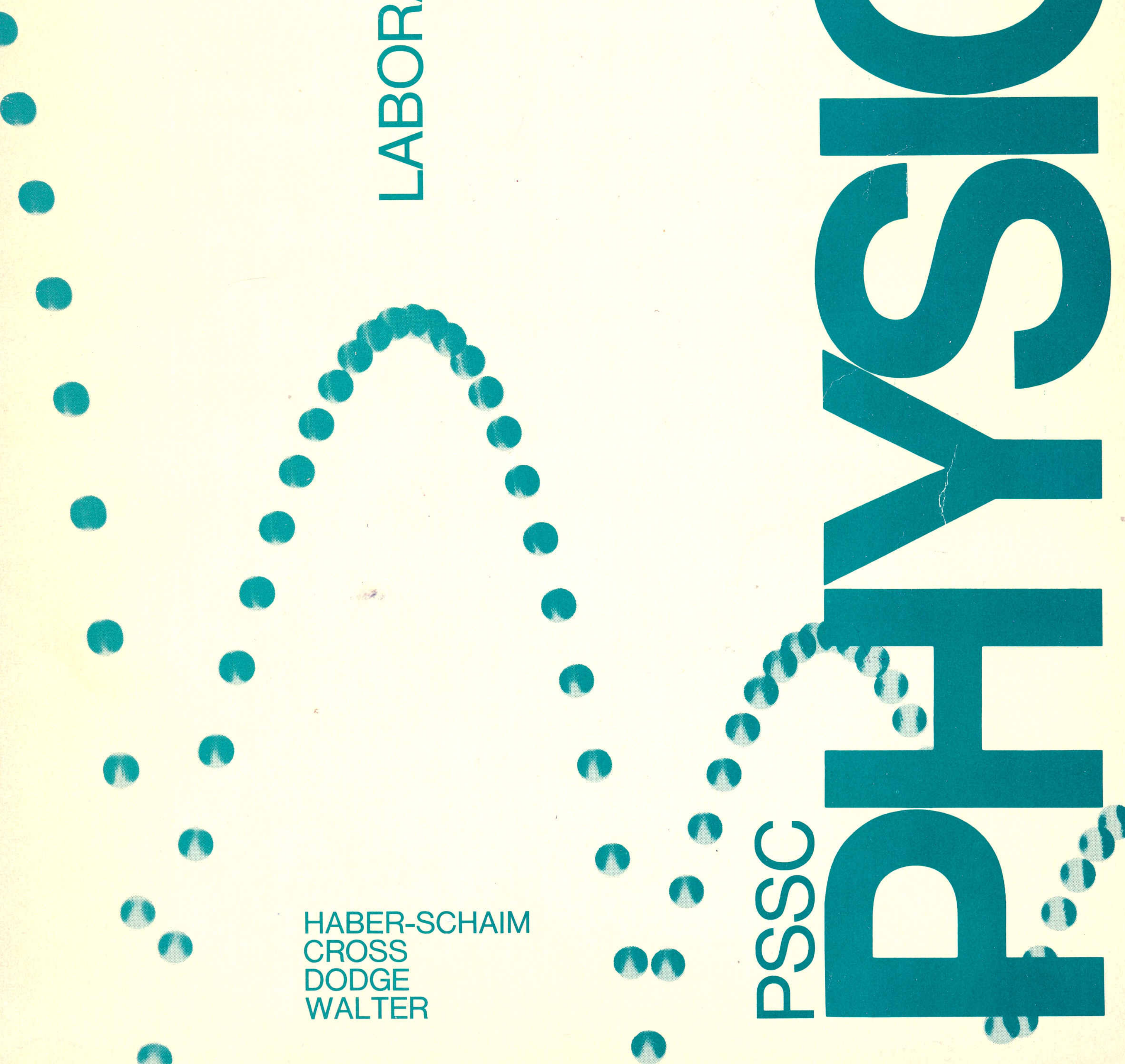
PSSC PHYSICS

THIRD EDITION

LABORATORY GUIDE



HABER-SCHAIM
CROSS
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WALTER

LABORATORY GUIDE

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THIRD EDITION

Uri Haber-Schaim
Judson B. Cross
John H. Dodge
James A. Walter

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Preface

The prefaces to the first and second editions of the PSSC Physics Laboratory Guide contained a strong plea for letting the student experiment in the laboratory without being told in advance what he is supposed to find. This general style of learning science is now so well established that we feel it would be superfluous to repeat the plea here.

As might be expected, much of the material in this Laboratory Guide has been taken over from the earlier editions. However, some significant additions, deletions, and revisions should be mentioned.

There are six new experiments. Several previous experiments have been rewritten, and ten were dropped. These changes were made to increase the cohesion between the experimental and theoretical parts of the course.

Like the textbook, the Laboratory Guide is the result of the efforts of many people. For detailed acknowledgments, see the Appendix to the textbook.

Uri Haber-Schaim
Judson B. Cross
John H. Dodge
James A. Walter
August 1970

To The Student

This Guide is designed to help you with your laboratory work. It provides a general introduction to the problems at hand, gives you technical hints, but leaves the thinking to you.

Throughout this Guide you will find many questions. Finding the answer to these questions may sometimes require a little thought about what you have done before, or it may require a short calculation. Sometimes more experimentation will be called for. It is up to you to decide what to do in each case.

Good working habits are useful. Always read the whole description of an experiment before you begin to work so you will have a clear understanding of what you are trying to do. Keep a clear record of your experiment as you perform it. Then you will have the data to refer to when needed, and sufficient information to know what you did.

In the course of an experiment, whenever necessary, repeat your measurements a few times. Several readings are usually better than one. You should decide when more measurements are needed.

Many of these experiments require the help of one or more partners. Discuss results with your partners. You may learn more by working together on an analysis than by going at it alone.

You will probably not find it possible to do all the parts of every experiment. Do not rush; you will get more out of doing half the things suggested in an experiment thoroughly than all of them superficially. Often, part of the analysis may be done at home.

The apparatus used in most experiments is quite simple. You can make many items yourself and experiment further at home.

Table of Contents

Analysis of an Experiment

1

The presentation and analysis of experimental results is an essential part of physics. In Table 1 are the results of an experiment. You are asked to present and analyze these results in a form which will enable you to predict the outcome of similar experiments.

The experiment was an investigation of the time it takes water to pour out of a can through a hole in the bottom. As you would expect, this time depends on the size of the hole and the amount of water in the can.

To find the dependence on the size of the hole, four large cylindrical containers of water of the same size were emptied through relatively small circular openings of different diameters. To find the dependence on the amount of water, the same containers were filled to different heights.

Each measurement was repeated several times, and the averages of the times (in seconds) that each container took to empty have been entered in the table. A stop watch operated by a human hand cannot be trusted to measure less than a tenth of a second. The last digit in each time entry in the table may be in error by one unit either way. Therefore, the relative (or fractional) error is larger for shorter times than for longer times.

	h in cm			
d in cm	30.0	10.0	4.0	1.0
1.5	73.0	43.5	26.7	13.5
2.0	41.2	23.7	15.0	7.2
3.0	18.4	10.5	6.8	3.7
5.0	6.8	3.9	2.2	1.5

Table 1
Times to Empty (sec)

All the information we shall use is in the table, but a graphical presentation will enable us to make predictions and will greatly facilitate the discovery of mathematical relationships.

First, plot the time versus the diameter of the opening for a constant height, say 30.0 cm. It is customary to mark the independent variable (in this case, the diameter d) on the horizontal axis and the dependent variable (here the time t) on the vertical axis. To get maximum accuracy on your plot, you will wish the curve to extend across the whole sheet of paper. Choose your scales on the two axes accordingly, without making them awkward to read.

Connect the points by a smooth curve. Is there just one way of doing this? From your curve, how accurately can you predict the time it would take to empty the same container if the diameter of the opening was 4.0 cm? 8.0 cm?

Although you can use the curve to interpolate between your measurements and roughly extrapolate beyond them, you have not yet found an algebraic expression for the relationship between t and d. From your graph you can see that t decreases rather rapidly with d; this suggests some inverse relationship. Furthermore, you may argue that the time of flow should be simply related to the area of the opening, since the larger the area of the opening, the more water will flow through it in the same time. This suggests trying a plot of t versus $1/d^2$.

