sides given in meters. What are sinθ and cosθ?



5. Suppose the sides of the previous triangle were in inches, but with the same numbers. (That is, a similar triangle.) What then are sinθ and cosθ?

6. A wheel 1 m in diameter is rolled without slipping along level ground until it performs 10 complete revolutions. What distance does it travel?

7. What are the SI units for mass? Force?

8. a. What is the weight of a 5-kg mass on the earth's surface?

   b. In outer space?

9. A 1-kg book is raised from the floor to a table top 1 m above the floor.

   a. How much work is done on the book?

   b. What is its change in potential energy?

   c. What is its potential energy on the table top?

10. The book is then nudged until it falls off the table.

   a. Just before it hits the floor, what is its potential energy?

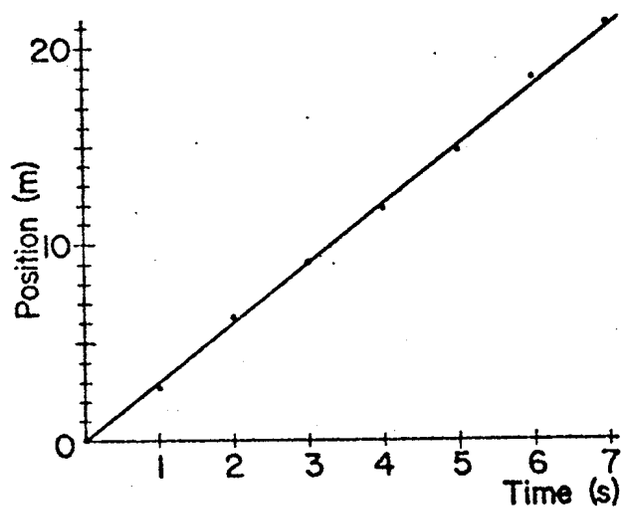
   b. At that point, what is its kinetic energy?

   c. What is its velocity?

   d. Where does this kinetic energy come from?

Answers to Prerequisites Test

1. a.



   b. Yes, approximately.

   c. $\underline{v}$ = 3 m/s.

2. a. No; only in the second half is it linear.

   b. Changing.

   c. $\underline{v}$ = 0 (It <u>is</u> constant during that time.)

3. Region A: 5 m/s.
Region B: 0 m/s.
Region C: -10 m/s.

4. $\sin\theta = 0.3$
$\cos\theta = 0.954$

5. $\sin\theta = 0.3$
$\cos\theta = 0.954$
(The size of the triangle makes no difference.)

6. 31.4 m

7. mass: kilograms (kg)
force: newtons (N)
$1\ N = 1\ kg\ m/s^2$

8. a. $\underline{Wt} = 5\ kg \times 9.8\ m/s^2$
$= 49\ N$

b. $\underline{Wt} = 0$

9. a. $\underline{W} = \underline{Wt} \times \underline{h}$
$= 9.8\ N \times 1\ m$
$= 9.8\ J$

b. $\Delta PE = 9.8\ J$

c. It is arbitrary. If one calls the potential energy at the level of the floor zero, then $\underline{PE} = 9.8\ J$ on the table.

10. a. Again, it is arbitrary. Let it be zero at the floor.

b. $\underline{KE} = \frac{1}{2}\underline{mv}^2 = 9.8\ J$

c. $\underline{v} = 4.4\ m/s$

d. As the book falls, the gravitational potential energy it had on the table top is gradually converted to kinetic energy.

THE BICYCLE

INTRODUCTION

The bicycle has been with us in various forms for quite a long time and it has had a far greater impact on human society than one might guess. Today it is assuming even greater importance in the affairs of man. In many countries, particularly in Asia and Africa, it is by far the most important means of transportation. Even in our own affluent society, we find people relying more and more on bicycles for basic transportation needs. Bikes are relatively cheap and readily available. In many cities, traffic problems are so severe that bicycles offer the quickest means of travel from point-to-point. They are non-polluting and actually healthful to the rider. And, perhaps most importantly, they are the most energy-efficient means of conveyance known. Not only do they not consume fossil fuels, electricity or nuclear power, but they use human-produced energy so well that a person on a bicycle is a more efficient transporter of mass than even fish or a horse, not to mention automobiles or airplanes. (During the 1976 Winter Olympics, the statement was made that ice skaters are more efficient than bicyclists in transporting mass. Whether or not this is true, bicycles seem a bit more practical for transportation purposes.)

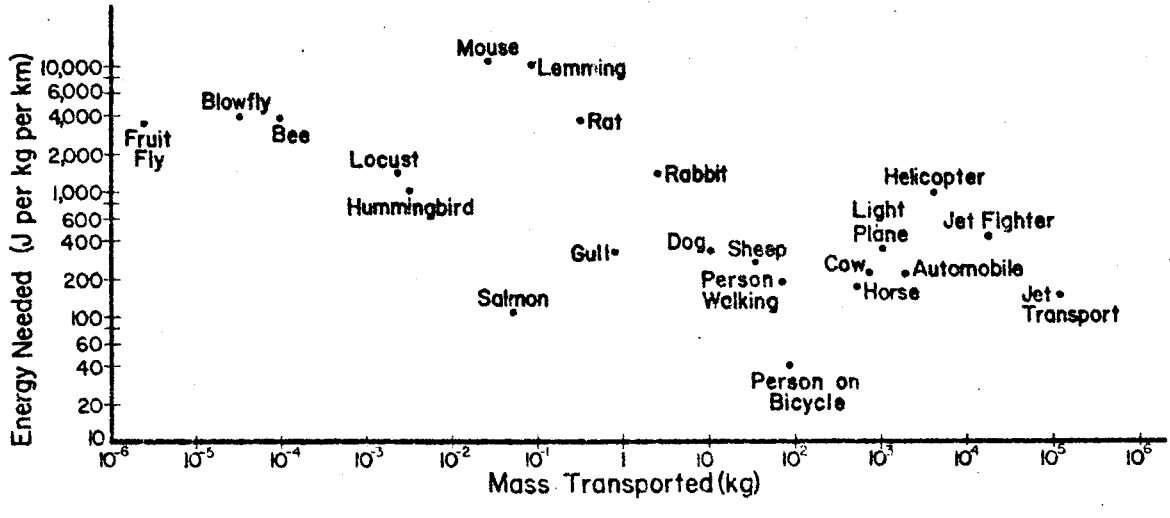


Figure 1. A person on a bicycle uses less energy to transport a kilogram of body mass one kilometer than does any animal or machine. (Courtesy Scientific-American.)

History

At about the same time Columbus sailed for the New World, Leonardo da Vinci was thinking about mechanical aids to locomotion. In addition to improbable flying machines, it is believed that he designed a foot-powered, land-traveling machine. However, more than three centuries elapsed before any such machines were built. In about 1816, a two-wheeled vehicle, propelled by pushing on the ground with one's feet, was invented. In 1839 a Scottish blacksmith made a bicycle which was driven by means of two pedals and a treadle mechanism, something like that of an old-fashioned sewing machine. In the latter half of the nineteenth century, the bicycle underwent numerous changes and improvements, including the development of the "high wheelers," those strange-looking vehicles with the huge front wheels. Such a large wheel was used so that each revolution of the pedals would take one a considerable distance, thus increasing the efficiency of the effort. Unfortunately, those bikes were rather unstable, especially on the rough roads of the day, and serious spills were common. In about 1885 in England, J. K. Starley pioneered a bike (called the "Rover") which resembled the modern-day bicycle, including a chain-and sprocket drive for efficiency. Although numerous important changes, such as changeable gear ratios, pneumatic tires, lightweight frames, brakes, and many others, have occurred since then, the basic design has remained the same.

Some Irrelevant but Interesting Facts

With bicycles, as with almost anything one can name, there have been those who felt compelled to make it biggest, fastest, smallest, widest, lightest, or some other superlative. For example, the world's longest bicycle was made in 1974 in Devon, England. It seats 32 people and is 61 feet 8 inches long.* On the other hand, in that same year a couple of Australian high-school boys made a ridable bicycle only 3 55/64 inches high.

In the realm of bicycle riding, the world records are even more astonishing. A bicycle with a very large front sprocket was ridden for a distance of three-quarters of a mile at an average speed of 140.5 mph in 1973. This was done behind a wind shield mounted on the back of a car, since air resistance at such speeds is extremely large. Unpaced—that is, with no means of reducing wind resistance—the greatest speed recorded is 42.21 mph for 200 meters (10.6 seconds). In terms of distance covered, in 1928 a Belgian traveled 76 miles, 604 yards in one hour, paced by a motorcycle. In 1932 an Australian covered a paced distance of 860 miles, 367 yards in 24 hours. Unpaced, the greatest distances traveled in one hour and 24 hours, respectively, were 30

*All of the records cited here were taken from the Guinness Book of World Records, Norris and Ross McWhirter, Sterling Publishing Company, (1976).

miles, 700 yards and 507 miles. The differences between paced and unpaced records give one an idea of the effect of wind resistance.

There are also records for cycling coast-to-coast across the United States, including a trip from Olympia, Washington, to Boston, Massachusetts, accomplished by an 11-year-old girl in 46 days during the summer of 1973. On the other hand, the champion slow-riding bicyclist is a Japanese who balanced on a stationary bicycle for 5 hours 25 minutes.

Impact on Technology

In addition to rather strange efforts like those mentioned above, the bicycle has been very important in the development of our present technological society. For example, the first really good paved roads were built in response to the needs of bicycle riders. Many of the other needs of automobiles were developed for use in bicycles. Among these are ball bearings, chain-and-sprocket drives, variable-speed transmissions, brakes, tubeless tires, spoked wheels, and even the differential gear which allows a car to turn corners without dragging one rear wheel. Many early automobile manufacturers began by building bicycles. The Wright brothers also got their mechanical know-how by working with bicycles. A lightweight, but strong, tubular frame, so important for early aircraft, was first developed for bicycles. If a practical manpowered flying machine is ever developed, it will surely be first cousin to the bicycle.

In addition, much of the technology of mass-production, which is so important to us today, was developed for the production of large numbers of bicycles. The manufacture, sale and maintenance of bicycles is now a very big business indeed, with nearly 40 million bicycles per year manufactured on a worldwide scale. Even a conservative estimate of cost puts the annual sales of new bicycles at two or three billion dollars, and the business done in parts, repairs, and maintenance is difficult even to guess at.

ABOUT THIS MODULE

In this module you will be learning mostly about work and energy by working directly with a bicycle. You will be introduced to new ideas primarily through experimental work with the bike. Often, questions will arise that are not answered immediately. The answers will be found in further experiment and in the written explanations that follow. Try to answer the questions asked in the module as you come to them; you will find that doing so will lead you to the answers to many of your own questions.

As you work through the module, imagine that you are an engineer working for a bicycle manufacturer. You know that the rider of a bicycle must do work in order to travel from point to point, but your assigned task is to design a bicycle which will

use that work even more efficiently. Before you can do so, you must understand how the work is used by present bicycles. That is, you will need to know where the energy input goes as the bicycle is being ridden. Learning that will be the main task of this module.

SECTION A

Force, Work, and Speed

EXPERIMENT A-1. Comparing Work Input and Work Output

For this experiment you will need a variable-speed bicycle with its tires inflated to the proper pressure, a variety of metric weights, and at least one metric spring balance.

A. Procedure: Holding the bike upright on the floor, hang a weight from one of the pedals. (See Figure 2.) Use a spring balance calibrated in newtons to measure the force impelling or pushing the bike forward. (Let's call this the "impelling force.") If you know your own body weight fairly accurately, you may do a trial by standing on one pedal while your partner steadies the bike.

Repeat this measurement for several different weights, for several pedal crank positions, and with the bike in at least two different gears.

Questions:*

1. For a given applied force (weight) and in a given gear, at what position of

*Throughout the module, the questions are intended to help you to think about the experiments. You may find that you cannot answer some of them completely until you have read the discussion following the experiment.

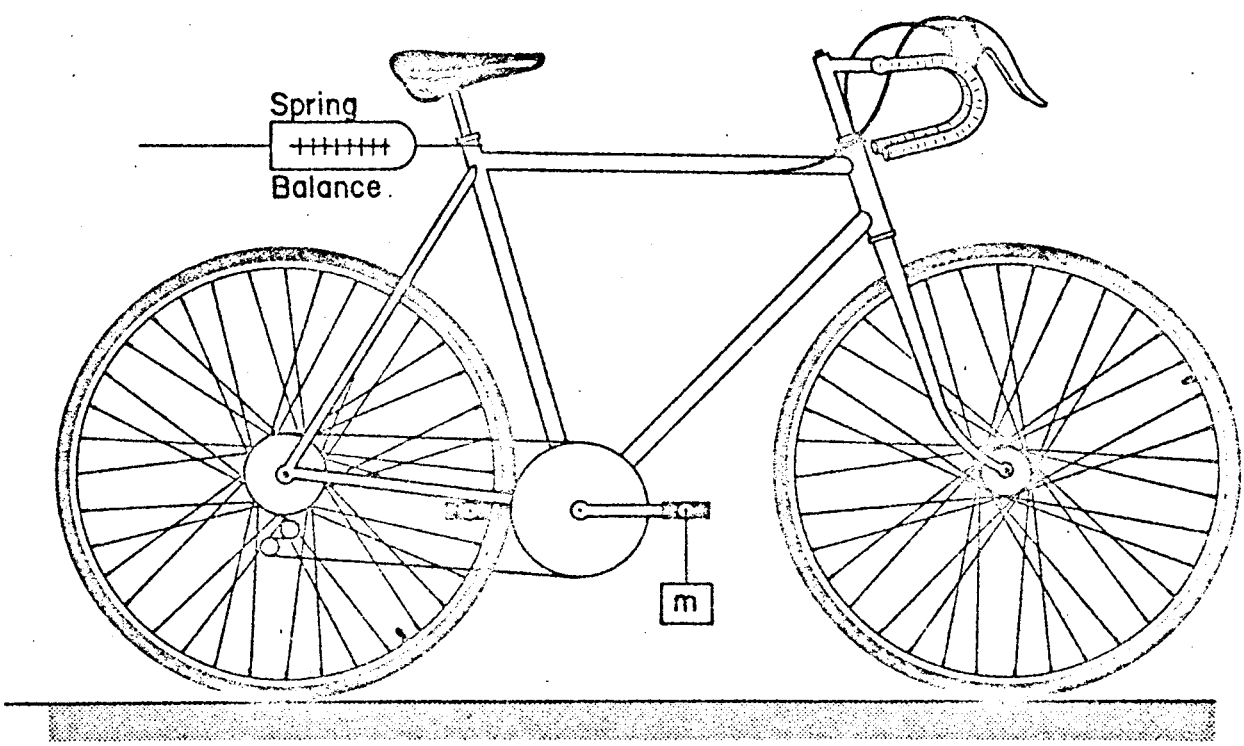


Figure 2. Hang a weight from one pedal and measure the impelling force produced.

the pedal is the impelling force maximum? Minimum? (What is the minimum impelling force?)

2. What is the appearance of a graph of the impelling force versus pedal crank position? (In fact, how will you measure pedal position?) If you can't answer this by thinking about it, try to plot an actual graph.

3. At the position of maximum impelling force for a given weight and gear, how does the impelling force vary as the weight is changed?

4. At the pedal position of maximum impelling force, how does the impelling force vary with different gears? That is, measure the impelling force with the bike in different gears, but with the same hanging weight. (Incidentally, to change gears, you must have the pedals turning. If you don't know about this, get some help.)

B. Procedure: With the bike in the same gear as for one of the previous trials, hold it upright on the floor and turn the pedals through one complete revolution, thus causing the bike to move forward. Measure the distance, D, the bike moves forward for one revolution of the pedals. Now compute the distance, ΔD*, that the bike moves forward when the pedals turn 1/100th of a turn (3.6°). The reason for doing this is that we would like to use the impelling force at each pedal position, but the pedal must move in order to do work. As a compromise, we think of the pedal moving a short distance about each position.

Questions:

1. As the crank turns 3.6°, what is the distance (Δd) the pedal moves in its circular path?

2. Using a force, F, equal to one of the hanging weights used in part A, what would be the force times the distance (F X Δd) the pedal travels for 3.6°?

3. Using this force and choosing a particular gear, what is the impelling force, F_i, for each of three pedal positions: with the crank vertical, with it horizontal, and with it halfway between?

4. Compute the work which would be done (F_i X ΔD) by each of these impelling forces in pushing the bike forward the distance ΔD that it moves with 1/100th revolution of the crank. How does F_i X ΔD for each case compare to F X Δd?

5. What might be a reason that the two calculations of work, F_i X ΔD and F X Δd, do not yield the same answers? (Note: Although it is very tempting to

*The Greek letter delta (Δ) is often used to denote small quantities or a change in a quantity. Thus, ΔD is a small distance moved.

blame inefficiencies in the chain and sprockets, these are not the villains. The losses there are quite small. It helps to think, in each case, about the direction of the force on the pedal as compared to the direction of the motion of the pedal.)

C. Procedure: Draw the path of the pedal (a circle), and find by careful measurement or by trigonometry that part of the force (the component) which is in the direction of the motion of the pedal. Figure 3 will give you an idea of how to do this.