To do this, add a column for the values of $1/d^2$ in your notebook and, again choosing a convenient scale, plot t versus $1/d^2$ and connect the points with a smooth curve. What do you find? Was your conjecture correct? Can you write down the algebraic relation between t and d for the particular height of water used?

To find whether this kind of relationship between t and d also holds when the container is filled to different heights, on the same sheet of graph paper plot the graphs of t versus $1/d^2$ for the other heights. What do you conclude?

Notice that the graph for $h = 1.0$ cm extends upward very slightly. Make a special plot of these data on a larger time scale so that you will use the whole sheet. What do you observe? On the basis of your data, what can you say about the algebraic relation between t and d for $h = 1.0$ cm?

Now investigate the dependence of t on h when the diameter of the opening stays fixed. Take the case of $d = 1.5$ cm, which is the first row. Make a plot in which h will be marked on the horizontal axis and connect your points by a curve. Extrapolate the curve toward the origin. Does it pass through it? Would you expect it to do so?

How can you use your plots of t versus $1/d^2$ to predict t for $h = 20.0$ cm and $d = 4.0$ cm?

There is no simple geometric consideration to guide us to the right mathematical relation between t and h. You can try to guess it from the curve. It may be helpful to rotate the graph paper through 90° and look first at h as a function of t, and then at t as a function of h. If you succeed, check by proper graphing to see if the same kind of relation between t and h holds for $d =$ 5.0 cm.

If you are familiar with logarithms, you can check to see if the relation belongs to a general class of relations, such as a power law, $t \propto h^n$. To do this, plot $\log t$ versus $\log h$ (or simply t versus h on log-log paper). What do you obtain? What is the value of n?

Can you find the general expression for time of flow as a function of both h and d? Calculate t for $h = 20.0$ cm and $d = 4.0$ cm and compare the answer with that found graphically. Which do you think is more reliable?

Reflection from a Plane Mirror

2

Hold a pencil vertically at arm's length. In your other hand, hold a second pencil about 15 cm closer than the first. Without moving the pencils, look at them while you move your head from side to side. Which way does the nearer pencil appear to move with respect to the one behind it when you move your head to the left? Now move the pencils closer together and observe the apparent relative motion as you move your head. Where must the pencils be if there is to be no apparent relative motion, that is, no parallax, between them?

Now we shall use parallax to locate the image of a nail seen in a plane mirror. Support a plane mirror vertically on the table by fastening it to a wood block with a rubber band. Stand a nail on its head about 10 cm in front of the mirror. Where do you think the image of the nail is? Move your head from side to side while looking at the nail and the image. Is the image in front of, at the same place as, or behind the real nail? Locate the position of the image of the nail by moving a second nail around until there is no parallax between it and the image of the first nail. In this way, locate the position of the image for several positions of the object. How do the distances of the image and object from the reflecting surface compare?

We can also locate the position of an object by drawing rays which show the direction in which light travels from it to our eye. Stick a pin vertically into a piece of paper resting on a sheet of soft cardboard. This will be the object pin. Establish the direction in which light comes to your eye from the pin by sticking two additional pins into the paper along the line of sight. Your eye should be at arm's length from the pins as you stick them in place so that all three pins will be in clear focus simultaneously. Look at the object pin from several widely different directions and, with more pins, mark the new lines of sight to the object pin. Where do these lines intersect?

We can use the same method to locate an image. On a fresh piece of paper, locate the position of the image of a pin seen in a plane mirror by tracing at least three rays from widely different directions. Mark the position of the mirror before removing it. Where do the lines of sight converge?

Draw rays showing the path of the light from the object pin to the points on the mirror where the light was reflected to your eye. What do you conclude about the angles formed between the mirror surface and the light paths?

Arrange two mirrors at right angles on the paper with a nail as an object somewhere between them. Locate all the images by parallax. From what you have learned about reflection in this experiment, show that these images are where you would expect to find them.

3 Refraction

It is convenient to study the refraction of light in terms of the angle of incidence and the angle of refraction. When light passes from air into water, for example, the angle of refraction is the angle between a ray in the water and the normal to the water surface. In this experiment we shall try to find the relation between this angle and the angle of incidence.

Use a pin to scratch a vertical line down the middle of the straight side of a semicircular, transparent plastic box. Fill the box half full of water and align it on a sheet of polar graph paper resting on soft cardboard as shown in Fig. 1, making sure the bottom of the vertical line on the box falls on the center of the graph paper. Stick a pin on the line passing beneath the center of the box as shown in the figure. Be sure the pin is vertical.

Now look at the pin through the water from the curved side and move your head until the pin and the vertical mark on the box are in line. Mark this line of sight with another pin. What do you conclude about the bending of light as it passes from air into water and from water into air at an angle of incidence of 0°?

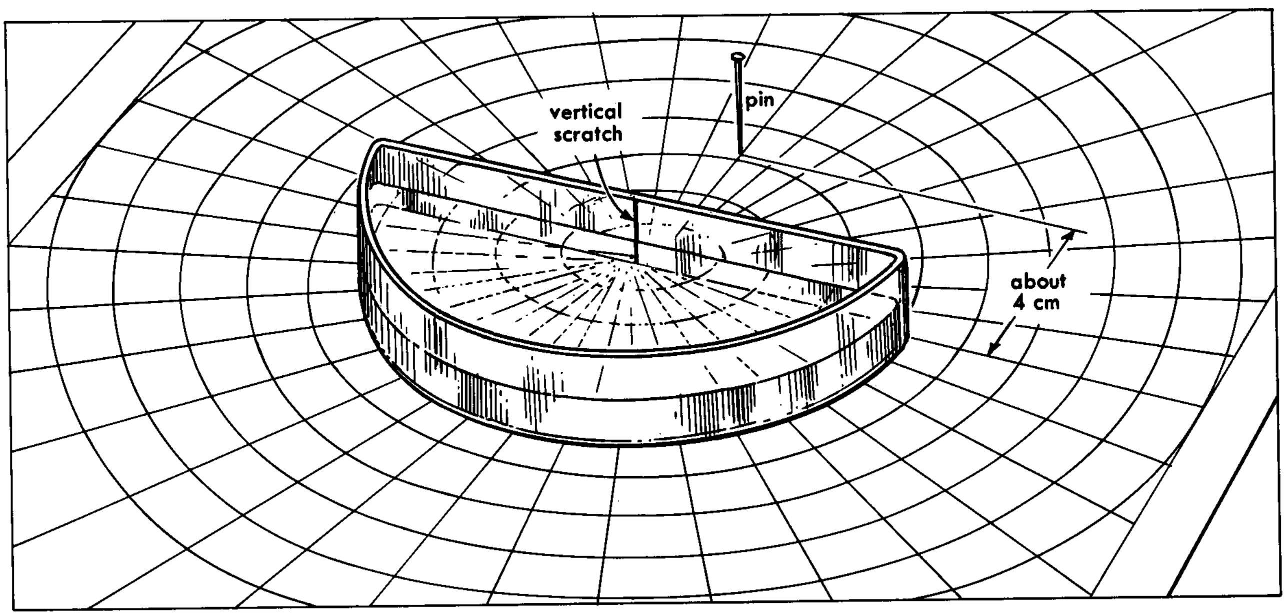


Figure 1

Change the position of the first pin to obtain an angle of incidence of about 20°. With the second pin, mark the path of light going from the first pin to the vertical line on the box and through the water. Repeat this for angles of incidence up to about 80°. To ensure a sharp image of the first pin at large angles, it should never be placed more than 4 cm away from the vertical line on the box. (The pinholes give a permanent record of the angles.)

Is the difference between the angles of incidence and of refraction constant? Is their ratio constant?

Plot the angle of refraction as a function of the angle of incidence. Also plot the sine of the angle of refraction as a function of the sine of the angle of incidence. What simple mathematical relation do you think best describes the refraction of light?

Is the path of the light through the water the same when its direction is reversed? Investigate this with your apparatus.

Can you predict how light will bend when it goes obliquely through a block of glass with parallel sides?

Repeat the experiment, using another liquid in the box; again plot the sine of the angle of refraction as a function of the sine of the angle of incidence." Does this liquid refract differently from water?

4 Images Formed by a Converging Lens

Look through a converging lens at an object. Is the image you see larger or smaller than the object? Is it right side up or upside down? Do the size and position of the image change when you move the lens with respect to the object?