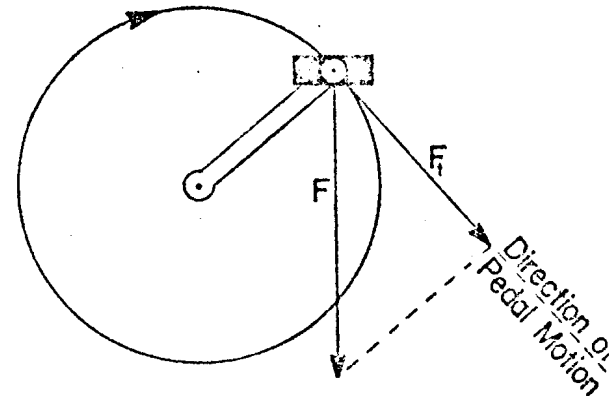


Figure 3. Finding the part of the force of the hanging weight which is in the same direction as the motion of the pedal at a given position.

Questions:

1. Calling the component of the hanging force which is in the direction of the motion F_t, how does F_t X Δd compare to the work done on the bike for a given pedal position?

2. What can you conclude about the work done by a force which is not in the direction of the motion? Can you write down an equation which is appropriate?

EXPERIMENT A-2. Calibrating the Speedometer

For this experiment you will need a bicycle mounted on a supporting frame, a stroboscope, and a motor with a driving wheel to drive the bike's front wheel. (See Figure 4.) The bike should have a speedometer to measure the speed of the front wheel.

A. Procedure: Mount the bike securely in the stand, with the wheels off the floor and able to turn freely. You will probably find that the front wheel is slightly out of balance and tends to rotate until the valve stem is at the bottom. Balance the wheel by wrapping some wire solder around the base of the spokes opposite the valve stem, until the wheel no longer tends to rotate.

Place the electric motor in front of the front wheel so that the drive wheel on the motor will cause the bike's wheel to rotate in the same direction it turns when the bike is moving forward. Turn the electric motor on and push it into position so that it drives the front wheel of the bike. "Rev up" the bike wheel to the highest speed possible.

With the front wheel going at full speed, "strobe" it until either the valve stem or a mark on the wheel is stationary. If the strobe light is blinking at such a rate that it produces a flash of light each time the stem is in the same position, the number of flashes per minute (fpm), as read from the strobe-light dial, will be the same as the number of revolutions per minute (rpm) of the wheel. You will probably find that under these conditions the strobe rate is so slow

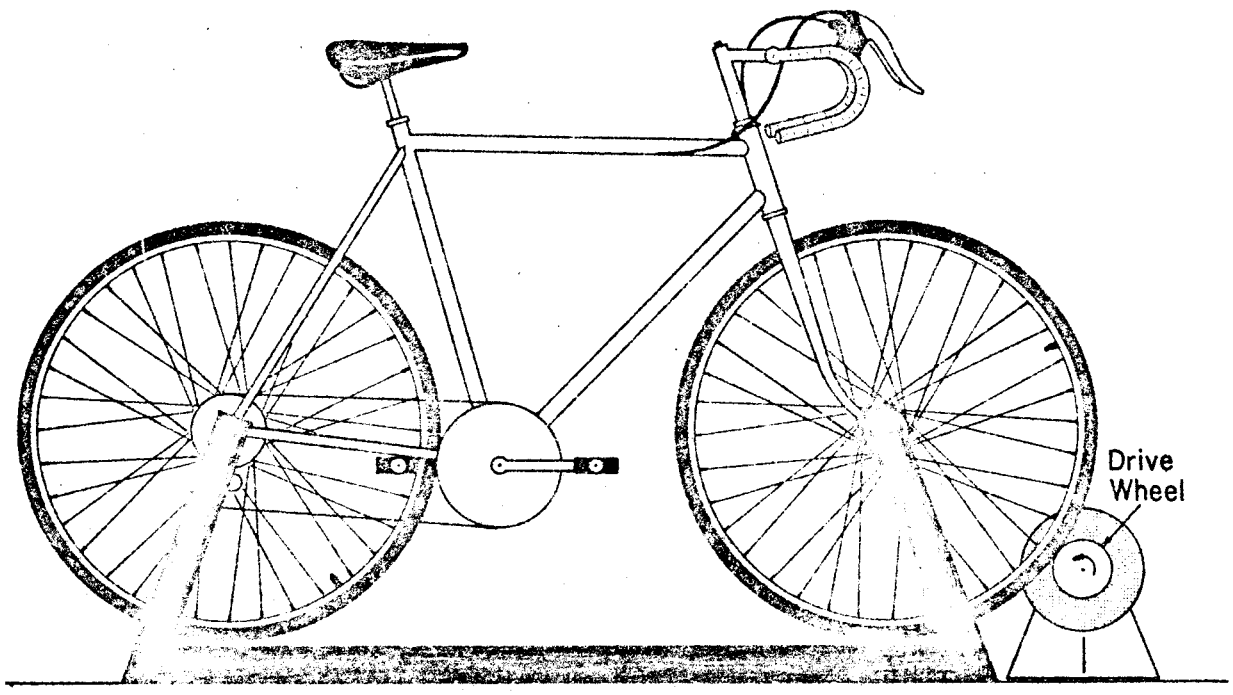


Figure 4. The front wheel is "stopped" with a strobe light.

that the image is difficult to make out. You can get around this by strobing at exactly twice the rate of turning.

CAUTION: Please be careful with this! When the wheel appears "stopped" by the strobe, it is awfully tempting to stick one's hand into it. This would get blood all over the spokes, because the wheel is not actually stopped.

Questions:

1. How can you tell for sure that the number of revolutions per minute is the same as the number of flashes per minute? (Hint: Change the fpm rate. At what rates is the wheel "stopped"?)

2. What do you see when the fpm is exactly twice the rpm? How about three times or four times the rpm?

With the front wheel going at full speed, read and record the speedometer reading. Now pull the electric motor away from the front wheel and let the wheel slow down, with the strobe light still flashing on it at the same number of fpm as before. At each instant the spokes "stop" momentarily, read the speedometer and record the reading. This will require one person to call out "now" at each appropriate moment and another person to read the speedometer. It also might take some practice.

Note: When you try this, you will probably get confused by watching the spokes. This is because they are "tangential" spokes, as on most modern bicycles, rather than "radial" spokes which go directly toward the center of the hub. Tangential spokes are arranged (Figure 4) so that, as the hub drives the wheel, the spokes pull rather than bend. With radial spokes, the only way to drive the wheel is by the sidewise (bending) force of the spokes.

To avoid the visual confusion caused by the spokes, mask off all but a few spoke nipples, as indicated in Figure 5. Black construction paper taped to the frame works well. Now, all you can see is the equally spaced nipples and the confusion is eliminated.

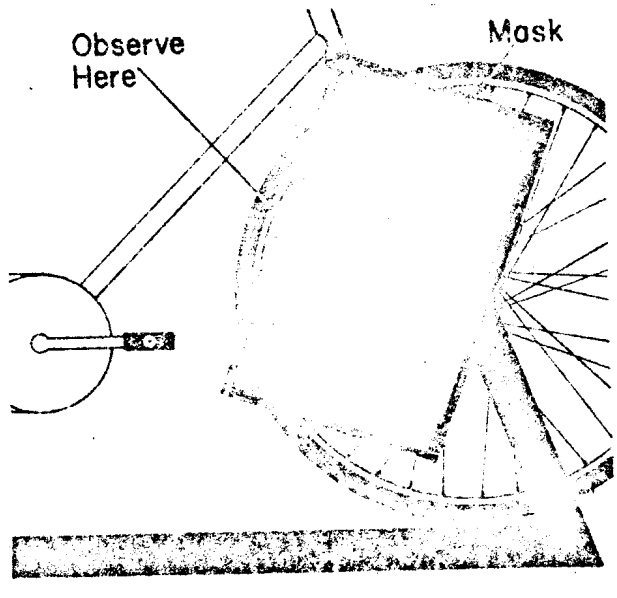


Figure 5. It is helpful to have a piece of construction paper masking off the spokes so that only a few nipples are visible.

Questions:

1. When the wheel is going full speed, the strobe flashes twice for each complete revolution. As the wheel slows down slightly, and the spoke nipples are "stopped" for the first time (but, the valve stem is not stopped), how far does the wheel turn between flashes? (Hint: It is less than half a revolution.)

2. At this instant, how does the rotational speed of the wheel compare to its speed when it was being driven? That is, what fraction of the full speed is it now running at? (Hint: How many spokes are there?)

B. Procedure: Measure the outer diameter of the wheel. Note that, when the wheel makes one revolution, the bicycle moves forward a distance equal to the circumference of the wheel. You can measure this by pushing the bike forward as the wheel makes one complete revolution, or you can compute it by taking π times the diameter.

Questions:

1. What is the top speed of the front wheel in miles per hour? (That is, if the bike were moving along the ground with the front wheel turning at that rate, what would be its speed?) How does this compare to the speedometer reading?

2. What is the speed of the wheel at each later time the spokes are "stopped" by the strobe?

C. Procedure: Plot a graph of the speed calculated for this last question versus the corresponding speed as read on the speedometer.

Questions:

1. Was the speedometer accurately calibrated in the factory?

2. Is the speedometer linear?

Discussion of Experiment A-1

Did you find that the maximum impelling force on the bike—for a given gear and a given hanging weight—resulted when the pedal crank was horizontal? For that position, and that position only, the force of the hanging weight acts in the same direction in which the pedal moves. Obviously, as indicated in Figure 6, when the pedal is at the bottom of its circle, the hanging force doesn't do anything except exert a force which is supported by the axle bearings. The same thing is true when the pedal is at the top.

What we know, then, is that in position A of Figure 6, the weight produces a maximum impelling force, and in positions B and C it produces no impelling force. How about the positions in between? If you drew your graph correctly, you found that the impelling

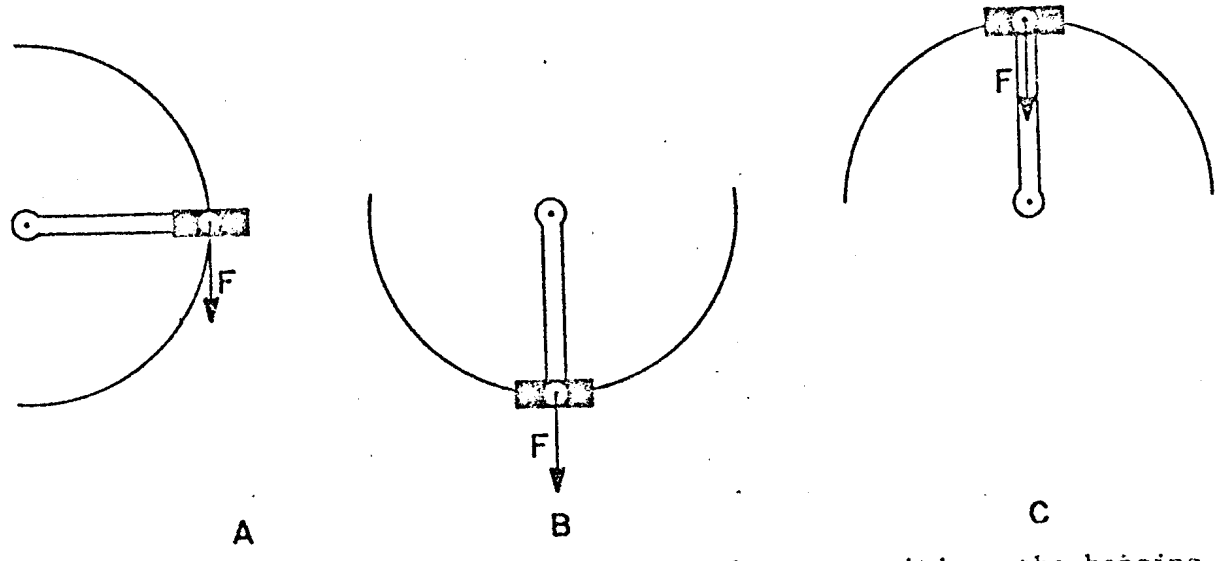


Figure 6. When the pedal is at its top or bottom position, the hanging weight produces no impelling force. When it is horizontal the impelling force is maximum for a given gear ratio and hanging weight.

force varies smoothly between those two extremes as shown in Figure 7A. The angle I used to plot this graph is measured from the horizontal position of the crank (See Figure 7). As the pedal moves from the top position to the bottom, this angle goes from 90°, through 0° at the horizontal position, and back to 90°. Just to keep them straight, I'll call angles above the horizontal positive and those below negative. Then the angle varies from +90° to -90°.

One way to sort this out is to consider the work done on the bike by the hanging weight. Letting the hanging weight move the pedal through a small angle moves the bike forward some distance. We use a "small angle" (3.6° earlier) so that the pedal motion is localized near some angular position in each case.

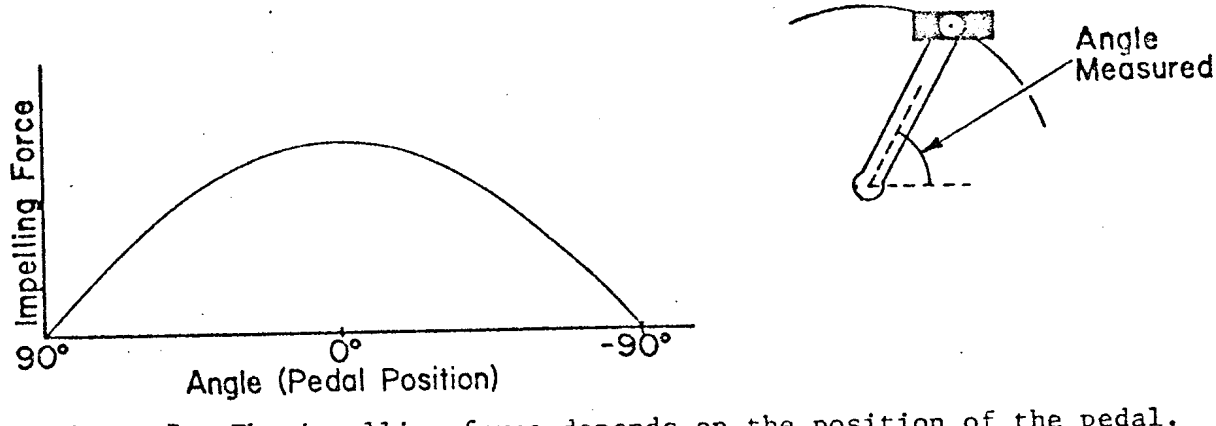


Figure 7. The impelling force depends on the position of the pedal. The angle is measured from the horizontal position of the pedal.

Then, in our thought processes, we can restrict the motion to a very tiny angle at each position. This avoids the complication of having the angle change appreciably during a given motion.

At any rate, if the pedal crank is slowly turned through a particular angle at each of many different positions, the forward distance the bike travels is the same each time. That is, the distance traveled by the bike doesn't depend on the position of the pedal, but only on how far the pedal moves along the arc. Since the work done on the bike is the impelling force, F_i, times the distance the bike moves, ΔD, and since ΔD is the same each time the pedal moves through the same small angle, the graph of work done on the bike versus the pedal position is indicated in Figure 8. In that figure, each rectangle represents a pedal motion of 3.6°, and the height of the rectangle is the work done, $F_i \times \Delta D$. If we then make the rectangles narrower and narrower, representing smaller and smaller angles, we end up with a smooth graph like that of Figure 9. Now let's look at the work input to the bicycle. You might guess that this would be the force (F) exerted by the hanging weight times the distance Δd the pedal travels along the arc. Since the weight F is constant, as is the distance Δd, this product does not vary as the pedal rotates. But the work done on the bicycle ($F_i \times \Delta D$) does vary as the pedal rotates. Apparently, the work done on the bike is not equal to the force exerted by the hanging weight (F) times the distance the pedal travels along the arc (d). Yet, in this orderly world, it is reasonable to think that the work input, except for frictional losses, should be the same as the work output. If this is the case, the work put into the pedal must depend upon the pedal position.

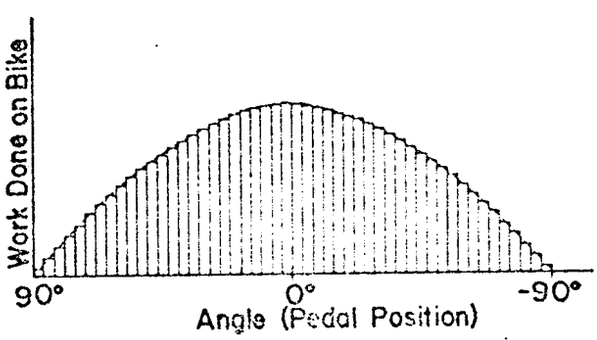


Figure 8. The work done as the pedal moves through a small angle depends on the pedal position.

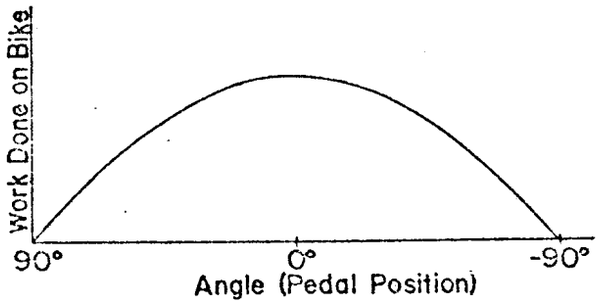


Figure 9. Work done on the bike depends on the pedal position.

The clue to what is going on involves thinking about direction. The force exerted on the pedal not only has a size, it has a direction—downward. The motion of the pedal also has a direction. It is tangent to the arc through which the pedal moves. When the pedal crank is horizontal (maximum work), the force and the motion are in the same direction. When the crank is vertical (zero

work), the force and the motion are at right angles to one another. Let's look again at these two quantities for some position of the pedal, as in Figure 10.

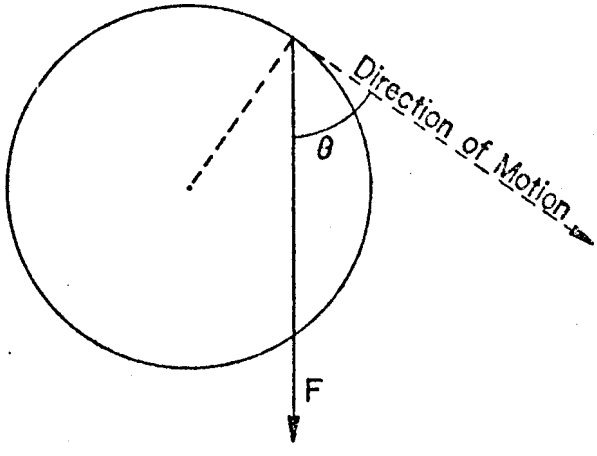


Figure 10. The directions of the motion and of the force differ by an angle θ.

As indicated in Figure 11, the force can be broken down into two parts, or <u>components</u>, one in the same direction as the motion and one perpendicular to that direction.* Using a little

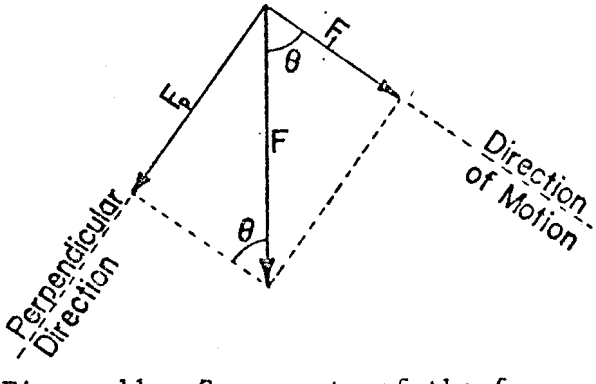


Figure 11. Components of the force.

*If this is new to you, you might like to stop for a minute and read Appendix A on vectors.

trigonometry on this gives the following results:

$$\underline{F}_t = \underline{F}\cos\theta$$

$$\underline{F}_p = \underline{F}\sin\theta$$

where $\underline{F}_t$ is the tangential component, the one in the direction of the motion. As before, when the pedal moves from the top position to the bottom, the angle θ goes from 90°, through 0° at the horizontal position, and then to -90°. A moment's thought should convince you that the angle θ shown in Figures 10 and 11 is the same angle θ defined in Figure 7. Getting out my handy-dandy calculator to find sinθ and cosθ for various angles, I am able to draw the two graphs in Figure 12.

Jackpot! The graph for $\underline{F}_p$ is not at all familiar, but the graph for $\underline{F}_t$ is precisely the same graph we already have seen a couple of times. One might very well suspect that the part of the force which is in the direction of the motion is closely related to the work done. In fact, whenever a force acts on an object, causing it to move, <u>only that component of the force which is in the direction of the motion does any work.</u>

Consider a simple example. Suppose you push a hockey puck along the ice with a stick, as indicated in Figure 13. The downward component of the force only serves to push the puck more firmly onto the ice; since there is no downward motion, that component does no work. The horizontal component, on the other hand, accelerates the puck, doing work in the process. The work done is equal to the horizon-

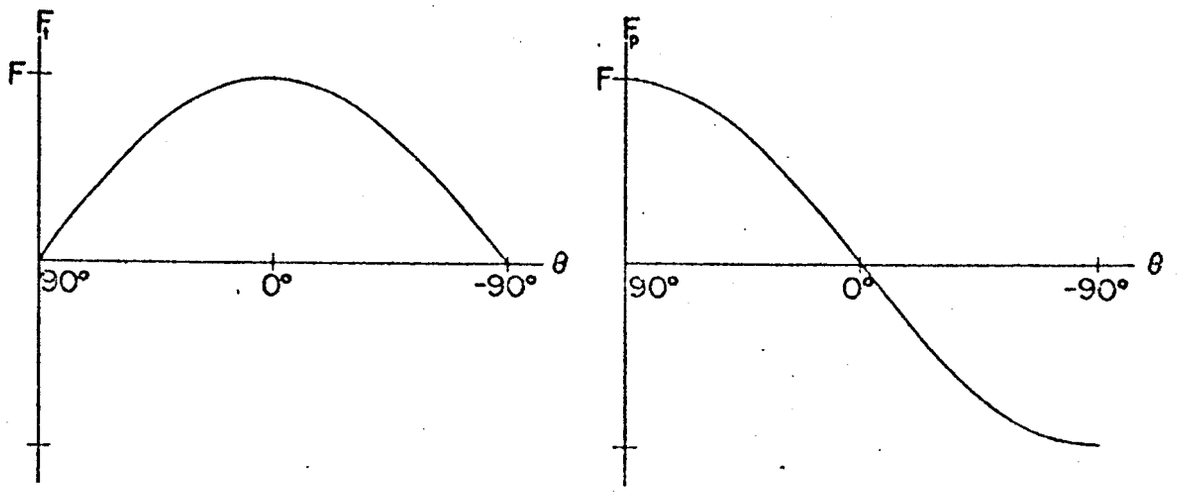


Figure 12. Graphs of the two components of the force of the pedal versus pedal position.

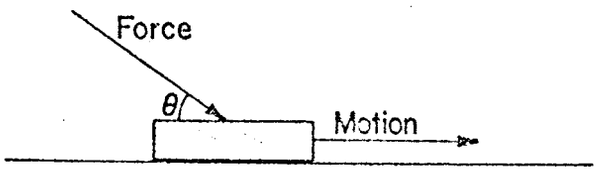


Figure 13. Pushing a hockey puck.

tal component of the force times the distance traveled. Using mathematical shorthand, this is ($F\cos\theta$) times d, where θ is the angle between F and d. Or, as it is usually written:

$$W = Fd\cos\theta$$

Mechanical Advantage

As you no doubt noticed in Experiment A-1, when you change gears, the same hanging weight—say at the horizontal pedal position—produces a different impelling force. The lower the gear, the greater the impelling force. Did you also notice that, in the lower gear, the distance the bike travels when the pedals move through a given angle is less than the corresponding distance in a higher gear? Take another look at your data from Experiment A-1. With the pedals in the horizontal position, what is the work done on the bike (F_i X ΔD) for each gear? You probably find that the answers are nearly the same for each gear. The small differences arise from experimental errors.

This result shows that, for a given force acting on the pedal, the impelling force depends on the gear the bike is in, but the work done for one turn of the crank is the same in every gear. If the gears are changed in a way that doubles the impelling force with a given force on the pedal, then the distance traveled for a turn of the pedals is halved, and the product of force and distance remains the same.

In other words, you can't get something for nothing. Whatever work you put in at the pedals—minus frictional losses in the chain, gears, bearings, etc.—is the same quantity of work which goes into the forward motion. What

you can control by changing gears is the impelling force you can produce. This helps in the following way: in the highest gear on your bike, you are able to produce some maximum impelling force, say 50 N, with the pedals horizontal and your whole weight on one pedal. You may even pull up on the handlebars to produce more force. This is fine when traveling downhill or on level ground. However, if you try a fairly steep hill, the 50-N impelling force is no longer enough to keep you moving. In the days of single-speed bicycles, this meant you had to get off and push the bike up hill. But, with a variable-gear bike, you may shift to a lower gear which increases the impelling force. However, you then pay the price of having to pedal more times to go the same distance. We say that you have gained in mechanical advantage—the ratio of the output force to the input force. (But you have not gained in the amount of work you must do.)

Mini-Experiment

If you have a multiple-speed bike equipped with a speedometer, you might like to try the following little experiment. On a smooth, level road, starting from a standstill, accelerate the bike by applying as much force as you can to each pedal in turn, say ten times. Notice what speed you attain. Now try the same thing in different gears. You may be surprised at how nearly the same the final speed is in each case. The work input is the same in each case, thus so is the resultant kinetic energy.

If you are particularly interested, you might try timing each of these trials. That is, in each case, how long does it take to go from zero to top speed? The answer might surprise you.

Discussion of Experiment A-2.

In this experiment, when you used a flash rate double the maximum rate at which the wheel turns, the wheel turned only one-half revolution between flashes. As indicated in Figure 14, this means that, any particular one of the spoke nipples appears to alternate back and forth across a diameter of the wheel with successive flashes. Of course, the flashes are of such short duration and come so close together that the spoke nipple appears to be stopped at each of the two positions.

Looking at this in a slightly different way, one can concentrate attention on a point in space near the rim where a spoke nipple seems to be stopped. In the time between flashes, exactly half of the spoke nipples pass the point of interest. For example, using a 36-spoke wheel, if nipple number 1 is at a given point at the time of the first flash, nipple number 19 will be at the same point at the time of the second flash and nipple number 1 will be back at the third flash.

When the wheel is allowed to slow down gradually, but the flash rate is kept the same, there is no longer a nipple at just the right place during successive flashes, and

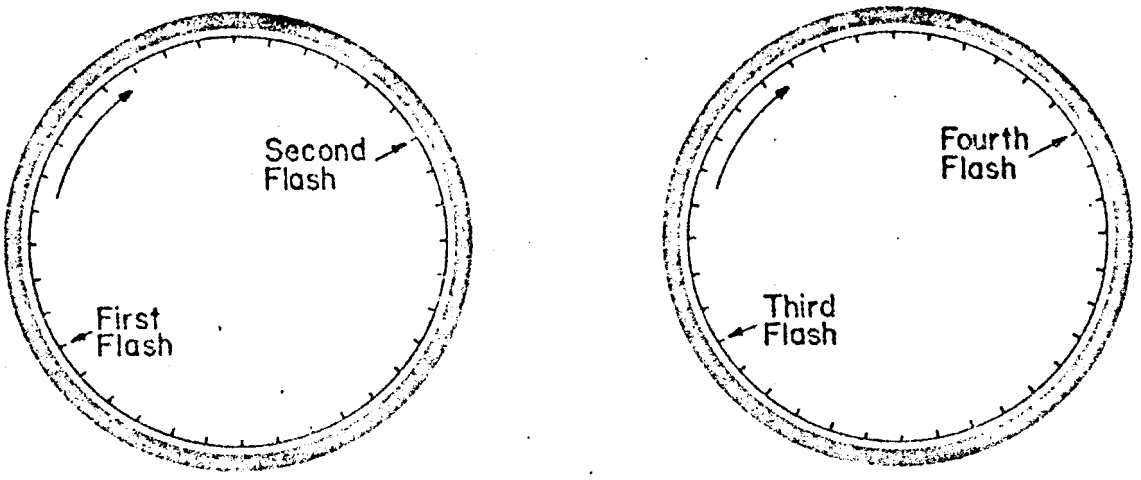


Figure 14. With the strobe flashing twice during each revolution, a spoke nipple seems to jump across a diameter.

the image blurs. However, as slowdown continues, the wheel reaches a speed at which nipple number 18 will be at exactly that position during a flash which was occupied by nipple number 1 during the preceeding flash. Once again, the nipples will appear "stopped," but now the wheel turns only 17/18 of a half revolution between flashes. As the wheel slows further, the nipples will again blur and eventually "stop," this time when there is 16/18 of a half revolution between flashes. This continues until the wheel actually does stop.* Since the amount of turning of the wheel between flashes is proportional to its speed, we may write the following equation for the speed, s_n, of the wheel at each instant that it is "stopped" by the strobe:

$$s_n = \frac{n}{18} s_{max}$$

*If the wheel is not well balanced, the irregularities in the motion may become confusing as the wheel speed nears zero.

Here s_{max} is the speed of the wheel as it is being driven by the motor, n is an integer ranging from 0 to 18, and s_n is the "stopped" speed corresponding to each n. Although we have not yet calculated s_{max}, we can now make a graph of speedometer reading versus n, which is proportional to the "stopped" speed. If this graph is a straight line, then the speedometer is linear, which it must be if it is ever to be calibrated accurately. A sample graph for a particular speedometer is shown in Figure 15.

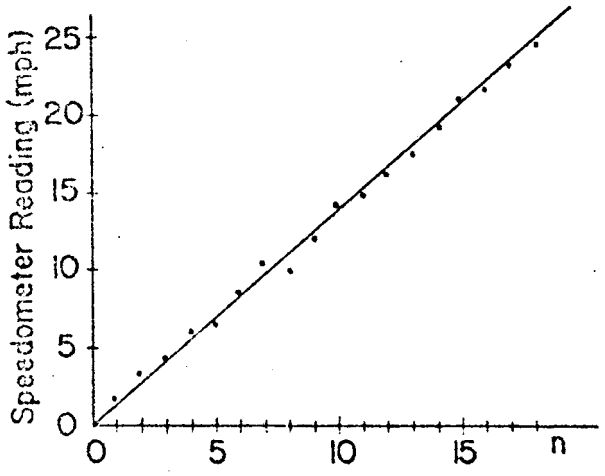


Figure 15. The speedometer represented by this graph is linear.

If your speedometer is linear, and thus useable, now is the time to see if it is accurately calibrated. That is, does the reading you get on the speedometer for any given speed correspond to the actual speed of the bike? If not, you will have to make corrections to any future speedometer readings.

If your speedometer is linear, any speed is as good as any other for checking accuracy, so you may as well use s_{max}. Then the rotation rate, in rpm, times the circumference of the wheel, in inches (π times the diameter), will give you the true speed in the strange units of inches per minute. Then you will have to change this to miles per hour. Below, I show how I did it for a wheel of 27-inch diameter which was driven at 320 rpm.

$$s_{max} = (\text{circum})\text{X}(\text{rotation rate})$$

$$= (85\text{in/rev})\text{X}(320\text{rev/min})$$

$$= 27{,}200 \text{ in/min}$$

This answer is not terribly helpful; we need to change it to miles per hour:

$$s_{max} = (27{,}200\text{in/min})\text{X}(1\text{ft}/12\text{in})$$

$$= (2{,}267\text{ft/min})\text{X}(1\text{mi}/5280\text{ft})$$

$$= (0.429\text{mi/min})\text{X}(60\text{min}/1\text{hr})$$

$$= 25.7 \text{ mph}$$

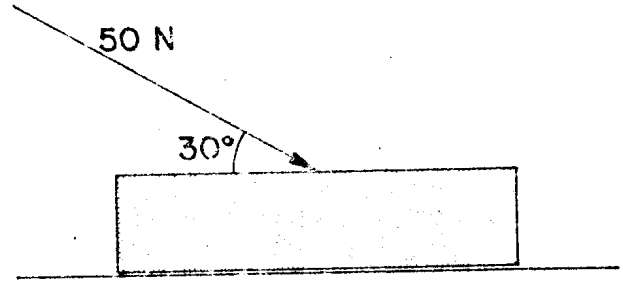
In this case, the reading on the speedometer was about 25 mph, so the calibration was reasonably good. How good is your speedometer?

GOALS FOR SECTION A

The following is a listing of the learning goals for Section A of the module. When you have completed the section you should have an understanding of the principle or technique specified in each goal. The question which accompanies each goal is a sample exam question. If you can answer it correctly, you probably have a good understanding of the required information. The answers to the questions appear at the back of the module.

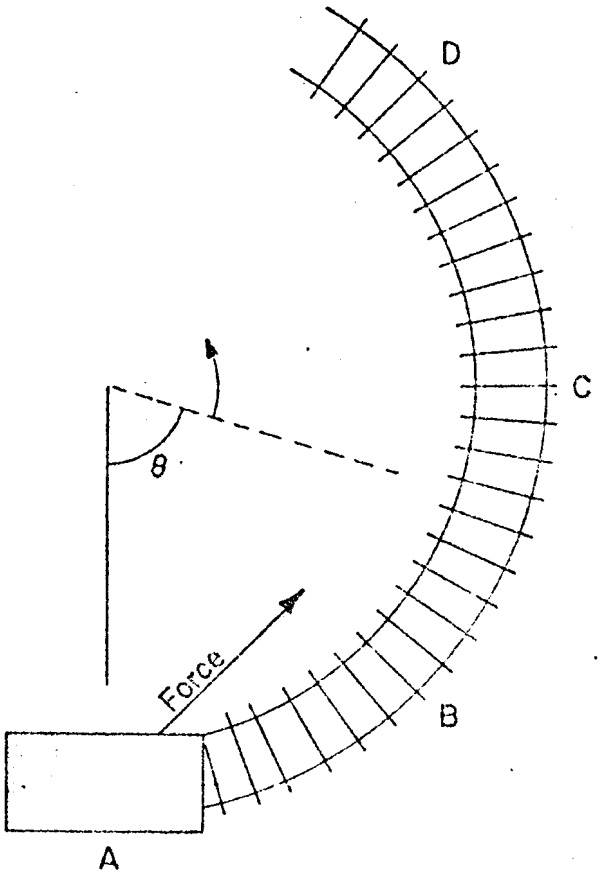
Goal 1. To be able to find the components of a force (or any other vector quantity) along two mutually perpendicular directions.

Question: If you push a hockey puck with the force shown, what is the component of the force parallel to the surface of the ice and what is the component pushing the puck down onto the ice?



Goal 2. To understand how work done depends on the directions of the force and the displacement.

Question: Suppose you push a toy train around part of a circular track with a constant force which is always in the same direction, as indicated in the figure.



a. For a given distance moved (say 1 cm), at which point(s) on the track is the work done the greatest?

b. At which point(s) is it least?

c. Sketch a graph of work done per cm of motion versus the angle θ.