To investigate the images formed by a converging lens, arrange a lens and a lighted flashlight bulb on a long strip of paper as shown in Fig. 1. Start with the bulb at one end of the paper tape and locate its image by parallax. Is the image right side up or upside down?

Now move the object toward the lens in small steps, marking and labeling the positions of both object and image as you go. Continue this until the image moves off the end of the tape and can no longer be recorded. How does the change in the position of the image compare with that of the object? Where (on your tape) do you expect the image to be when the object is at least several meters away? Check it. With the object far away, you may find it easier to locate its image on a piece of paper. The location of the image when the object is very far away is the principal focus of the lens. How can you convince yourself that the lens has two principal foci, one on each side and at the same distance from the center?

Now place the bulb as close to the lens as possible and again locate the image by parallax. Is it upside down or right side up? Again move the object away from the lens in small steps, marking and labeling the positions of object and image until the image is no longer on the tape.

Measure S_o and S_i, the distance from the principal foci to the object and image respectively, for the pairs of points. (The distance S_o is measured from the principal focus on the object side of the lens and S_i is always measured from the principal focus on the opposite side from the object.) Since S_i clearly decreases when S_o increases, try plotting S_i as a function of $1/S_o$. What do you conclude about the mathematical relation between S_o and S_i?

Where will the image be if the object is placed at the principal focus? Can you see it?

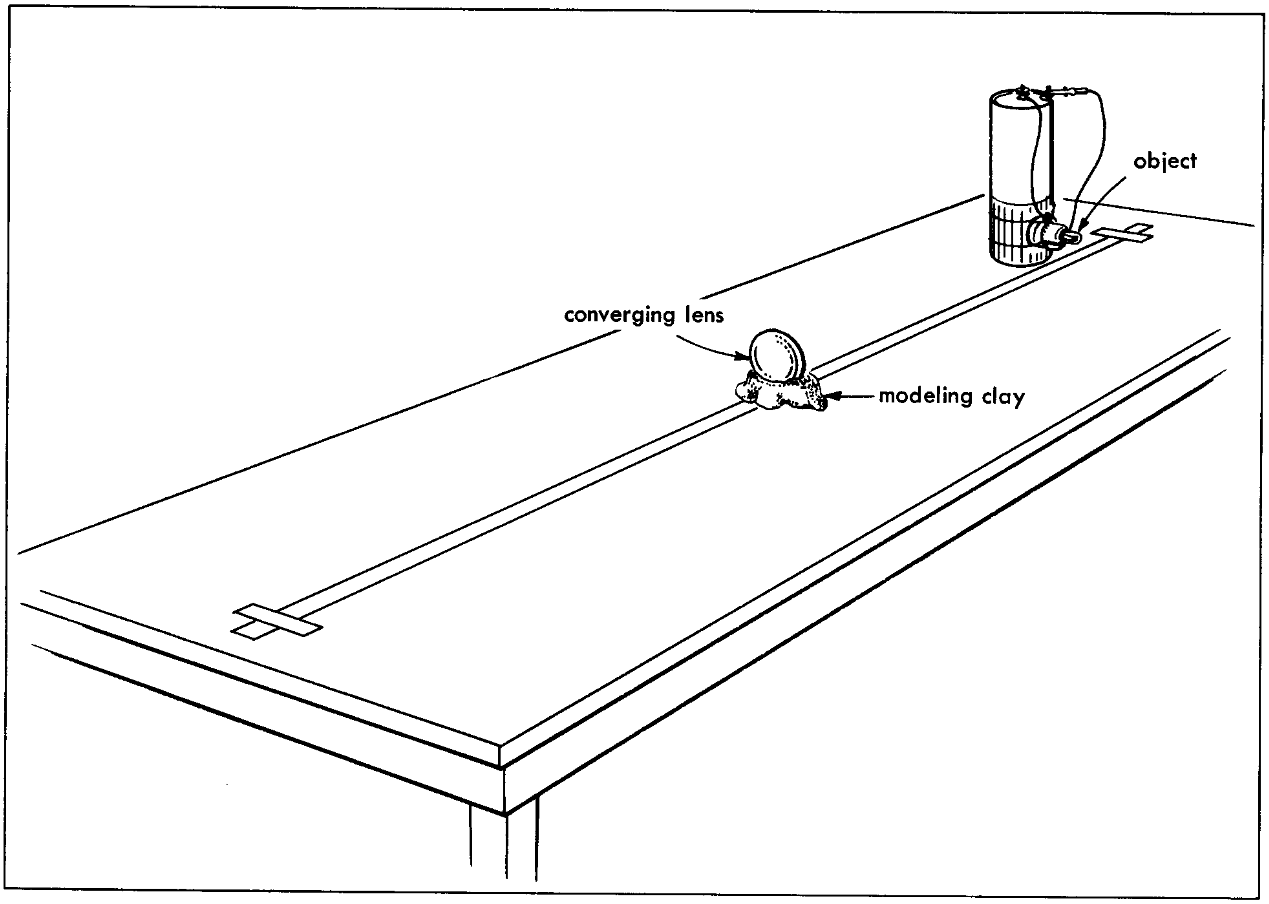


Figure 1

5 A Diverging Lens

You could study the properties of a diverging lens by forming images as you did with the converging lens in Experiment 4. However, you may also investigate the properties of a lens by observing its effect on a parallel light beam. You can use a light bulb placed at the principal focus of a converging lens to get the parallel beam. It is best to work with a narrow beam which you can make by mounting the converging lens directly behind a barrier with a circular hole. Both lens and barrier can be supported by a piece of plasticine as in Experiment 4.

Now let the parallel beam pass through the diverging lens and strike a piece of paper. Measure the diameter of the light circle for different distances from the paper to the lens. Plot the diameter of the circle as a function of the distance. From the graph can you find the principal foci? Can you get a magnified image with a diverging lens? Can you get a real image with a diverging lens?

Now place the light bulb at one of the principal foci. Try to estimate the size of the image compared to the size of the object and to find how far behind the lens the image is formed. See if you can support your conclusions by theoretical considerations, for example, by sketching a few rays from the top of the object.

The "Refraction" of Particles

6

A steel ball rolling across a horizontal surface moves in a straight line at nearly constant speed. If the ball intercepts a slope obliquely, the speed it gains as it rolls down the slope will change its direction. At the bottom of the slope it will move off in a straight line in a direction different from its original direction.

The path of a ball moving this way resembles the path of light as it is refracted in going, for example, from air into glass. In going from the top to the bottom of the slope, the ball changes direction; at the interface between two media, light changes direction. In the model, therefore, the upper level corresponds to one medium (air); the lower level corresponds to the other medium (glass); the slope corresponds to the interface between them.

Examine the paths of "refracted" particles to see if they change direction according to Snell's law, with the apparatus shown in Fig. 1. Letting the steel

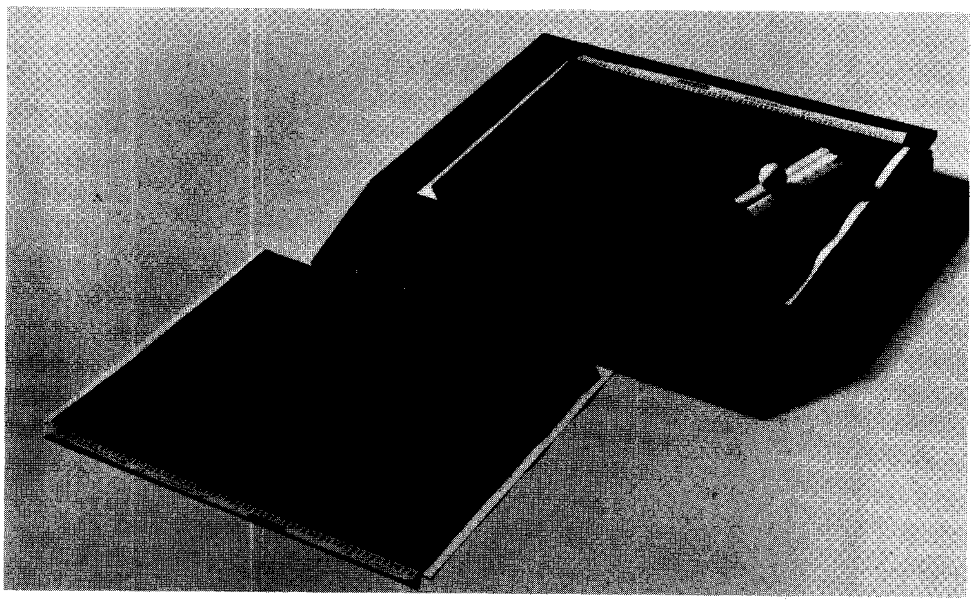


Figure 1
Arrange two horizontal surfaces connected by a short slope. Make sure that the two surfaces are level. Tape a sheet of white paper on each surface so that the edges coincide with the top and bottom edges of the slope and place sheets of soft carbon paper over the white sheets.

ball roll down the full length of the launching ramp in different directions will correspond to different angles of incidence. Remove the carbon paper and, for identification, label the tracks made by the ball on the upper and lower planes after each run. Why must you be careful to start the ball from the same point on the launching ramp?