Goal 3. To understand the principles of strobing to "stop" repetitive motion.

Question. A printer prints labels on a long, continuous strip of self-sticking paper at a very high speed. As the labels come off the machine and are rolled onto big spools, it is desired to spot check them with a strobe light. Suppose that the speed at which the paper runs is 4 m/s, that the labels are 5 cm apart and that every tenth label should be illuminated at the same position, making the paper appear "stopped." What should be the flash rate?

SECTION B

Energy and Frictional Losses

In this section you will look at how the work put into the bike by exerting forces on the pedals is converted into kinetic energy, and what happens to that kinetic energy as the bike moves.

EXPERIMENT B-1. Rotational Kinetic Energy of the Rear Wheel

For this experiment the bicycle should be mounted on a stand to raise the rear wheel. You will need a set of balanced weights which can be mounted on the rear wheel near the rim, and a means of measuring the maximum speed of the rear wheel. This latter might be a speedometer or a stroboscope.

A. Procedure: With the bike on the stand, and with the pedal crank horizontal, allow a weight to pull the pedal down to the bottom position, causing the rear wheel to turn in the process. Measure the maximum speed attained by the rear wheel.

The weight used to push the pedal down may be a chunk of lead or standard masses. However, you may find it easiest to use your own body weight. You can do this by balancing your weight on the two pedals with the cranks horizontal, then suddenly lifting your back foot, allowing all of your weight to act on the forward pedal. Try not to lean on your hands very much.

Questions:

1. How much work did you put into the bike on each trial? (Remember that the work done is just the weight times the height through which it falls.)

2. What is the maximum speed of the wheel for each trial?

3. Take the wheel off and find its mass (or, get the figure from your teacher.) Making the approximation that all of the mass is located at the rim, what is the kinetic energy of the wheel at its maximum speed?

4. Is the kinetic energy you calculate this way equal to the work put into the bike? If not, how do you account for the discrepancy?

B. Procedure: Repeat the experiment using one or two different gears.

Questions:

1. How does the maximum speed of the wheel compare when different gears are used? How do you explain this result?

2. Can you distinguish any difference in the rate at which you do the work? That is, when you stand

on the pedal, does it fall equally fast for the different gears?

C. Procedure: Now add the extra weights to the wheel near the rim. As indicated in Figure 16, use balanced sets and place them opposite one another so that the wheel remains reasonably well balanced. Be sure you know the masses of the added weights and their distances from the axle. Repeat the experiment using the gear you first used.

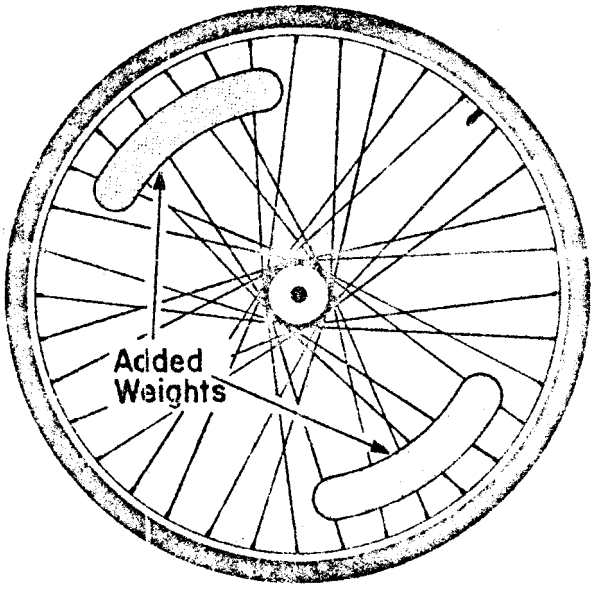


Figure 16. Adding weights to the rear wheel increases the kinetic energy at a given speed.

You will probably find that the maximum speed attained by the wheel is less than it was without the added weights. To compensate for this, add enough extra work so that the weighted wheel reaches the same maximum speed the unweighted wheel did. You can do this in one of two different ways: Either add more weight to the weight pulling the pedal down, or start the pedal from a higher position with the same weight acting. If you did the first part by standing on the pedal, you will find the latter method much easier to do.

Questions:

1. At the maximum speed of the wheel, what is the kinetic energy of the added weights?

2. How does this kinetic energy compare with the extra work put into the bike to get the wheel up to the same maximum speed?

3. What difference would it make if the weights were attached to the wheel at a place which is nearer to the axle? (You may want to try this as an optional experiment.)

EXPERIMENT B-2. Energy Losses to Friction

Just to keep a bicycle moving at a constant speed on level ground requires a continual output of energy on your part. In this experiment, you will find out where some of that energy goes. You will need the bicycle, the stand to lift it off the floor, a tire pump, a good pressure gauge, and a smooth, level place to ride the bike. For this last, either a long corridor or a gymnasium floor works best, if available. If not, a paved, level parking lot works well on a day with no wind. If there is wind, try to keep it at your side, or do the trials in pairs—one with and one against the wind—taking the average of each pair.

A. Procedure: With the bike on the stand, do work on the rear wheel, as before, by letting a weight push the pedal downward. (Alternatively, you can get it up to a known speed with the motor you used earlier on the front wheel.) Record the maximum speed of the rear wheel and count the number of revolutions the wheel makes in slowing from that maximum speed to a stop. For this purpose, it will help to place a piece of masking tape on the tire as a marker.

Questions:

1. What is the maximum kinetic energy of the wheel? Hence, what is the energy loss to friction as the wheel comes to rest?

2. If the bike had been moving on a level surface, how far would it have traveled for the number of revolutions which the wheel performed?

B. Procedure: Now do what physicists like to call a "thought-experiment." This is a way of simplifying the situation to a greater degree than can be done in real life. Think of coasting on a smooth, level surface with the only force slowing the bike being that of the rear wheel bearings. If that force were also missing, you would coast forever. Start with a speed such that the total kinetic energy of you and the bike is just equal to the original kinetic energy the rear wheel had in Part A of this experiment.

In this simplified view, the forces in the rear wheel bearings are solely responsible for the energy loss—that is, for stopping the bike. It is reasonable (and in fact quite accurate) to assume that the bearing forces are about the same for the ridden bike as for the freely rotating wheel. Thus, the same amount of energy is lost for each revolution of the wheel in either case.

This means that, to bring the bike to a stop, the rear wheel must make the same number of turns as it did in your real experiment of Part A.

Another way of viewing this is that the frictional forces in the rear wheel bearings, acting through the wheel, exert an effective "drag" force to slow the bike down to a stop. The drag force times the distance the bike moves is the work done

by the frictional forces. Thus, it is the kinetic energy which was lost.

Questions:

1. In your thought experiment, how far does the bike move before coming to rest? What was its initial kinetic energy?

2. What is the "drag" due to the rear wheel bearings?

C. Procedure: Rev up the front wheel with the driving motor, measure its speed, and count the number of revolutions it makes in coming to rest.

Questions:

1. What is the initial kinetic energy of the front wheel? How many revolutions did it make?

2. What is the "drag" produced by the front wheel bearings?

D. Procedure: Now take the bicycle to that smooth, level surface and try the experiment. Starting with the pedal at the horizontal, suddenly stand on it and measure the distance traveled before the bike comes to a stop. Just to be completely convinced about the effect of changing gears, you might try this for a different gear, as well. Take the average of several trials. (Note: A skillful rider can, by energetic manipulations of the handlebars, keep the bike upright practically forever and even add substantially to the distance traveled. Try not to do this. Rather, when the bike is practically stopped, let it stop there. It will take some judgement to decide where it stops.)

Questions:

1. How much work did you put into the bike?

2. How far does the bike travel before stopping?

3. What is the drag force? How does this compare to the drag of the wheel bearings? How do you explain this result?

4. How does the distance traveled compare when you change gear ratios? How do you explain this result?

E. Procedure: To examine this result a little more closely, reduce the pressure in each tire by 5 lb/in^2 or so, and repeat. That is, if the normal pressure is 70 lb/in^2, try this at 65 lb/in^2 for each trial. Continue at successively lower pressures, but stop before the tires get so low that they are crushed by the rims.

Questions:

1. In each trial, how far did the bike go? Hence, what was the total average force slowing the bike down each time?

2. Make a graph of the frictional force due to the tires versus the tire pressure, p. In making this graph, how can you compensate for the frictional force of the bearings?

3. If the previous graph is definitely not a straight line, try one of force versus $1/p$. Is this possibly a straight line?

4. If the tires could be inflated to extremely high pressures, what do you suppose would happen to the frictional force due to the tires? How could you approximate this result with real tires?

Discussion of Experiment B-1.

Mass on the Rim of a Wheel

When a single mass moves in a straight line at a velocity v, the kinetic energy is just $\frac{1}{2}mv^2$. When the same mass moves in a circle, the kinetic energy is still $\frac{1}{2}mv^2$, but it is a bit trickier to see what the velocity is.

For example, suppose a mass moves on the rim of a wheel which is turning at the rate of ω (Greek letter omega) revolutions per second (Figure 17). Then, in one revolution of the wheel the mass travels at a distance of one circumference, $2\pi r$. In ω revolutions the mass travels a distance $2\pi r\omega$ and, since the wheel turns ω times per second, the speed is $2\pi r\omega$ in, say meters per second. That is:

$$v = 2\pi r\omega$$

Numerical Example

Suppose a 1-kg mass is on the rim of a bicycle wheel with a diameter of 68 cm (about 27 inches), which is

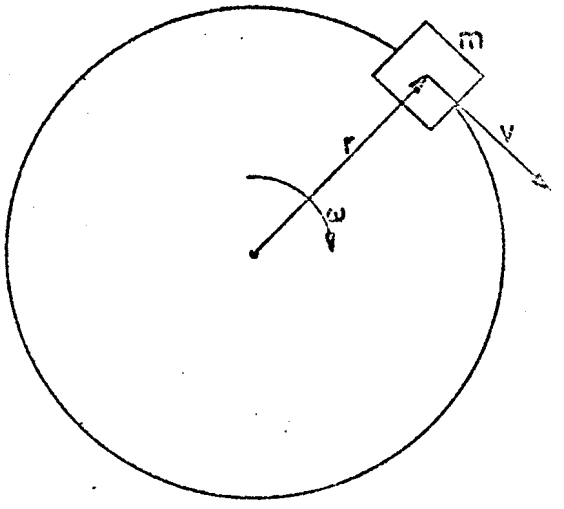


Figure 17. A mass on the rim of a turning wheel travels at a speed which depends on the rate at which the wheel turns and on the radius of the wheel.

turning at a rate of 3 revolutions per second (equivalent to about 15 mph). What is the kinetic energy of the mass?

Known quantities:

$m = 1$ kg

$\omega = 3$ rev/s

$r = 0.34$ m

Then:

$$v = 2\pi r\omega$$
$$= 2\pi \times 0.34 \text{ m} \times 3 \text{ rev/s}$$
$$\approx 6.4 \text{ m/s}$$

And:

$$KE = \tfrac{1}{2}mv^2$$
$$= \tfrac{1}{2} \times 1 \text{ kg} \times (6.4 \text{ m/s})^2$$
$$\approx 20.5 \text{ kg m}^2/\text{s}^2$$
$$\approx 20.5 \text{ J}$$

Using the Speedometer

One helpful point is that, when the bicycle wheel rotates at any particular rate on the stand, a point on its outer surface travels at the same speed as would the bike if it were moving on a road with the wheel turning at the same rate. Thus, if one can attach a speedometer to the rear wheel, the speedometer reading gives directly the approximate speed of any point on the tire. To get that speed into the units of meters per second, one can use the conversion factor 1 mph = 0.45 m/s.

Several Masses on the Rim of a Wheel

Suppose that you take the mass which was on the rim of the wheel, divide it into two equal parts, and put those halves at opposite ends of a diameter, as in Figure 18. Now, since the two masses are on the same wheel, turning at the same rate, and at the same distance from the center, they have the same speed. The total kinetic energy is thus the sum of the kinetic energies of the two masses:

$$\underline{KE} = \underline{KE}_1 + \underline{KE}_2$$

$$= \tfrac{1}{2}\,(\tfrac{1}{2}\,\underline{m})\,\underline{v}^2 + \tfrac{1}{2}\,(\tfrac{1}{2}\,\underline{m})\,\underline{v}^2$$

$$= \tfrac{1}{2}\,\underline{mv}^2$$

This is just the same energy as before, but the mass is in better balance and the wheel turns more smoothly.

Suppose you now separate the mass into many parts, say 20, distributed at equal intervals along the rim, as in Figure 19. Now each piece has 1/20th the original kinetic energy, and the total kinetic energy is still $\tfrac{1}{2}\underline{mv}^2$.

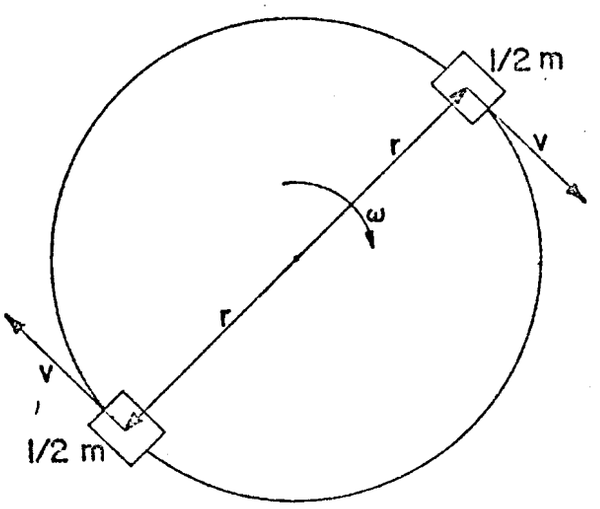


Figure 18. Two equal masses on the rim of a wheel.

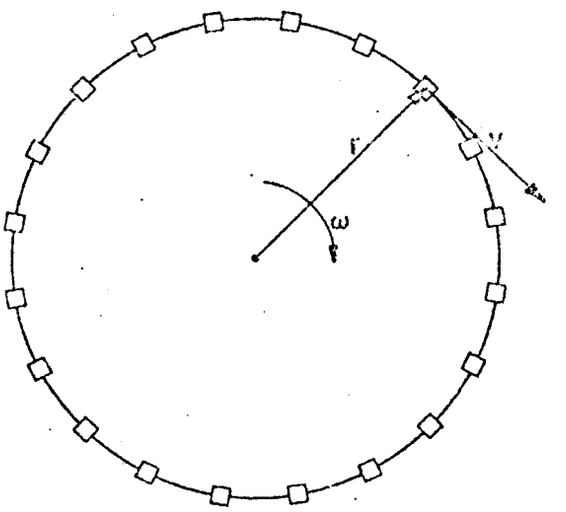


Figure 19. When the mass on the rim is divided up into many parts, the total kinetic energy remains the same.

A "Tire" on the Rim

Finally, suppose that you divide the mass up into so many parts that they form one continuous "tire" on the rim, as in Figure 20. The kinetic energy is still the same.

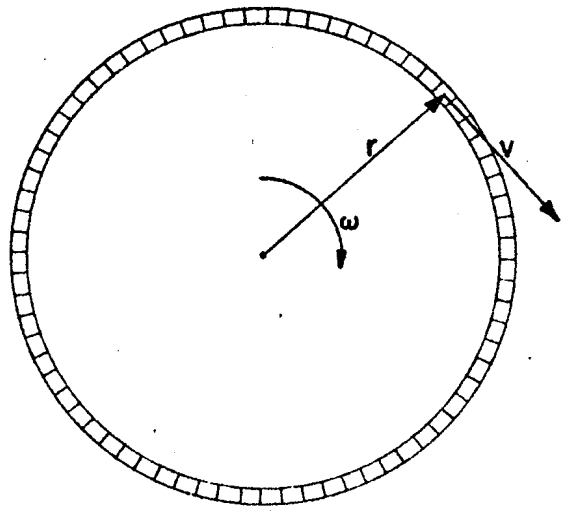


Figure 20. The kinetic energy of a continuous tire is still the same for the same speed and total mass.

That is

$$KE_{tire} = \tfrac{1}{2} m_{tire} (v_{tire})^2$$

Numerical Example

Approximately what is the kinetic energy of a 2.3-kg bicycle wheel rotating at a rate equivalent to 20 mph?

Known quantities:

$m = 2.3$ kg

$v \simeq 20$ mph $\simeq 9$ m/s

Note that v is given approximately. This is because the mass is not really located at the outer surface of the tire, where the speed is 20 mph. More about this later. For now, assuming that the mass is concentrated there,

$$KE \simeq \tfrac{1}{2} m v^2$$

$$\simeq \tfrac{1}{2} \times 2.3 \text{ kg} \times (9 \text{ m/s})^2$$

$$\simeq 93 \text{ J}$$

Adding Mass to the Wheel

When you attach lead weights to the bike wheel, the situation changes. As indicated in Figure 21, the wheel itself has the same kinetic energy as before, and the moving weights have an additional kinetic energy of $\tfrac{1}{2} m' v'^2$. Here m' is the total mass of the added weights and v' is their speed. Note that v' is different from v because the added weights are only a distance r' from the center. In fact, since ω is the same for the rim as for the added weights:

$$v' = 2\pi r' \omega$$

$$v = 2\pi r \omega$$

Thus:

$$\frac{v'}{v} = \frac{r'}{r}$$

So:

$$v' = \frac{r'}{r} v$$

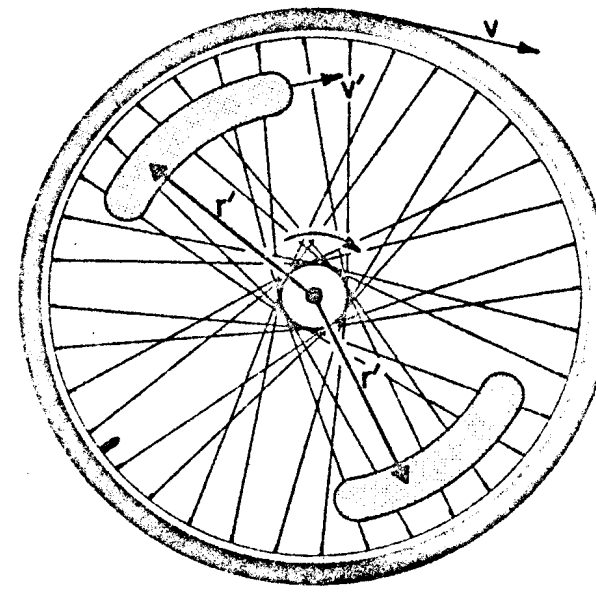


Figure 21. The kinetic energy is that of the wheel plus that of the added weights.

Numerical Example

Suppose you add a total mass of 1 kg to the bicycle wheel, attached at a distance of 20 cm from the center, and cause the wheel to rotate at the 20 mph rate. What is the kinetic energy of the added weights?

Known quantities:

$\underline{v}$ = 20 mph ≈ 9 m/s

$\underline{r}$ = 34 cm

r' = 20 cm

m' = 1 kg

Thus:

$$\underline{KE} = \tfrac{1}{2}\underline{m}'\underline{v}'^2$$

$$= \tfrac{1}{2}m'\left(\frac{\underline{r}'}{\underline{r}}\right)^2 \underline{v}^2$$

$$= \tfrac{1}{2}\ \text{kg}\left(\frac{20\ \text{cm}}{34\ \text{cm}}\right)^2 (9\ \text{m/s})^2$$

$$\approx 14\ \text{J}$$

Solid Disc

When a wheel is a solid disc, it is obvious that not all of the mass is concentrated at the rim. Analysis of this problem involves adding up pieces which are thin rings like the one shown in Figure 22. Every part of a given ring is the same distance from the axle. The addition is more complicated than that done for masses on the rim of the wheel and requires the methods of calculus.

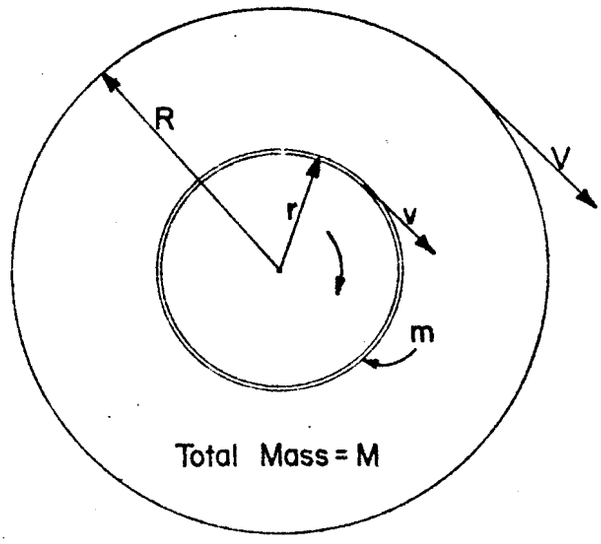


Figure 22. To find the kinetic energy of a rotating disc, one must add up the energies of all possible thin rings of mass.

We simply state the result. As indicated in Figure 23, the kinetic energy of the disc is $\tfrac{1}{4}Mv^2$, where $\underline{v}$ is the speed of a point on the rim, and $\underline{M}$ is the total mass of the disc. This may be interpreted in one of two ways: the disc has the same kinetic energy as

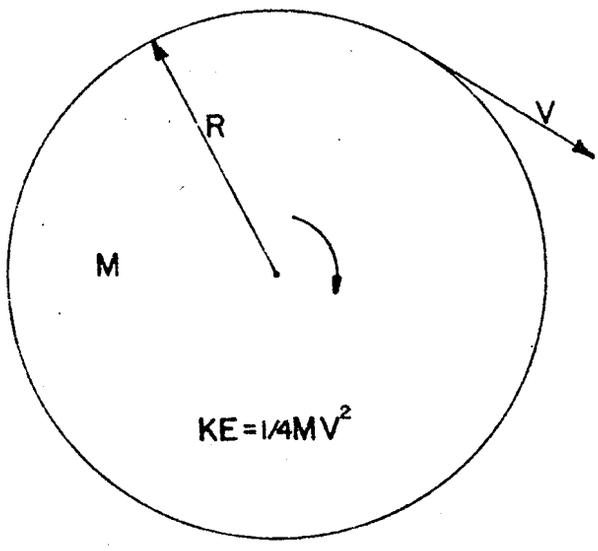


Figure 23. The rotating disc has a kinetic energy of $\tfrac{1}{4}Mv^2$.

would half the mass concentrated at the rim or, alternatively, the disc has the same kinetic energy as would the same mass concentrated at a distance $R/\sqrt{2}$ from the axle.

The rear wheel of a bicycle is much more like the wheel with the mass all concentrated at the rim than it is like the solid disc, but it is not exactly like either. The tire and rim constitute most of the mass and the spokes are negligible. The hub and gears have a fairly substantial mass, but they are so close to the hub that, because of their smaller speed, their contribution to the kinetic energy is small. As a good approximation, one can assume that only the mass at or near the rim of the wheel contributes to the kinetic energy.

Work Put Into the Wheel

Where does the kinetic energy of the rear wheel come from, anyway? When you apply force to the pedal, say by standing on it, you do work which, through the chain and gear mechanism, acts on the wheel to increase its kinetic energy. However, since the force acting on the pedal is not always in the same direction as the motion of the pedal, this work is hard to compute. It is much easier to use the change in potential energy of the weight as it falls and does work on the pedal. Except for frictional losses, the change in potential energy is equal to the work done.

As you know, the change in potential energy of a mass which moves from a higher point to a lower one depends only on its weight and the vertical distance moved. This is indicated in Figure 24.

The potential energy lost by the weight turning the pedal depends only on the starting and ending heights of the pedal, as shown in Figure 25.

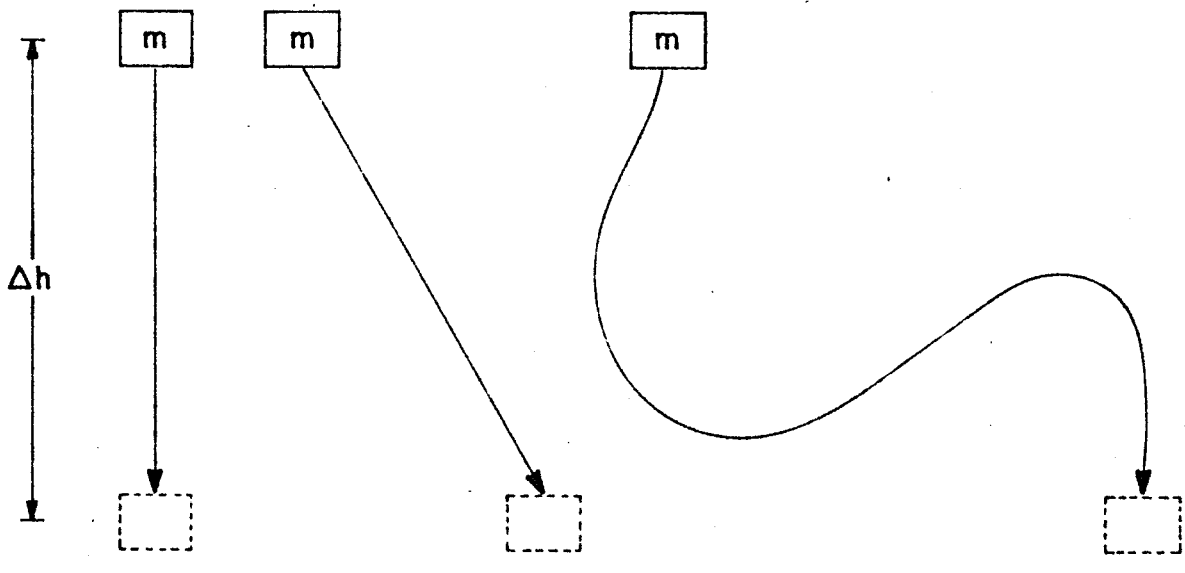


Figure 24. The change in potential energy of the mass depends only on its initial height and final height and not on the path it takes. In the three cases shown, the change in potential energy is the same.

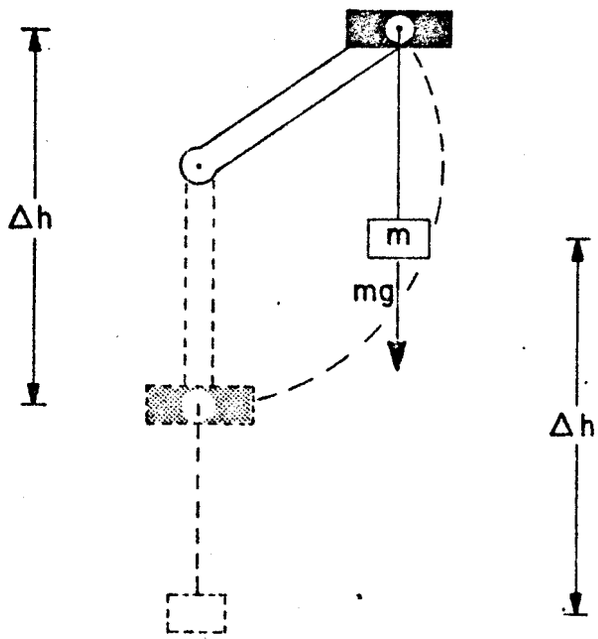


Figure 25. The weight on the pedal falls a distance Δh and loses potential energy $mg\Delta h$.

Since the weight of the mass m is mg and the change in height is Δh, the change in potential energy is $mg\Delta h$.

What is the work done in this case? The only source of work is that change of potential energy of the mass and, if the frictional losses are negligible, the work done must just be equal to the change in potential energy. That is:

$$W = mg\Delta h$$

Numerical Example

a. Suppose a 60-kg person suddenly stands on a bicycle pedal, starting at the horizontal pedal position and falling to the bottom of its swing. If the pedal crank is 15 cm from the hub to center of pedal, what was the change in potential energy?

Known quantities:

$m = 60$ kg

$\Delta h = 15$ cm

The change in potential energy is:

$$\Delta(PE) = mg\Delta h$$

$$= 60 \text{ kg} \times 9.8 \text{ m/s}^2 \times 0.15 \text{ m}$$

$$= 88 \text{ kg m}^2/\text{s}^2$$

$$= 88 \text{ J}$$

b. If the mass of the rear wheel is 2.5 kg, mostly concentrated at the rim, and if all of the work is converted into kinetic energy, approximately what is the maximum speed the wheel reaches?

Known quantities:

$KE \approx 88$ J

$m = 2.5$ kg

In terms of the speed (at the rim) the kinetic energy is:

$$KE \approx \tfrac{1}{2}mv^2$$

This can be solved for speed:

$$v \approx \sqrt{\frac{2\ KE}{m}}$$

$$\approx \sqrt{\frac{2 \times 88 \text{ kg m}^2/\text{s}^2}{2.5 \text{ kg}}}$$

$$\approx 8.4 \text{ m/s}$$

$$\approx 19 \text{ mph}$$

The Effects of Different Gears

Did you notice that in the preceeding discussion and example, no mention is made of the gear used? This may be a bit surprising. However, if there are no losses to friction, the only place for the loss in potential energy to appear is in increased kinetic energy of the rear wheel. Thus, whatever the gear, the rear wheel should reach the same speed each time one stands on the pedal in the same manner.

In fact, you probably found that, to the best of your ability to measure, this was the case in the higher gears. However, if you were standing on the pedal and no extra weights were added to the wheel, you probably found that the maximum speed, and thus the kinetic energy of the wheel, was considerably less for the lowest gears.

Where did the energy which was lost in this process go? This is an effect well known to bicycle racers who find that, if the gear ratio is too low for the conditions, the rear wheel tends to "get ahead" of the pedals.

To get some insight into this, think of riding a bicycle on a level road in the lowest gear, so that the pedalling is very easy. If you pedal as fast as possible, you soon reach a speed where you can no longer keep up with the wheel. Then you put work into the bike only infrequently—as it slows down a little. If you want to increase your speed, you must then shift to a higher gear ratio.* Something akin to this happens when the resistance to acceleration—the inertia—of the rear wheel isn't sufficiently great at a given gear ratio as you stand on a pedal. Then energy is lost.

This still doesn't do much to explain where that lost energy goes. When some students tried this experiment, the rear wheel reached a speed of 20 mph for the highest gear and only about 10 mph for the lowest. Since the kinetic energy depends on the square of the speed, this means that the kinetic energy of the wheel was only about one-fourth as great in the latter case. Where did the other three-fourths of the energy go?

The answer can be found in a related experiment. Suppose you jump from a table onto the floor. As your body falls, it loses potential energy and gains kinetic energy, going faster and faster. Then you hit the floor and suddenly are stopped. You have lost both the initial potential energy and the kinetic energy gained on the way down. However, if we believe in "conservation of energy," it wasn't destroyed but must merely have changed into other forms. One of these forms is obvious; you can hear some of the converted energy. A

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*With all this talk of "gear ratio," you may have some curiosity about the meaning of the term more precisely than it is used here. If so, read the appendix at the back of the module. If not, don't; it isn't really necessary for an understanding of the module.

larger fraction is converted into heat, which is not quite so easily detected but which is there, nonetheless. If you foolishly jump from too great a height, some energy may go into cracking bones. Even the stinging sensation on the soles of your feet is an indication of the conversion of kinetic energy into other forms.

If you were suddenly to stand on a bicycle pedal with the chain removed, much the same thing would happen. When you hit bottom, your kinetic energy is changed to heat, sound and possibly other forms. When you connect the pedal to the bicycle wheel, which has inertia, at least some of the energy is converted into kinetic energy of the rotating wheel. In the higher gears—or when you increase the inertia of the wheel by adding weights to it—most of the initial potential energy goes into kinetic energy of the wheel. Do your experiments bear this out?

Power

You probably noticed that, even in gears where most of the energy goes into the rear wheel, the rate at which this happens is different in the different gears. In a lower gear, the pedal falls more rapidly than it does in a higher gear. In each case the maximum speed of the wheel is the same, but in one case it gets to that speed sooner. The rate at which work is done is called power. That is, power is work per unit time:

$$P = \frac{W}{t}$$

Numerical Example

A bicyclist rides a bicycle, from a standing start, so that the average impelling force is 80 N. In 10 s he travels 50 m. What was the power—the rate at which he did work?

Known quantities:

$$F_i = 80 \text{ N}$$

$$D = 50 \text{ m}$$

$$t = 10 \text{ s}$$

The amount of work is:

$$W = F_i \times D$$

$$= (80 \text{ N}) \times (50 \text{ m})$$

$$= 4000 \text{ J}$$

The power:

$$P = W/t$$

$$= 4000 \text{ J}/10 \text{ s}$$

$$= 400 \text{ W}$$

(One joule per second is a watt—W.)

Incidentally, this power output is within the capability of most adult bicyclists. One horsepower is about 750 watts, and even that is possible for some adults for short periods of time.

Power and Velocity

Since the work put into the bike is:

$$W = F_i \times D$$

and since the power is W/t, then:

$$P = F_i \times D/t$$

But D/t is just the average velocity and

$$P = F_i v_{ave}$$

Note that this does not depend on whether the bike is accelerating or moving at constant speed. In the case where the impelling force is constant and the speed is constant, one may write simply:

$$P = F_i v$$

Since the impelling force depends on the force applied to the pedals and since the maximum pedal speed depends on the speed of the rear wheel, the power produced depends on both the force applied to the pedals and the speed with which the pedals turn. For a given rate of doing work on the bike, the bicyclist may choose to do it with a greater force and the pedals turning less rapidly—a high gear ratio—or with a lesser force and the pedals turning more rapidly—a low gear ratio. Experiments indicate that bicyclists are most efficient in the use of bodily energy for extended periods when the rate of pedalling is about 50 revolutions per minute. Thus, for the speed at which you wish to travel, you should choose a gear ratio which will accomplish this. Actually, since the difference in bodily efficiency is small until one gets below 30 or above 70 rpm, one should choose a rate which is comfortable.

Discussion of Experiment B-2

The measurements you made in this experiment are difficult to analyze. First, when the free wheel slowed from some initial speed to a stop, we assumed that the slowing was caused solely by bearing friction. However, when a wheel turns, it drags along the nearby air, as indicated in Figure 26.