Measure and record the angles of incidence and refraction as measured from normals to the horizontal edges of the slope. Can this change in direction of the ball be described by Snell's law? What does this particle model of light predict about the speed of light in water compared with its speed in air?

Could you make a "lens" that would focus rolling balls?

Waves on a Coil Spring

7

You probably have seen various kinds of waves but have not experimented with them. With this experiment you will begin a detailed study of waves.

While your partner holds one end of a coil spring on a smooth floor, pull on the other end until the spring is stretched to a length of about 10 meters. With a little practice you will learn to generate a short, easily observed pulse. Look at the pulse as it moves along the spring. Does its shape change? Does its speed change?

Shake some pulses of different sizes and shapes. Does the speed of propagation depend on the size of the pulse? To find the speed more accurately you can let the pulse go back and forth a few times, assuming that the speed of the pulse does not change upon reflection. How do you check this assumption?

Change the tension in the spring. Does this affect the speed of the pulse? Would you consider two springs of the same material stretched to different lengths to be the same or different media?

You and your partner can send two pulses at the same time. What happens to the pulses as they collide? Try it with pulses of different sizes and shapes, traveling along the same side and along opposite sides of the spring.

When the pulses meet, how does the maximum displacement of the spring compare with the maximum displacement of each pulse alone? You can determine the largest displacement of an individual pulse by moving your hand a measured distance as it generates the pulse. A third partner can mark on the floor with chalk the largest displacement of the spring when the pulses meet.

You can investigate the passage of waves from one medium to another by tying together two coil springs on which waves travel with different speeds (Fig. 1). Send a pulse first in one direction and then in the other. What happens when the pulses reach the junction between the two springs?

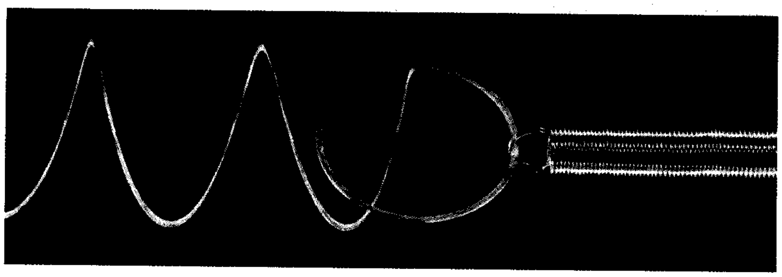


Figure 1

Tie a spring to a long, thin thread (Fig. 2). How does a pulse sent along the spring reflect when it reaches the thread? How does this reflection compare with that at a fixed end? Is the speed of the pulse on the thread greater or less than that on the spring?

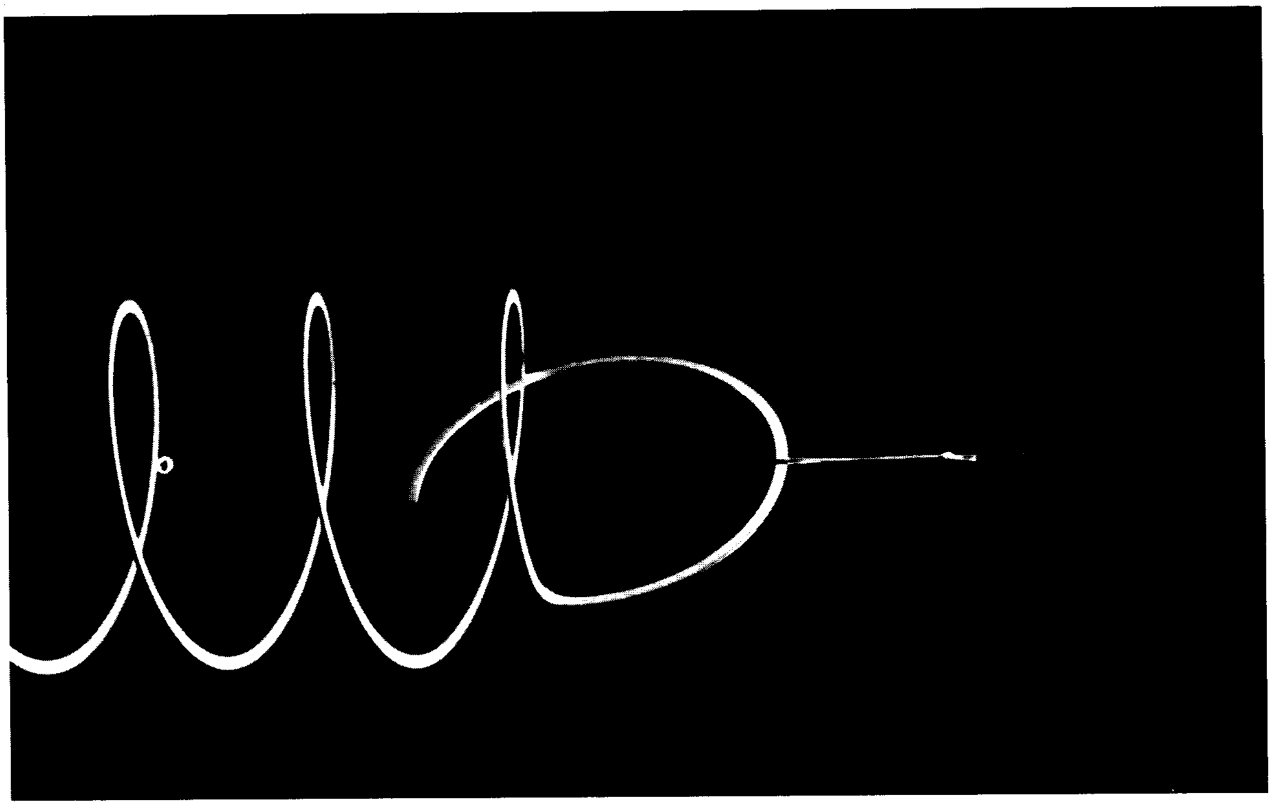


Figure 2

Pulses in a Ripple Tank

8

Set up a ripple tank, screen, and light source as shown in Fig. 1. Fill the tank with water to a depth of ½ to ¾ cm and measure the depth at all four corners to be sure the tank is level.