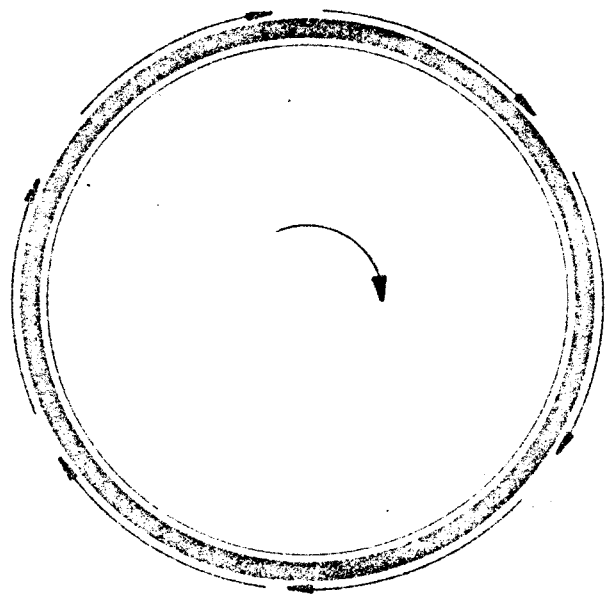


Figure 26. When a wheel turns, air near it is dragged along.

In fact, the drag force of the air may be as great or greater than the bearing friction force, especially at higher speeds. However, the two effects are difficult to separate, so we shall continue to work with "bearing friction."

A second difficulty is that rubber is not an "engineering material." That is, there are so many variables

in the design and manufacture of tires that it is not possible to devise accurate theoretical formulas for their behavior. In fact, as you saw, even a small change in tire pressure produces a large difference in the slowing forces.

"Bearing Drag"

From the first part of Experiment B-2, you can calculate the combined effect of bearing friction and air drag on the wheel by knowing the equivalent distance the bike would have rolled with the wheels turning the same number of revolutions. That is, if the distance is D_1, the work done by the bearings in slowing the wheel is $F_B D_1$, where F_B is the average force which would slow the bike if the bearing friction and air drag were the only forces involved, and D_1 is the distance the wheel would have rolled for the same number of turns as in your experiment. However, the work done in slowing the wheel from some initial speed to a stop must be equal to the initial kinetic energy. Thus:

$$KE_{wheel} = F_B D_1$$

and

$$F_B = \frac{KE_{wheel}}{D_1}$$

A similar calculation can be done for the other wheel, and the sum of the two forces is the net "bearing drag" on the bike.

Tire Friction

When discussing tire friction, nice neat theoretical formulas are not available, but it is possible to get some insight into why there is tire friction and where the lost energy goes.

To begin, it is important to realize that both the tire and the road surface determine the so-called "rolling resistance." First, consider what might be the easiest rolling of all wheels—a perfectly hard wheel rolling on a perfectly hard surface. Trains approximate this with steel wheels rolling on a steel track. Some of the early bicycles—the ones with huge front wheels—had iron tires. Considering the condition of the roads in those days, it is no wonder that they were called "bone-shakers."

To get an idea of where the energy goes when a rubber tire rolls, look first at the simplified situation depicted by Figure 27. Here, a perfectly rigid wheel is in a rigid hole. As the tire starts to climb out of the hole, the edge of the hole exerts the force shown. This force has a backwards component, or a "drag" which must be overcome in order for the wheel to move forward.

If a rigid tire rolls on soft ground (Figure 28) there is still a drag, but now the lost energy goes into squashing the ground down into a rut.

Consider again a rigid road and a rigid tire, but one with a flat place on the bot-

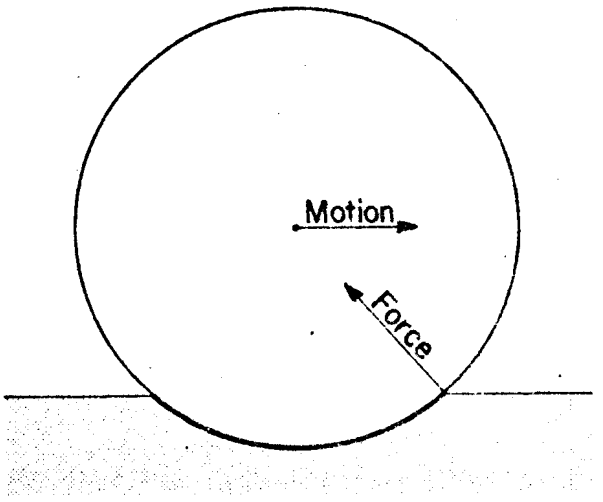


Figure 27. Work must be done on the wheel to cause it to climb out of the hole.

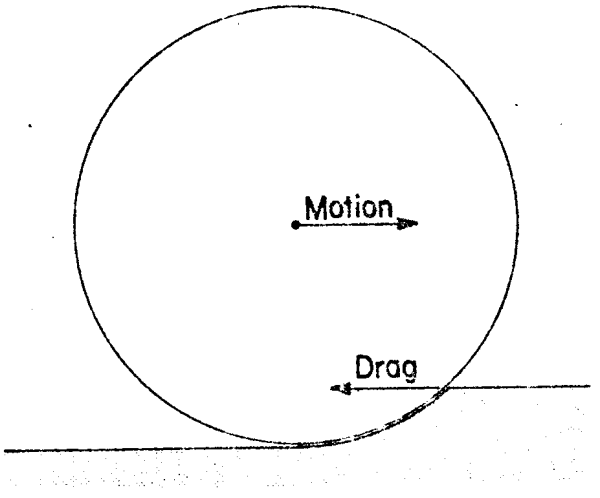


Figure 28. Energy is required to make a rut in soft ground.

tom, as in Figure 29A. This is somewhat more like a real tire. Once again, if the tire turns without slipping, drag is involved and energy is used to raise the center of the wheel to the position of Figure 29B. Just for fun, think of riding on wheels like the one shown in Figure 30. The ride would be pretty bumpy. Every time the center of the

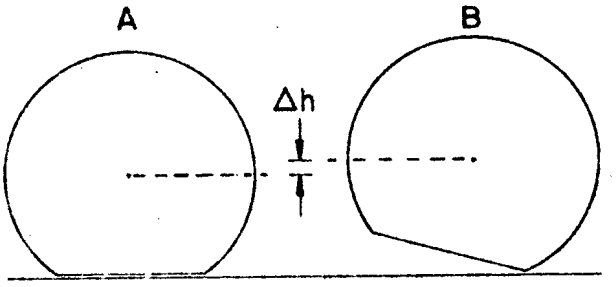


Figure 29. A tire with a flat place on the bottom also causes "drag."

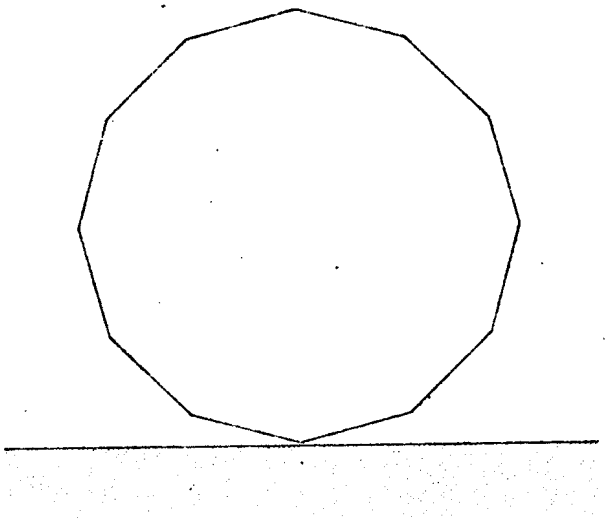


Figure 30. A "wheel" which is a regular polygon.

wheel was raised, as a vertex of the polygon came to the bottom, the bike would lose energy. Theoretically, that lost energy would be regained each time the center of the wheel came down to its lowest position, but in the real world this would not happen. As the flat part of the wheel smacked the pavement each time, energy would be converted to heat and sound. Also, this bike would rattle your teeth, so polygonal wheels are not recommended.

Although making the polygon with a greater number of sides would improve the smoothness of the ride and decrease the energy loss, there would always be some energy loss, and thus drag, until the wheel is made perfectly round and perfectly hard.

However, real wheels and road surfaces, even the steel ones used by trains, are always deformed to some degree by the weight on them. The amount of deformation, and thus the amount of energy loss, depends on the circumstances. A pneumatic (air-filled) tire looks something like Figure 31 as it rolls along. There is a flat place on the bottom and a bulge at the front and rear of the flat place, with the front one being more pronounced. The tire keeps the same shape as it rolls, but the bulge at the front gets pushed around the circumference. To push it requires a force and, since it moves through a distance as a result, work is required. The rear bulge actually puts work into the tire as the part of the tire just behind it continually regains its circular shape. However, as is evident from experiment, more work goes into the tire than is retrieved. One says that the tire is not "perfectly elastic." Where does the lost energy go? The answer, once again, is that it goes mostly into heat. You can easily test this by touching an automobile tire after it has been running at high speed for a while—it is obviously warmer than the surrounding air.

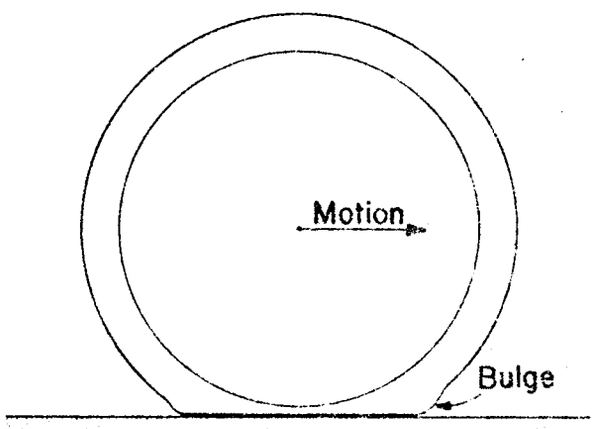


Figure 31. A pneumatic tire.

Finding the Rolling Resistance

Using the data for Experiment B-2, you can get a good approximation to the drag produced by the tires—the rolling resistance. If, for a given trial, the bike plus rider rolls a distance D_2 before stopping, the work done against all the frictional forces is $F_T D_2$, where F_T is the total force holding the bike back. This must (approximately) be equal to the work done on the bike when you stand on a pedal. That is:

$$W = F_T D_2$$

Then, since you know the retarding force of the bearings on the wheels, F_B, the rolling resistance must be:*

$$R = F_T - F_B$$

*This assumes that the air resistance is negligible at these speeds. In Section C, you will find out if this is a reasonable assumption.

Calculating Rolling Resistance

Although theoretical formulas cannot reasonably be derived for tire rolling resistance, empirical formulas—formulas which are just "cooked up" to fit the experiments—have been devised. Empirical formulas find frequent use in science and engineering, for they often can fill a large gap between an inadequate theory and the realities of experiment. One such formula, for 27-inch bicycle wheels, is the following:**

$$R = 5 \times 10^{-3} Wt + (0.15 + 3.5 \times 10^{-5} v^2) \frac{Wt}{p}$$

where Wt is the weight of the bike plus rider, p is the tire pressure in pounds per square inch and v is the speed in miles per hour. If Wt is measured in newtons, R will also be in newtons (or both may be expressed in pounds).

Notice that, in the parentheses, there is a term which depends on the square of the velocity. This says that the faster the bike travels the greater the rolling resistance, and it is a considerable complication. However, if one compares the two terms in the parentheses, at 5 mph the velocity-dependent term is only about half of one percent of the other term and, even at 20 mph, it is less than 10 percent. Therefore, it is safe to ignore it for your experiment, since the speeds were low.

When we neglect the term including velocity, the formula simplifies to:

$$R = (A + B/p)Wt$$

where $A = 5 \times 10^{-3}$ and $B = 0.15$. (Actually, $B = 0.15$ lb/in^2, and these units will cancel the lb/in^2 in which p is expressed.)

A graph of R versus p will look like Figure 32. Did your graph resemble this?

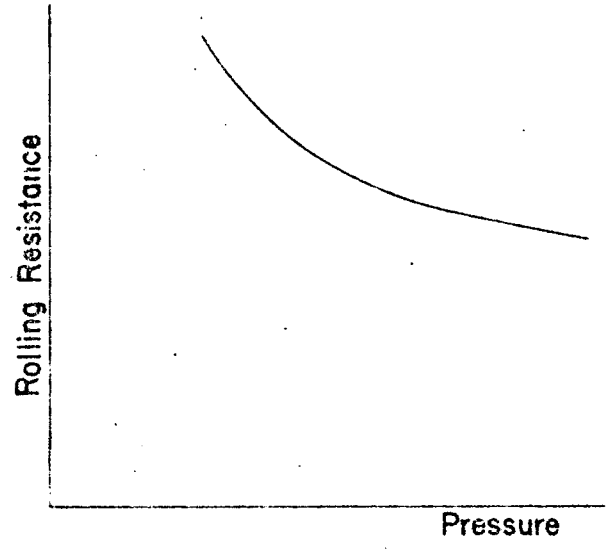


Figure 32. A graph of rolling resistance versus tire pressure.

Figure 33 shows an accurate graph, derived from the formula, of the rolling resistance versus the inverse of pressure. Here the vertical axis is R/Wt, the force per unit weight. Thus, if the bike plus rider were 800 N, the scale would have to be multiplied by 800 to get the rolling resistance in newtons. Note that it makes no difference how the weight is distributed on the front and rear

**Bicycling Science, F. W. Whitt and D. G. Wilson, MIT Press (1974).

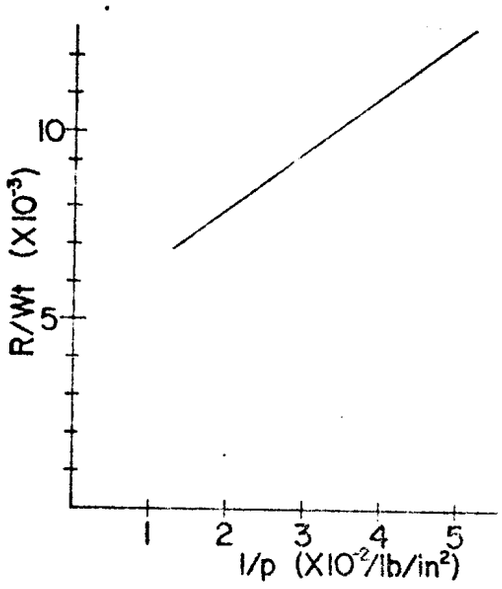


Figure 33. Rolling resistance versus the inverse of pressure.

tires; multiplying by 800 has the same effect as doing each tire separately and adding the two forces.

If you will now graph your data on the same scale as Figure 33, you can compare the two directly by laying your graph on the figure. They will be in agreement if they have the same slope. If it should happen that your data do not match the formula very well, don't be distressed. After all, the formula was just cooked up to fit the data for tires that may have been quite different from yours. In fact, other empirical formulas exist which differ quite a bit from the one given.

However, all of the formulas agree that the rolling resistance varies as the inverse of the pressure, so your graph should approximate a straight line. If it doesn't, you might look for possible errors.

We will now leave the topic of frictional losses and examine some of the other ways in which energy is used to keep the bike moving.

GOALS FOR SECTION B

Goal 1. To be able to find the kinetic energy of mass undergoing circular motion.

Question. A 2 kg mass is attached to a bicycle wheel at a distance of 15 cm from the axle, and the wheel is turned at the rate of 15 revolutions per second. What is the kinetic energy of the mass?

Goal 2. To be able to find the speed imparted to an object by doing work on it.

Question: With a bicycle on a stand so that the rear wheel is free to turn, you stand on a pedal suddenly. The pedal goes from a height of 35 cm above the floor to a height of 15 cm above the floor in the process. If your mass is 65 kg and the mass of the bicycle wheel is 1.5 kg, concentrated mainly at the rim, what is the final speed of the rim? Assume no loss of energy. (Note that it is not necessary to specify the diameter of the wheel!)

Goal 3. To be able to calculate the power generated by a force.

Question: In order to keep a car moving at 25 m/s, an impelling force of 1,000 N is required. What is the power requirement?

Goal 4. To be able to compute frictional forces from work input and distance traveled.

Question: Some students got the following data for Experiment B-2:

Work Input = 100 J (each trial)

"Distance" traveled by free wheel = 256 m

Distance rolled with tires at 70 psi = 40 m

Assuming that this was done at such low speeds that the wind resistance was negligible, approximately what was the frictional force of the tires?

Goal 5. To be able to compare the measured force of rolling resistance to the prediction of the empirical formula.

Question: For the trial of the previous question, the mass of the bike and rider was about 80 kg. Approximately what force of rolling resistance does the formula predict?

SECTION C

Other Losses

In this part of the module, we shall look at the work done against some of the forces other than rolling friction which are present when riding a bicycle. One of these, the force of air resistance, we cannot eliminate; it is always there, to be contended with by the rider. The other force is the one used to drive the generator to make the lights work. Perhaps you will discover whether a generator-operated light is worth the extra effort.

EXPERIMENT C-1. Air Resistance

In this experiment you will find the force exerted on you and the bike by using a device which measures the force on a piece of plastic. You will need the "wind-force measurer," a Polaroid camera, a bike with a well-calibrated speedometer, and a fearless rider. It is best to do the experiment indoors, but it can be done outside on a still day.

A. Procedure: Adjust the wind force measurer on the front of the bike so that, when the bike is upright and stationary, the pointer is on zero. (See Figure 34.) Ride the bike at various speeds, simultaneously noting both speed and the location of the pointer on the wind-force measurer. The scale on the device is calibrated in degrees, and the pointer reading gives directly the angle which the plastic sheet makes with the vertical. You should be able to estimate the angle to the nearest degree. When you make measurements, try to ride in a straight line and at a constant speed long enough so that any jittering of the pointer settles down and the reading is at a steady value. This will take some practice.