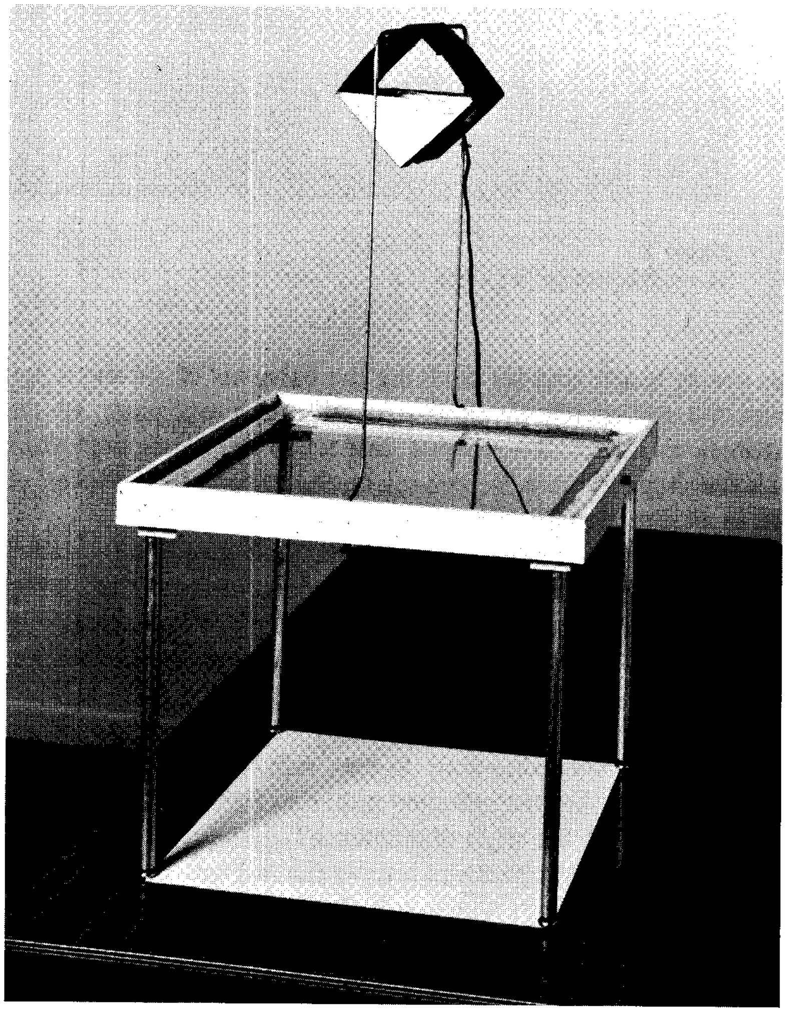


Figure 1

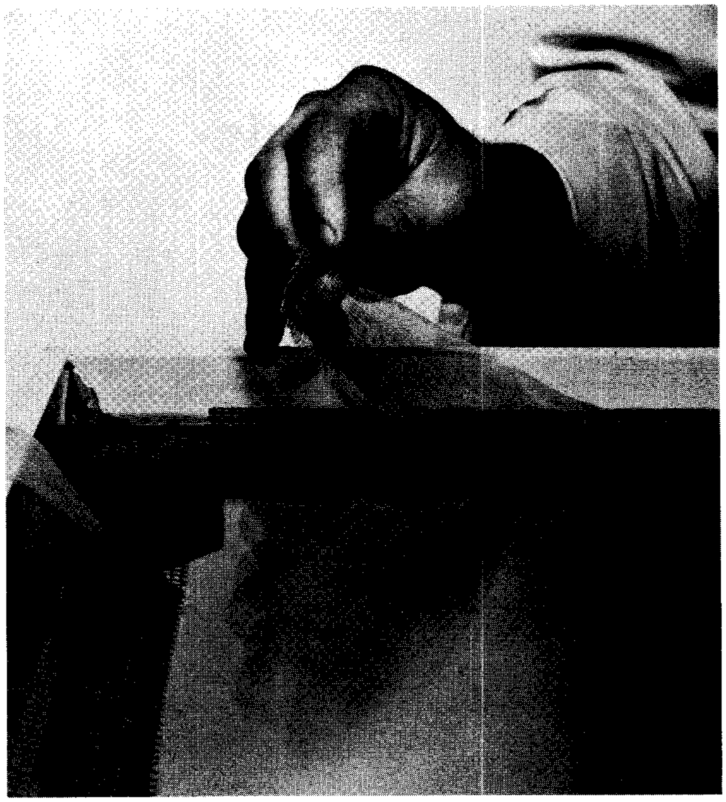


Figure 2
Shows how damper is placed.

You now have a very handy tool for studying the behavior of waves; it has an advantage over the coil spring, since the direction of propagation of the waves is not restricted to a line. To see this, just touch the water with your finger tip. What is the shape of the pulse you see on the screen? Is the speed of the pulse the same in all directions?

You can also generate straight pulses in the ripple tank by rolling a dowel through a fraction of a revolution in the water. (Place your hand flat on top of the rod and then move it forward about a centimeter.) Practice making such pulses until you can make them give sharp, bright images on the screen. Do the pulses remain straight as they move along the tank?

Place a straight barrier in the tank and generate pulses that strike it at an angle of incidence of 0°. In what direction do they reflect? Reflect pulses at different angles of incidence. Are the reflected pulses straight? How does the angle of reflection compare with the angle of incidence?

Instead of making direct measurements to answer the last question quantitatively, you can study a few situations which will clearly demonstrate the relation between the two angles. For example, observe how a circular pulse reflects from a straight barrier. Can you locate the virtual source of the reflected pulse (the image of the source of the incident pulse)? How would you explain the result?

Parabolic Mirrors 9

Try using a rectangular strip of metal bent in the form of a parabola to reflect straight pulses and periodic waves in a ripple tank. Have the waves reflect from the concave side of the parabola. How are the waves reflected? Try to follow the motion of several small segments of the wave fronts. How would you indicate the direction of motion of each segment? Draw a diagram using rays to show the path of the waves. How far from the parabola along the axis do the waves converge? What happens if you dip your finger in still water at the point where the waves converge?

If the metal parabola is placed on a white paper you can observe the paths of light waves reflected from it. Use a distant bright lamp as a source of light. How does the reflection of the light compare with that of the water waves? Does the light converge at the same point as the water waves?

10 Periodic Waves

The relation $v = f\lambda$ for the speed, frequency, and wavelength of a periodic wave holds for all periodic waves. We shall now apply this relation to waves in the ripple tank and on a coil spring.

Set up the straight wave generator as shown in Fig. 1 (the water should be ½ to ¾ cm deep). Practice using it at various frequencies. Look at the waves through your stroboscope (2 or 4 open slits) and "stop" their motion.

Adjust the wave generator to a low frequency and have your partner help you measure the frequency of rotation of the stroboscope while you "stop" the waves. How is this frequency related to that of the waves?

To find the wavelength, "stop" the wave pattern with the stroboscope and have your partner place two pencils or rulers parallel to the waves and several wavelengths apart.

Make several measurements of frequency and wavelength and calculate the

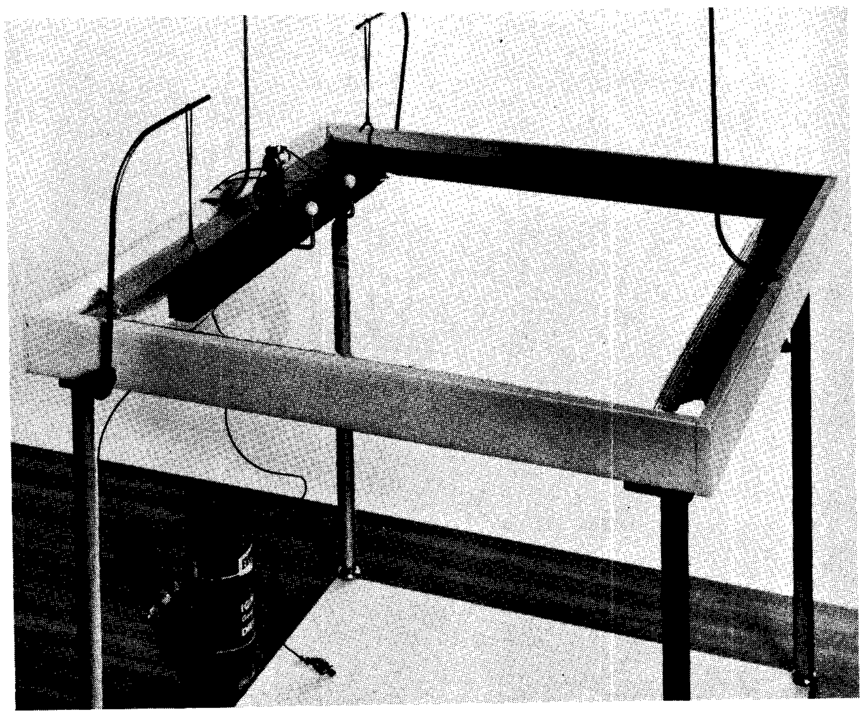


Figure 1

speed of propagation. How accurate is your determination of the speed? Notice that you have measured the wavelength of the image of the waves on the screen. How is this apparent wavelength related to the true wavelength of the water waves?

The wave pattern may also be stopped by placing a barrier in the middle of the tank as shown in Fig. 2. The incident and reflected waves superpose

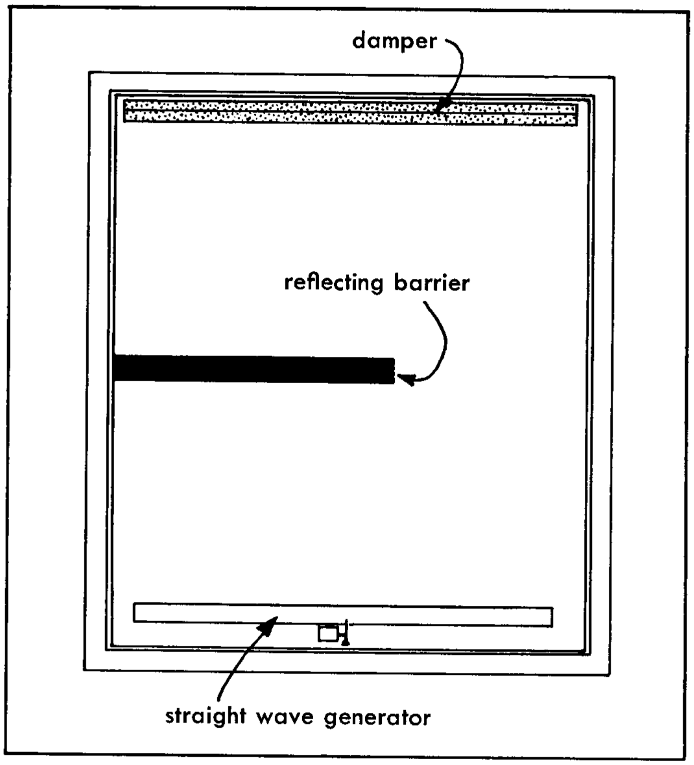


Figure 2

to give a stationary pattern—that is, a standing wave. How does the distance between two bright bars in the standing wave compare with that in the traveling wave? Can you measure the wavelength from the standing wave pattern?

Can you detect a change in speed when the depth of water is changed to about 2 cm?

Moving your hand only slightly, shake a periodic wave into a coil spring. Adjust the frequency until a standing wave builds up. By measuring wavelength and frequency, determine the speed of the wave on the spring.

Without changing the length of the spring, can you produce standing waves of any wavelength you choose?

If you have two coil springs on which pulses travel at different speeds, hook them together, end to end. Try to generate a standing wave in both. Fix one end of the pair and shake the other end. How do the frequencies, the wavelengths, and the speeds in the two media compare?

11 Refraction of Waves

In Experiment 10 we found that the speed of water waves depends on the depth of the water. Two different depths of water therefore constitute two different media in which waves can be propagated. This suggests that water waves can be refracted, for example, by allowing them to travel from deep water into shallow water.

Support a glass plate in the ripple tank so that its top surface is at least 1.5 cm above the bottom of the tank. Add water to the tank until it is no more than 0.2 cm deep over the glass plate. Be sure the depth of the water is uniform over the glass plate.

What do you predict will happen if straight periodic waves originating in the deep water cross into the shallower water when the boundary between the two media is parallel to the wave generator (Fig. 1)? Using very low-frequency waves, check your prediction with a stroboscope.

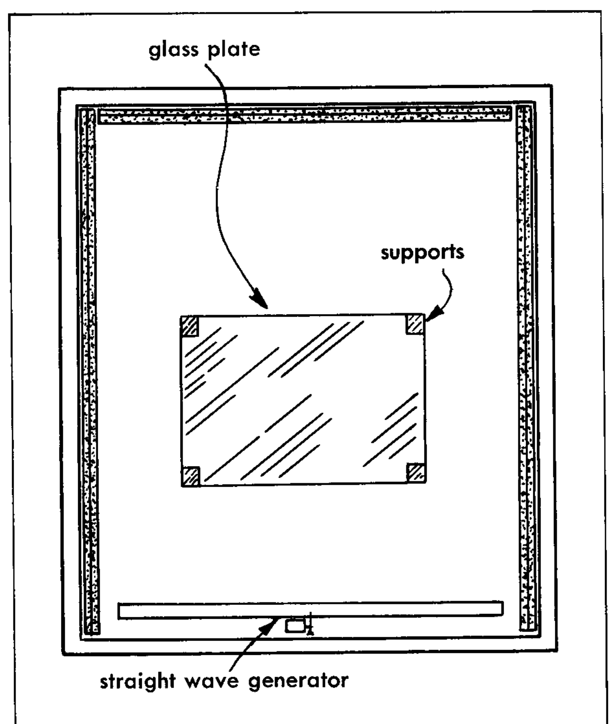


Figure 1

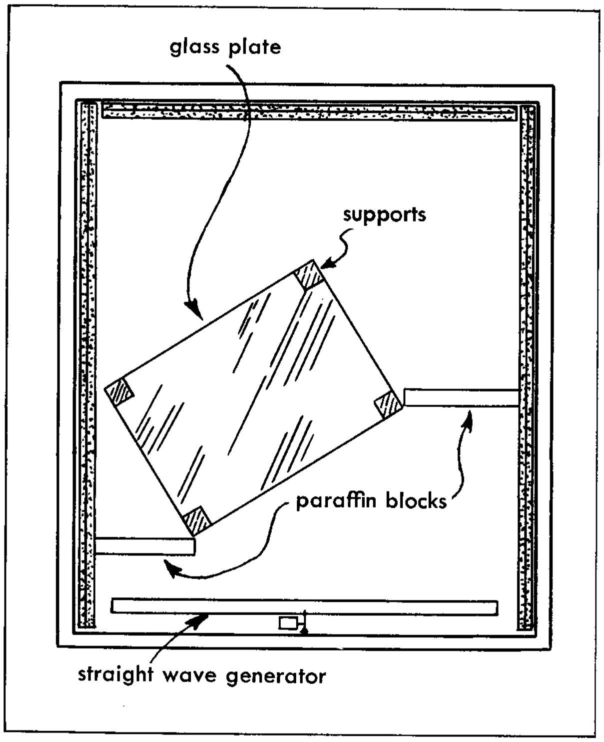


Figure 2

Now turn the glass plate so that the boundary is no longer parallel to the incident waves (Fig. 2). Are the refracted waves straight? How does the angle of refraction compare with the angle of incidence? How do the wavelengths in the two sections compare? What about the speeds? While keeping the generator running (to keep the frequency constant) try other angles of incidence.

Does a wave model agree with the refraction of light better than a particle model if we consider in which medium the speed of light is greater?

To establish the quantitative relation between the angles of incidence and refraction requires considerable care. Keeping the frequency constant, you can measure the angle of refraction for four or five different angles of incidence. Over what range should you choose the angles of incidence? What do you conclude from your results?

12 Waves and Obstacles

An opaque object placed in the path of a parallel beam of light will cast a sharp shadow on a screen behind it. The shadow will be the same size as the object. What happens when we place an obstacle in the path of a straight wave?

Place a small, smooth paraffin block in the ripple tank about 10 cm from the straight wave generator (Fig. 1), and generate periodic waves of long wavelength. Do the waves continue in their straight path on both sides of the block? Could you sense the presence of the block by looking at the pattern only near the far end of the screen? Does the block cast a sharp shadow?

How is the pattern behind the block affected when the wavelength is reduced by increasing the frequency? (To obtain clean waves at high frequency, the generator must be very smooth. Make sure there are no bubbles on its edge.) At high frequency the pattern is best seen by viewing it through the

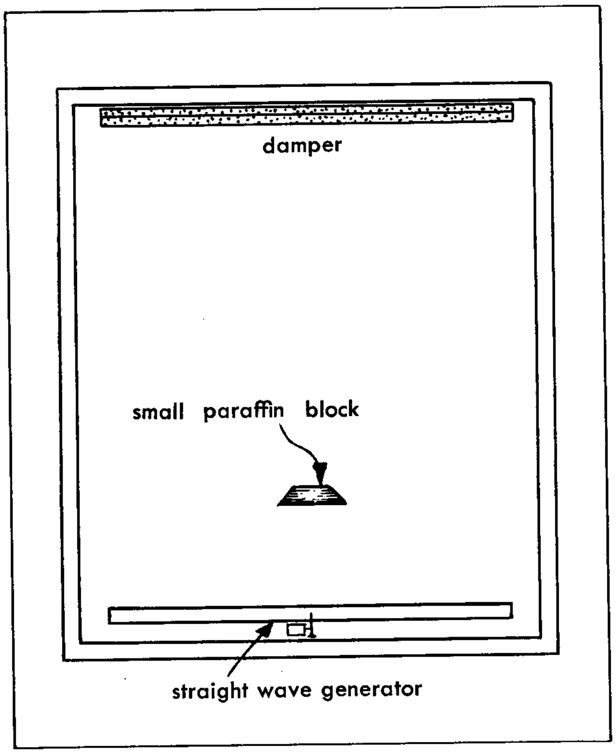


Figure 1

stroboscope with all slits open. Under what conditions would you expect the block to cast a sharp shadow?