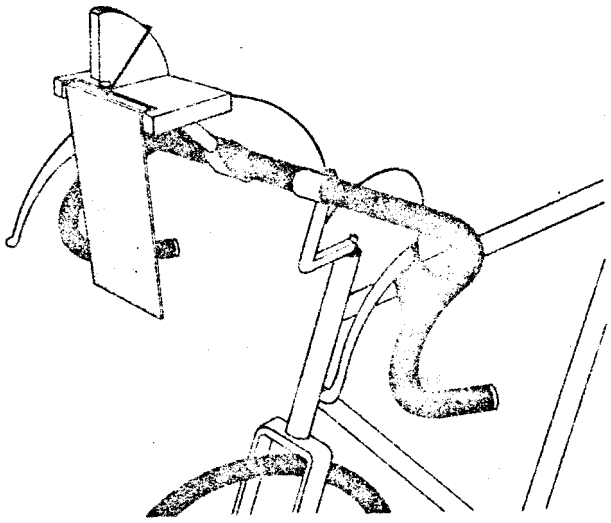


Figure 34. The wind-force measurer on the bike.

Now calibrate the wind-force measurer in the manner shown in Figure 35:

> Attach a string to the hole in the center of the plastic sheet. Hang a weight from the string, after passing it over a pulley, and measure the force (mg) required for each deflection of the pointer.

With the bike on its stand, take a front-view Polaroid picture of the rider on the bike and the wind-force measurer. Be sure the picture

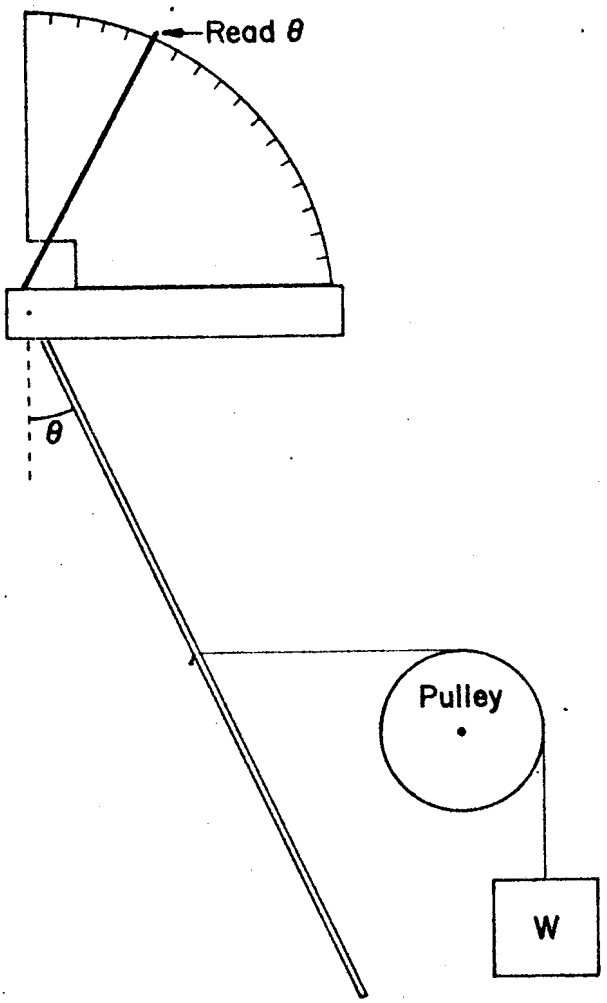


Figure 35. Calibrating the wind-force measurer.

is directly head on and that the rider is in the usual riding position.

Questions:

1. How does the front-view area of the bike and the rider compare to the area of the plastic sheet of the measurer? One easy way to get the ratio of the two areas is to trace them, using carbon paper, onto graph paper as in Figure 36. Then just count the squares on the graph paper.

2. What happens to the "effective area" of the plastic sheet of the wind-force measurer when it is deflected? (The "effective area" is the area you would get in a picture taken from the front when the plastic is deflected.) How can you account for this in comparing the area of the plastic sheet to the area of the bike and rider?

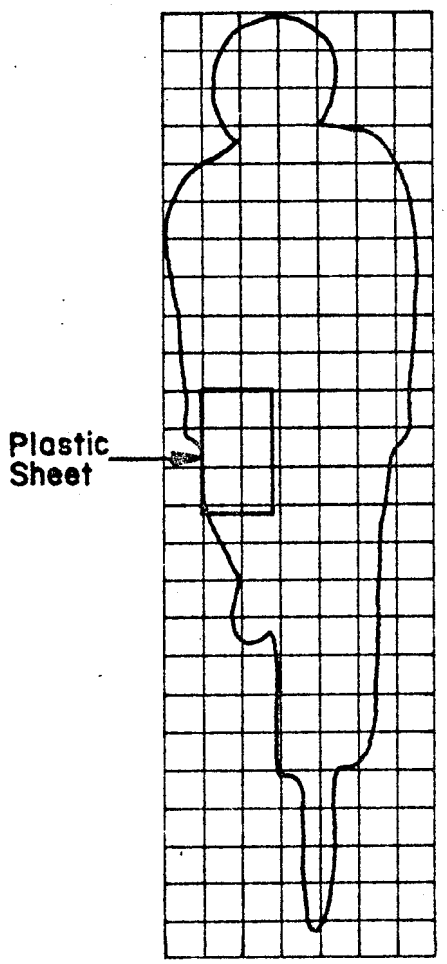


Figure 36. Comparing the frontal area of the bike and rider to that of the plastic sheet of the wind-force measurer.

3. For each speed at which you made a measurement, what was the total force of air resistance on the bike and rider?

B. Procedure: When you have the answer to the previous question, try making a graph of air resistance (force) versus speed. If you have trouble doing this, you can get some help from the discussion of this experiment.

Questions:

1. Is your graph of air resistance versus speed linear?

2. If the answer to the previous question is no, can you find a relationship which is linear? (Hint: try graphing air resistance versus speed squared.) What do you conclude?

EXPERIMENT C-2. Generator Power (OPTIONAL)

For this experiment you will need a generator and lights attached to the bicycle and a switch that will disconnect the lights from the generator.

A. Procedure: First, as a "bench mark," you should repeat one trial of Experiment B-2, Part D, standing on a pedal suddenly and letting the bike coast until it comes to a stop. Then, by knowing the work input and the distance traveled, you can compute the average retarding force.

Now adjust the generator so that the wheel turns it, but with the lights disconnected, and do another run or two. This will tell you how much of the work goes into the frictional forces associated with turning the generator.

Questions:

1. How far does the bike go with the generator turning? Thus, what is the total force slowing the bike?

2. What force is required to turn the generator?

B. Procedure: Now connect the lights to the generator and do about two more trials.

Questions:

1. What force is now needed to turn the generator?

2. How does this force compare with the force required to turn the generator with the lights disconnected?

3. If all the work done in producing electrical power is considered to be "useful," what is the efficiency of the generator? (Efficiency is useful output energy, divided by work input, times 100%.)

Discussion of Experiment C-1.

If you did your measurements carefully, you probably found that there is a linear relationship between the air resistance and the square of the speed. Actually, theory and careful measurements show that the force of air resistance, to a good approximation, obeys an equation like the following:

$$F = Av + Bv^2$$

where A and B are constants.

The first term of this expression gives the effect of viscosity for smooth (laminar) flow of the air, and the second term results from the irregular (turbulent) flow which occurs when the air flow breaks up into eddies and whirls. The two kinds of flow are indicated in Figures 37 and 38.

Viscosity is a measure of the "thickness" or resistance to flow of a fluid. For example, maple syrup is more viscous than water, which is more viscous than air. Viscosity not only impedes the flow of a fluid, it also impedes the movement of another body through the fluid.

Airplane wings and bodies, automobiles, and trains are "streamlined" to reduce the air resistance. However, if examined closely,

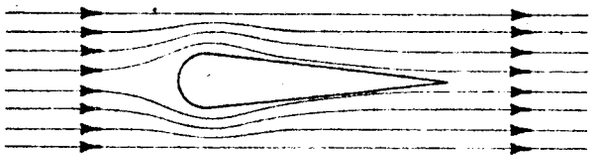


Figure 37. Laminar flow of air.

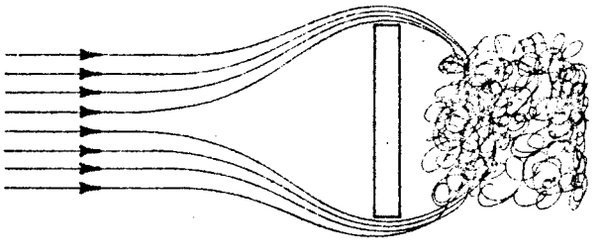


Figure 38. Turbulent flow of air.

almost all flow of air is turbulent. Laminar flow, as indicated in Figure 37, occurs only for tiny particles moving at very low speeds. One can see this by looking at the smoke rising from a stationary cigarette. At first the flow is laminar and the smoke rises in a smooth stream. Higher up, however, it breaks up into turbulence.

For a bicycle rider turbulent flow is the much greater effect, and the force is proportional to the square of the speed. You see this effect, for example, when a car passes and litter is lifted off the street and whirled around behind the vehicle.

Considering the first term to be negligible at the speeds involved, and referring to the theory for more detail, the force can be written as:

$$\underline{F} \approx \underline{K}\underline{A}_e \rho \underline{v}^2$$

Here, $\underline{K}$ is a constant, which the handbooks give as about 0.7 for a disc moving in air, $\underline{A}_e$ is the effective area presented to the air, ρ (Greek letter rho) is the density of the air, and $\underline{v}$, of course, is the speed.

The Wind Measurer

Before proceeding further, we need to know how to find the effective area of the wind-measuring device for any given angle at which it may hang. Figure 39 shows that the actual area presented to the wind is reduced as the wind holds the plastic sheet at some angle θ to the verti-

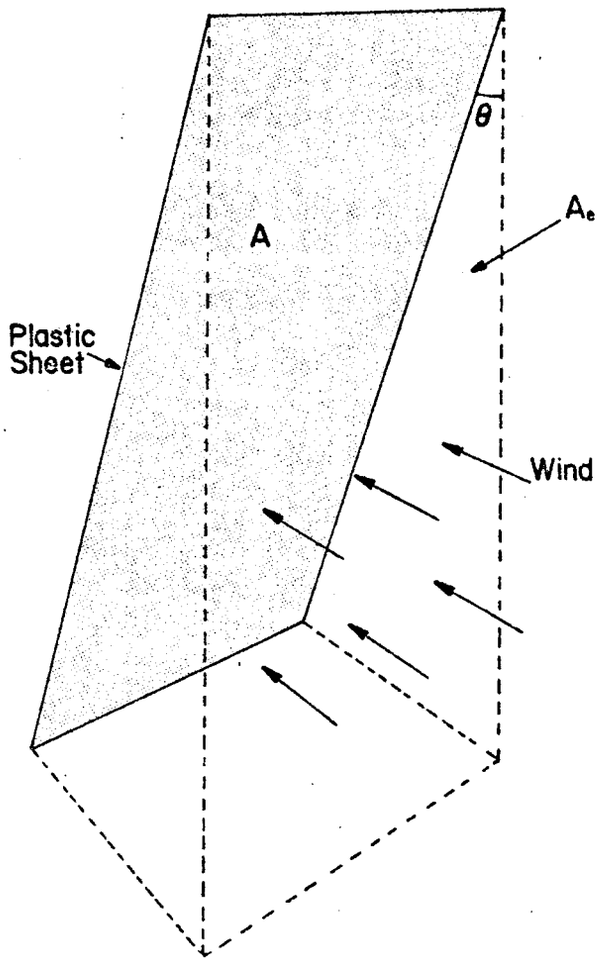


Figure 39. The effective area of the plastic sheet.

cal. That is, the area the wind "sees" is the area that would be measured on a photograph taken from directly in front of the device with the plastic sheet at the same angle to the vertical. In Figure 40, the same thing is shown in cross-section.

Since the width of the effective area is the same as the width of the actual area, the ratio of the two is just the same as the ratio of the lengths shown in Figure 40. That is:

$$\frac{A_e}{A} = \frac{L_e}{L} = \cos\theta$$

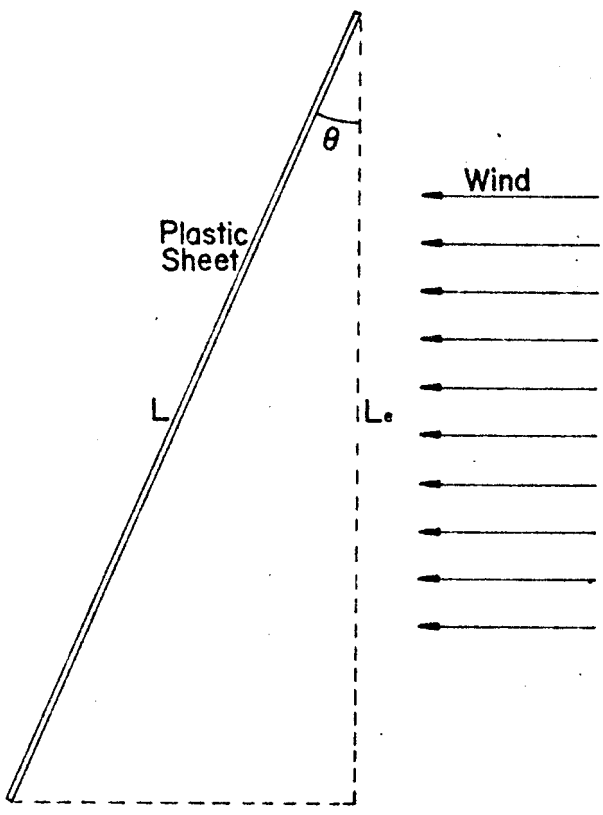


Figure 40. A cross-sectional view of the plastic sheet of the wind measurer.

where A is the actual area and A_e is the effective area of the sheet.

Or:

$$A_e = A \cos\theta$$

Numerical Examples

1. In doing a measurement of wind force, a couple of my students found that, at a speed of 10 mph (4.5 m/s), the plastic sheet was deflected to an angle of about 20°. This corresponded to a deflecting force of about 0.24 N, as measured by hanging a weight over the pulley. The dimensions of the plastic sheet were 10 cm by 20 cm. What was the effective area, and roughly what force would the theory predict?

Known quantities:

$A = 10 \text{ cm} \times 20 \text{ cm} = 0.02 \text{ m}^2$

$\theta = 20^\circ$

$F = 0.24 \text{ N}$

$\rho = 1.1 \text{ kg/m}^3$ (From a handbook)

Then, the effective area is:

$$A_e = A \cos\theta$$
$$= (0.02 \text{ m}^2)\cos 20^\circ$$
$$= 0.02 \times 0.94 \text{ m}^2$$
$$= 0.019 \text{ m}^2$$

Using the rough approximation that $K = 0.7$, as it

is for a disc, the calculated value of force on an area A_e is:

$$F = KA_e \rho v^2$$

$$= 0.7 \times 0.019\ m^2 \times 1.1\ kg/m^3 \times (4.5\ m/s)^2$$

$$\approx 0.30\ N$$

which corresponds fairly well to the measured value of 0.24 N. (This is known as a "ballpark" calculation.)

2. Using the measured value for force given above at 10 mph, what would be the net force on the bike and rider?

Known quantities:

$F = 0.24\ N$

$A = 0.020\ m^2$

$\theta = 20^\circ$

From Figure 36, the ratio of the total area of the person and bike (A_T) to the area of the plastic sheet of the wind measurer is:

$$\frac{A_T}{A} \approx \frac{89}{5.5} \approx 16$$

Thus, the total force on the person and bike will be:

$$F_T = \frac{A_T}{A_e} F$$

$$= \frac{A_T F}{A \cos\theta}$$

$$= (A_T/A)\ (F/\cos\theta)$$

$$= 16 \times 0.24\ N/0.94$$

$$\approx 4.1\ N$$

Table I shows the results of some student data, and Figure 41 is a graph of those results.

As you can see, those students got a pretty good straight-line relationship between F_T and v^2.

Questions:

1. From your experiments, how does the force of wind resistance at riding speeds

Table I - Wind Force Measurements

Speed (mph)	Speed (m/s)	v^2 (m^2/s^2)	Angle (Degrees)	Force on Plastic (N)	Total Force (N)
5	2.2	4.8	5	0.05	0.8
6	2.7	7.3	7.5	0.08	1.3
7	3.1	9.6	11	0.12	2.0
8	3.6	13.0	13.5	0.15	2.5
9	4.0	16.0	17	0.20	3.3
10	4.5	20.3	20	0.24	4.1
11.5	5.1	26.0	27	0.30	5.4

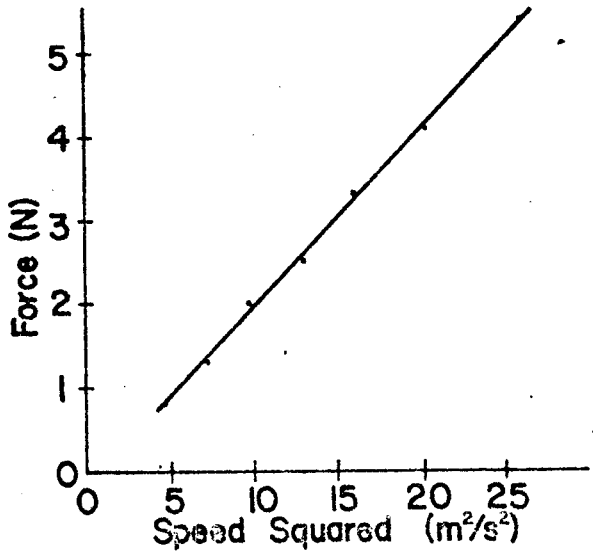


Figure 41. Force on the bike and rider versus speed squared from the data of Table I.

compare with the frictional forces of bearings and tires?