We can let a parallel beam of light pass through a small opening. If a screen is held behind the opening, we shall see a light spot equal in size to the opening.

You can produce an analogous situation in the ripple tank (Fig. 2). Are waves of long wavelength still straight beyond the slit? Do the waves continue to move in their original direction? What happens when you decrease the wavelength step by step? Show in a few sketches how the pattern changes.

Now that you have observed the effect of the wavelength on the wave pattern behind the slit, how does changing the width of the slit affect the pattern? Try it with a medium wavelength. How must you adjust the wavelength to compensate for the change in pattern?

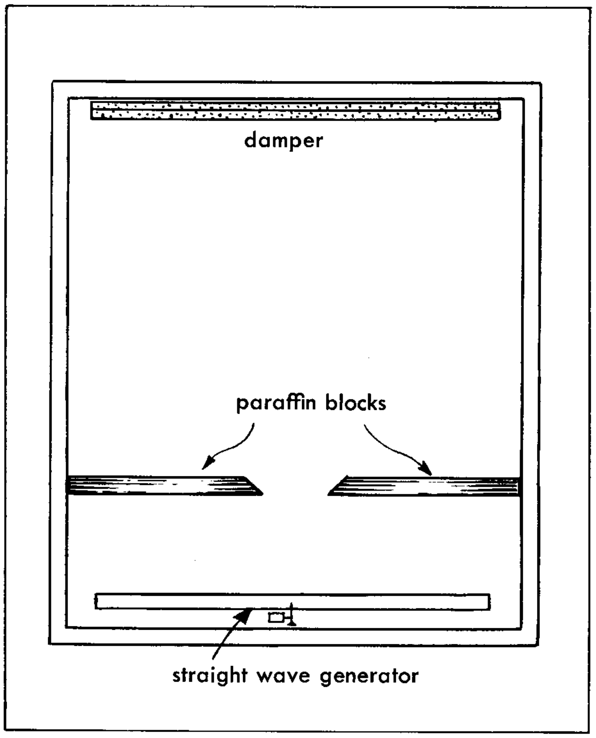


Figure 2

13 Waves from Two Point Sources

What will happen if two point sources near each other generate periodic waves of the same frequency? Try it in the ripple tank with the two point sources attached to the straight wave generator about 5 cm apart. How would you describe the resulting pattern? Are there regions where the waves from the two sources cancel each other at all times? How does the pattern change when you change the frequency?

Change the distance between the sources without stopping the motor (to keep the frequency as nearly constant as possible). How does this affect the pattern?

By applying the principle of superposition you have learned that for two point sources in phase, the direction of the nth nodal line far from the sources is given by

$$\sin\theta_n = \frac{x}{L} = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}.$$

Check this prediction by finding the wavelength from the above relation, measuring x, L, n, and d, and comparing it with a direct measurement of the wavelength.

You will recall that straight waves passing through a narrow slit are strongly diffracted. If the slits are narrow enough, they will act like point sources. Can you repeat the present experiment, using the straight wave generator and two slits made with an arrangement of paraffin blocks?

Interference and Phase 14

In the last experiment we investigated the interference pattern produced by two point sources in phase. In this experiment we shall learn how a change in the phase delay between the two point sources affects the direction of the nodal lines in the interference pattern.

A generator in which the phase delay can be adjusted is shown in Fig. 1. Choose a separation between the sources and a wavelength similar to those used in the preceding experiment, and set the sources in phase. Do you obtain the same kind of pattern you obtained with your regular generator?

Now change the phase in small steps and observe the change in the direction of the nodal lines. Using the in-phase pattern as a reference, how does the position of the first nodal line change as you change the phase delay from zero to one? How does the position of the second nodal line change?

How would you expect the interference pattern to look if you could change the phase of the sources while the generator operates?

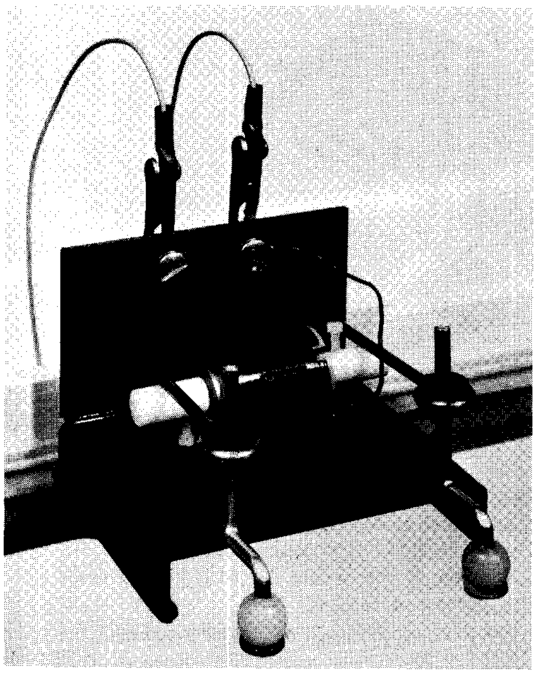


Figure 1
The two plastic dowels on both sides of the motor are mounted off center. If both plastic set screws are up at the same time, the sources are in phase. If one is up when the other is down, as shown in the photograph, the phase delay is one-half.

15 Young's Experiment

We have seen the interference pattern made by two point sources in the ripple tank. If we looked at two light sources in phase, we would expect to see light of maximum intensity in certain directions and no light in other directions (the directions of the nodal lines). From the direction of the nodal lines and the separation of the sources we can calculate the wavelength of light.

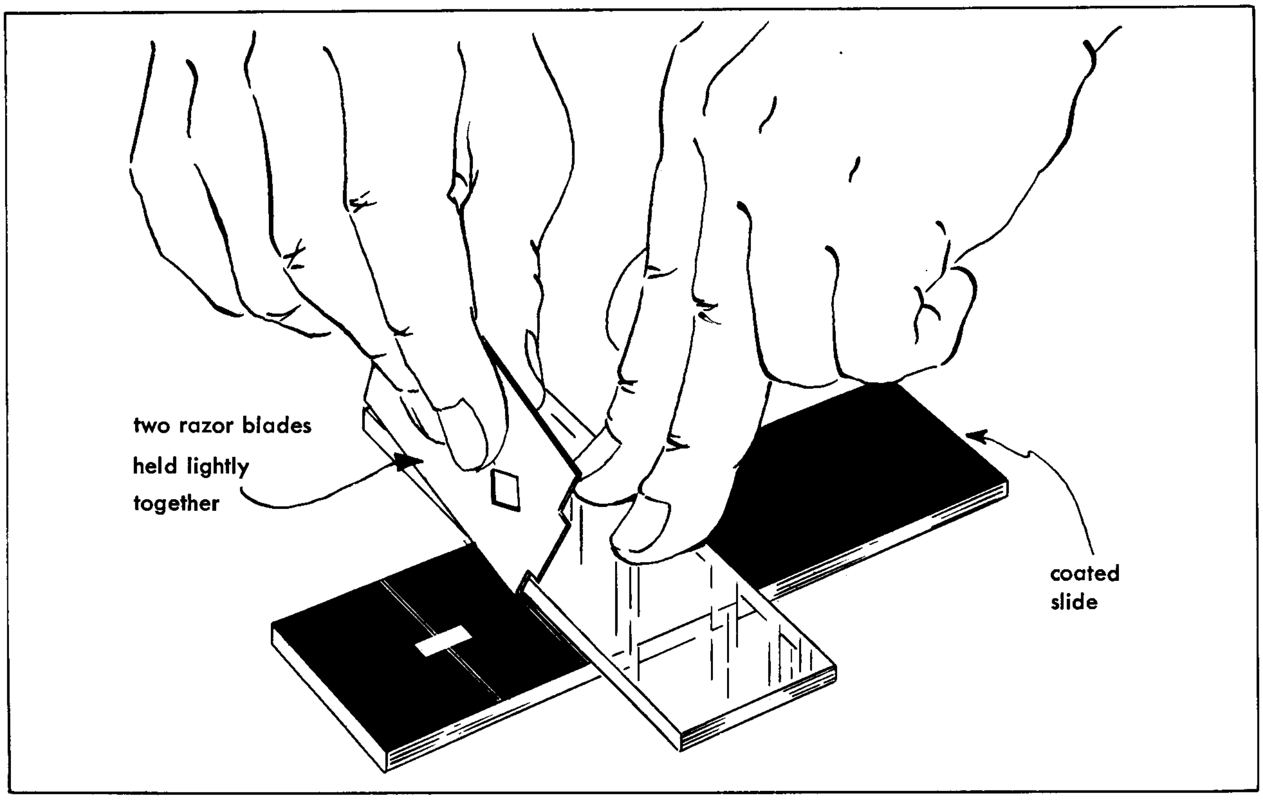


Figure 1
Coat a glass slide with a colloidal suspension of graphite and let it dry. Scratch a pair of slits as shown, holding the razor blades tightly together and using hard pressure. Make several pairs of slits. Select for use those which show at least three clear, white lines when you look at the showcase lamp. Scratch a "window" across each pair of slits. This will enable you to see the pattern through the slits and read a scale at the same time.
To prevent damage to the slits, it is worthwhile to cover the coated slide with a plain slide and tape them together.