2. At roughly what speed are the two kinds of forces the same?

3. What force would an unpaced racing bicyclist face if he were able to go at 100 mph?

Discussion of Experiment C-2.

This experiment is quite straightforward and needs little discussion. The calculations of forces are done exactly as in earlier experiments.

It might be appropriate, however, to say a bit about the "efficiency" you calculate in the experiment. In general, the efficiency of a device is:

$$\text{Eff} = \frac{E_{out}}{E_{in}} \times 100\%$$

where E_{in} is the energy input and E_{out} is the useful work or energy output. For example, E_{out} would be the usable heat produced by a furnace or the work done by a crane in lifting a large weight.

In this experiment, the "efficiency" you measured was surprisingly high, probably about 80%, but this is a bit misleading. The useful output is in the form of light and, in an incandescent lamp, most of the input energy goes into heat. Thus, although about 80% of the work you do on the generator goes into electrical energy, most of that is wasted in the form of lost heat. Measurements which separate the light energy from the heat energy are possible, but they are difficult to do.

Questions:

1. How does the work needed to run the generator compare with the work which goes into frictional and wind resistance forces?

2. Is a generator-operated light worth the extra effort for the occasional bike rider? For the racer?

Energy Transformations

In this module you have learned that the work one puts into a bicycle is converted into kinetic energy, potential energy, heat, sound, and sometimes electrical energy. The principles you have learned here have far wider applicability than just to the bicycle. In general, one may do work on an object or set of objects and change its energy in some way. Conversely, an

object with kinetic or potential energy may be made to do work on other objects, changing their energy states. In such processes energy is changed (transformed) from one form to others, but the total energy remains the same.

A good example of energy transformation is the sequence of changes that occur when electricity is produced in a coal-fired power plant and eventually used in your home. Chemical energy stored in the coal is changed into heat which produces high pressure steam. The steam does work on turbines which drive generators to produce electric power. Work is done on electrons, through transmission wires and transformers, and finally electric power is available in your home. There it is transformed into light, heat, kinetic energy, sound, etc.

As you noted with the bicycle, when work and energy are converted from one form to another, some of it is always "lost." When you pedal a bike, some of the work goes into useful kinetic energy and potential energy, but much of it is lost in overcoming tire rolling resistance and air resistance. In fact, when riding on a level road at a steady speed, all of the work input is "lost" in this sense. It isn't destroyed, but it doesn't contribute toward increasing the speed or raising the bike up a hill. Likewise with the generation and use of electric power: at every stage of conversion some of the energy is lost. These losses are quite substantial. For example, if your home is heated electrically, you get only about 30% of the heat that you would get if you burned the coal the power plant had to burn for that purpose. The rest is wasted, mostly in the form of heat escaping to the atmosphere, in the power plant, in the transmission lines, in the transformers, and so on. But, once the electrically produced heat is delivered to the house, almost all of it goes into useful heating, whereas half of the coal-produced heat escapes without heating the house. Thus people who can afford it may well choose the convenience of electric heat over the economy of coal heat. (Besides, only power plants can buy coal these days.)

As a student I know is fond of saying, "such is life. There is no device which is 100% efficient in converting work or energy into another usable form of energy; there are always losses. In a technilogical society, engineers—and society itself—must often make judgements about how much loss is tolerable as the price for a certain convenience. If the loss becomes intolerable, then more efficient devices must be designed or the convenience abandoned.

Postscript

At this point you should have a good understanding of why one must continuously put work into a bike just to keep it going at constant speed and where that work goes. Even if you don't ever get into the business of designing more efficient bicycles, perhaps this knowledge will help you as a bicyclist. I hope that acquiring it has been fun.

Good riding.

GOALS FOR SECTION C

Goal 1. To be able to compare the forces of wind-resistance with the frictional forces on a bicycle.

Question. From your graph of wind resistance versus speed squared, at about what range of speeds is the wind resistance small compared to the frictional resistance for fully inflated tires? At about what speed are they the same? At what range of speeds is the wind resistance force much the greater of the two?

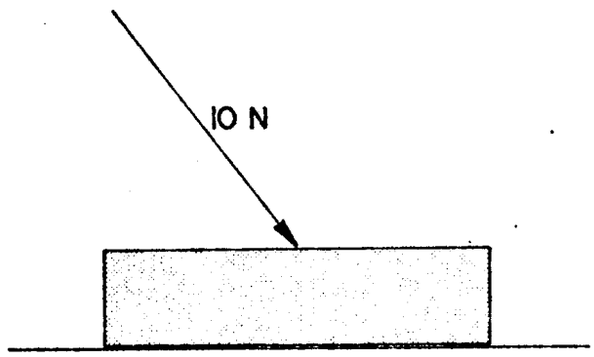
Goal 2. To be able to compute the efficiency of a machine.

Question. A crane at a junk yard raises a 1000-kg car a distance of 10 m. Careful measurements show that, in the process, the engine does 5×10^5 J of work on the crane. Approximately what is the efficiency of the crane under these conditions?

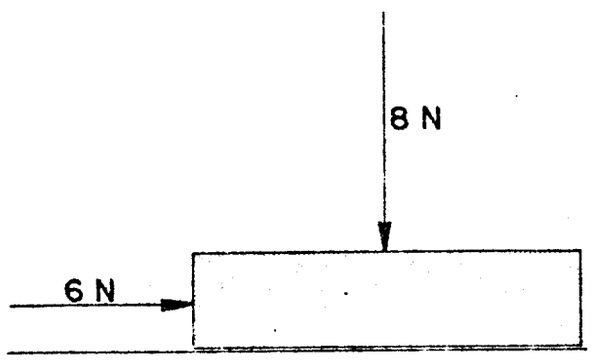
APPENDIX A

Components of Vectors

In dealing with vector quantities, such as a force, it is often useful to examine the effect of the vector in some direction other than the one in which it acts. The problem in the module of a force acting on a hockey puck is a good example. Look at a force like that in the drawing below.

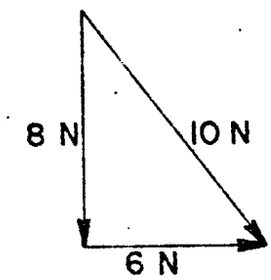


In this case, we wish to examine the components of the force which are parallel and perpendicular to the direction in which the puck slides. I chose the size and direction of the force in the figure above so that it is exactly equivalent to the net result of the two forces shown below:



In other words, the motion of the puck due to the 10 N force will be precisely the same as the motion produced by the combination of the 8 N and 6 N forces. Thus we know, for example, that if the puck is pushed 10 m along the ice by the 10 N force, 60 J (6 N X 10 m) of work is done on it. Looking at the component forces, the 8 N force does no work because the puck does not move vertically, and all of the work is done by the 6 N component.

Since it is customary to draw vectors with lengths which are proportional to their sizes, we can form a triangle with the original vector and its two components:



We say that we have added the 8 N and 6 N forces—vectorially—to produce the 10 N force.

Let me point out that there is nothing magical about the directions I chose. Any two directions would work (three directions for three-dimensional problems). However, to simplify matters, one usually chooses directions which are perpendicular to one another and which have some simple relation to the problem being studied.

APPENDIX B

Gear Ratio

Generally, the term gear ratio refers to the number of teeth on the driving gear divided by the number on the driven gear. For example, if the driving gear has 52 teeth and the driven one 26, the gear ratio is 2. In this case, each revolution of the driving gear will cause the driven gear to make two revolutions.

In the case of bicycles, there is a special definition of "gear ratio" that is related to how far the bike will travel for one revolution of the pedal. Since this depends on the wheel size, the definition of gear ratio (G.R.) includes it:

$$G.R. = \frac{N_1}{N_2} \times d$$

Here, N_1 is the number of teeth on the front sprocket, N_2 the number on the rear sprocket, and d is the diameter of the wheel in inches. Thus, for a 27" bike with a 52-tooth front sprocket and a 26-tooth rear sprocket, the gear ratio is 54.

For bicycle riding, a gear ratio of 100 is about tops and is hard to push. Racing bicyclists will often use a gear ratio of about 80 or 90, while "tourists" will usually be satisfied with 60 or 70. A ratio of about 40 is needed for steep terrain and heavy loads. Apparently, most casual bicyclists tend to gear too high and thus pedal too slowly for top efficiency. Research indicates that the bicyclist is most efficient for sustained effort when pedalling at a rate of about 50 revolutions per minute.

If you wish, you can calculate your speed on a bike by knowing the gear ratio and the frequency with which the pedals turn. The distance a bike travels in one turn of the pedals is π times the gear ratio. If the frequency of pedal revolution is f turns per minute, the speed is just:

$$v = \pi f(G.R.)$$

However, this is in units of inches per minute, not the most convenient in the world. To convert to miles per hour, multiply by the factor 9.5×10^{-4}. Then, the speed is:

$$v = 9.5 \times 10^{-4} \pi f(G.R.)$$

For example, as I rode my 10-speed to work this morning, I was in a gear with 52 teeth on the front sprocket and 16 on the rear, giving a G.R. = 88. I found 70 revolutions per minute to be a comfortable pedalling rate, so my speed was:

$$v = 9.5 \times 10^{-4} \pi \times 70 \times 88$$

$$= 18 \text{ mph}$$

One of the students who helped on this module is a racing bicyclist. On the level, he is able to sustain a pedalling rate of 90 revolutions per minute with a gear ratio of 90 for long periods of time. This gives him a speed of about 23 mph.

Answers to Questions Accompanying Goals

Section A.

1. Parallel to ice:

   $\underline{F}$ = 43.3 N.

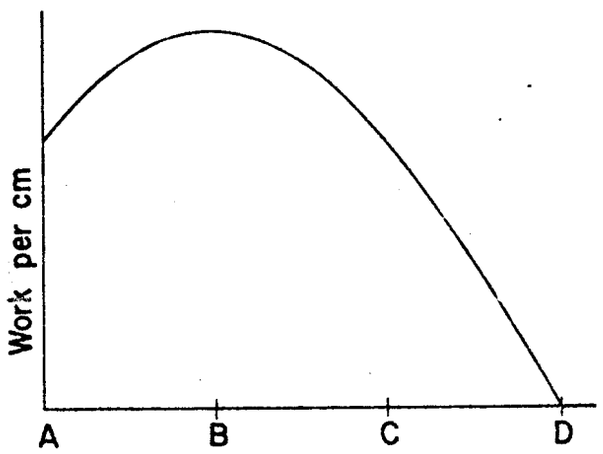
   Perpendicular to ice:

   $\underline{F}$ = 25 N.

2. a. Point B.

   b. Point D.

   c.



3. 480 fpm.

Section B.

1. 200 J.
2. 13 m/s.
3. 2.5×10^4 W. (About 33 horsepower.)
4. 2.1 N.
5. 5.6 N. (Not in very good agreement with your measurements, is it?)

Section C.

1. From the data of Table I, the wind resistance is about half the frictional force at 5 mph. They are about equal (2.5 N) at a speed between 6 mph and 7 mph. And wind resistance is much greater above about 10 mph. Of course, using your data you may get a somewhat different answer.
2. About 20%.

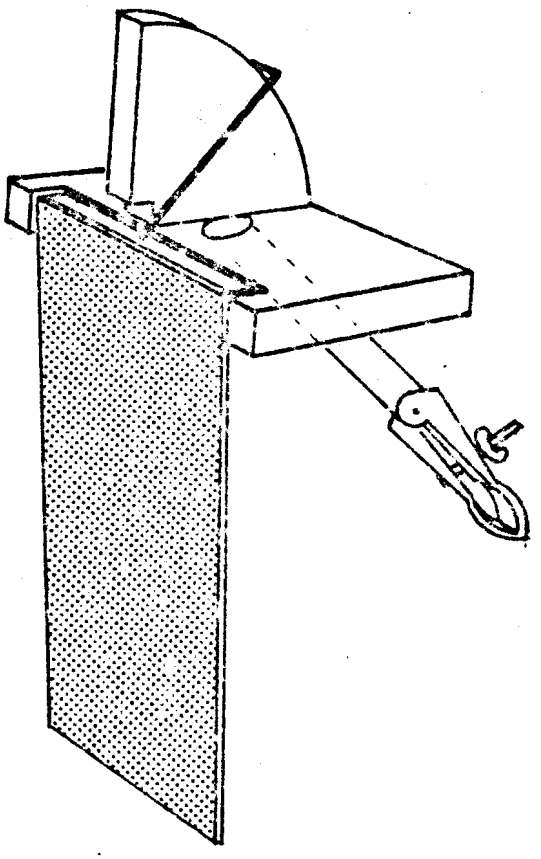
The "Wind Force Measurer"

Here are a couple of drawings of the device, as I built it. You will probably think of improvements, and I would be grateful to know about them.

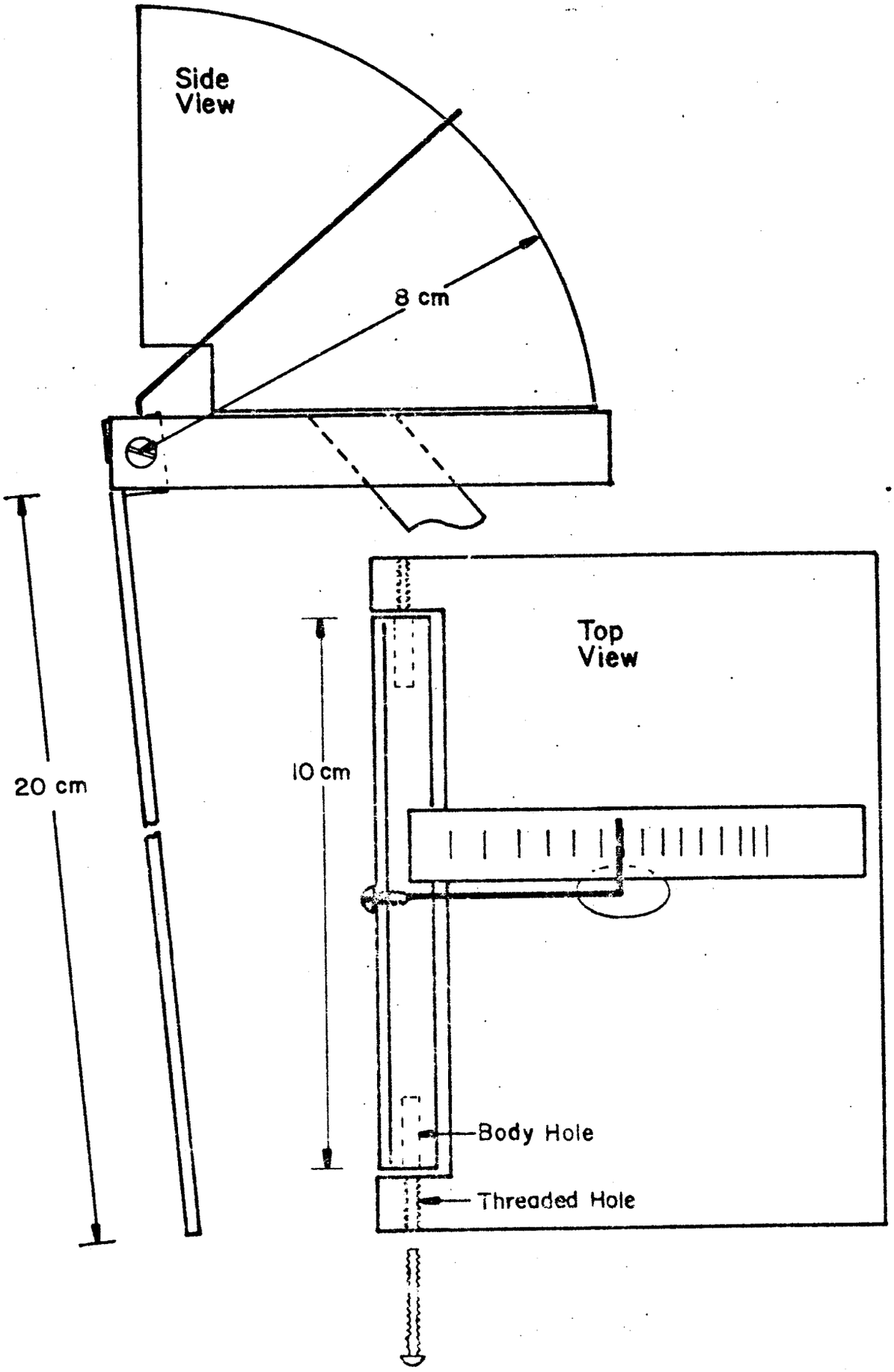
The gadget is made of plastic, and the working part is a sheet of thin (about 3/32") plastic which hangs from a pivot. The top of the sheet is attached to a plastic bar, about 3/8" X 1/2" X 10 cm, and holes in the ends of the bar rest on two screws coming in from the sides. This forms the pivot.

None of the dimensions are critical!

The device is attached to the bike by means of a standard laboratory clamp which fits the hole in the support piece. Keep the clamp as short as possible to avoid a long lever arm which shakes the device excessively when the handle bars jitter a bit. Another helpful hint is to lay out the scale on a piece of paper by using $r\theta$, where r is the radius of the scale and θ is in radians. For a scale radius of 8 cm, it comes out to about 0.7 cm for 5°. To minimize parallax--the rider sees the scale from above--the zero should be about where the pointer is shown in the drawings.



WIND FORCE MEASURER



(Full Scale)