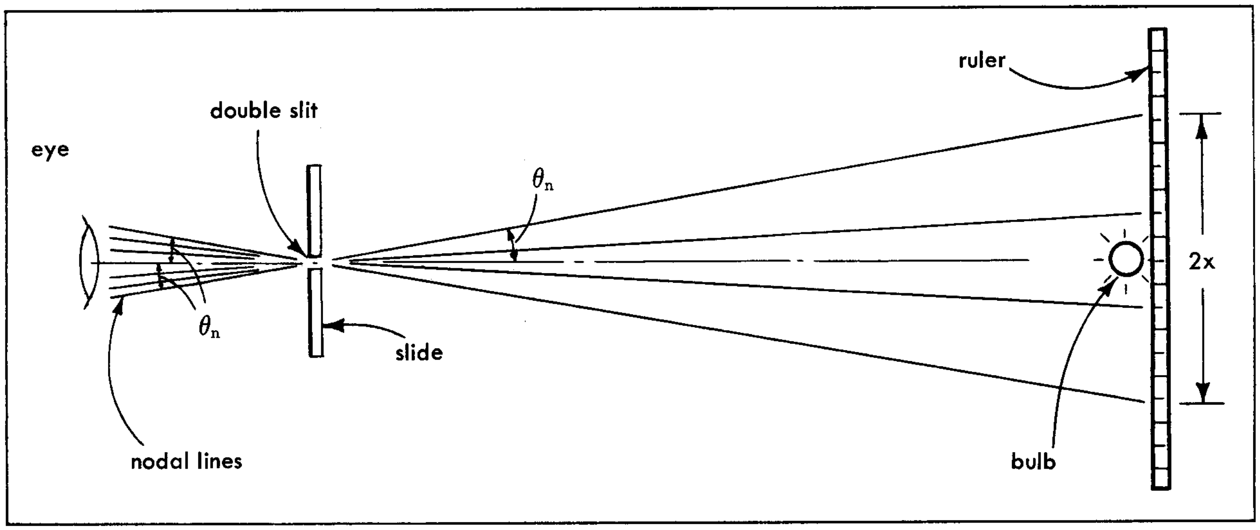


Figure 2

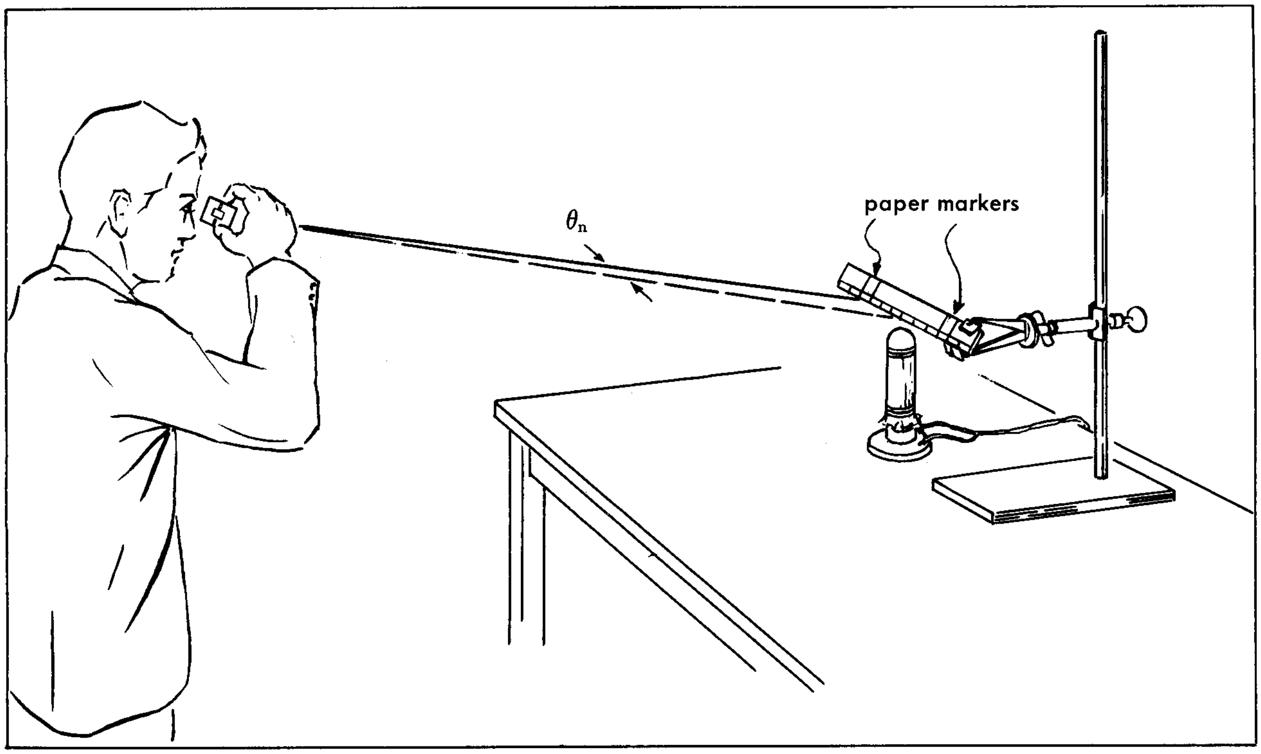


Figure 3
The interference pattern and the paper markers on the ruler can be seen simultaneously by looking through the slits and the "window" at the same time. The cellophane is held by rubber bands.

Two narrow slits illuminated by a showcase lamp will provide the two sources. Their preparation is explained in Fig. 1. Look through the slits toward the filament of the light bulb from a distance of about 2 meters. By using Fig. 2, explain why you see dark and light bars.

Can you suggest why the bars near the end of the pattern are colored? How does covering part of the bulb with red cellophane affect the pattern?

Now cover the whole bulb with red cellophane and place a ruler slightly above it as shown in Fig. 3. How will you determine sin θ_n for the farthest nodal line that is easily visible? By measuring the thickness of one of the razor blades with a micrometer, you can determine the separation of the slits. What is the wavelength of red light?

Repeat your measurements to find the source of the largest error. How accurate is your determination of the wavelength?

Cover part of the bulb with red cellophane and part with blue. Which color has the shorter wavelength?

How is the interference pattern affected when you turn the slide to form a horizontal angle of about 30° with the line of sight, instead of 90°? How do you explain this?

Diffraction of Light by a Single Slit

16

In preparing the double slits for Experiment 15 you may have made some single slits inadvertently and noticed that they also showed a pattern of dark and light bars. To study them further, scratch several single slits, using both a needle and a razor blade.

Compare the pattern obtained with the double slits with the pattern of the single slits. Use both white and red light. As you look at the bulb through a double slit, try blocking off one slit of the pair by holding a razor blade behind the slide. What happens?

It is quite difficult to measure the width of the slits directly. However, you can determine it by using the value you found for the wavelength of red light, and the theory of single-slit interference.

17 Resolution

We can study resolution qualitatively by looking through small apertures at two small light sources that are close together. The light sources can be tiny holes in aluminum foil placed in front of a ripple-tank bulb, and the apertures we look through can be holes of different sizes punched in another piece of aluminum foil. Figure 1 shows such a setup.

Look at the two sources with one eye from a distance of about one meter. Be sure that bright light, directly from the filament, reaches your eye through the two holes that make up the two sources. Can you resolve the sources into

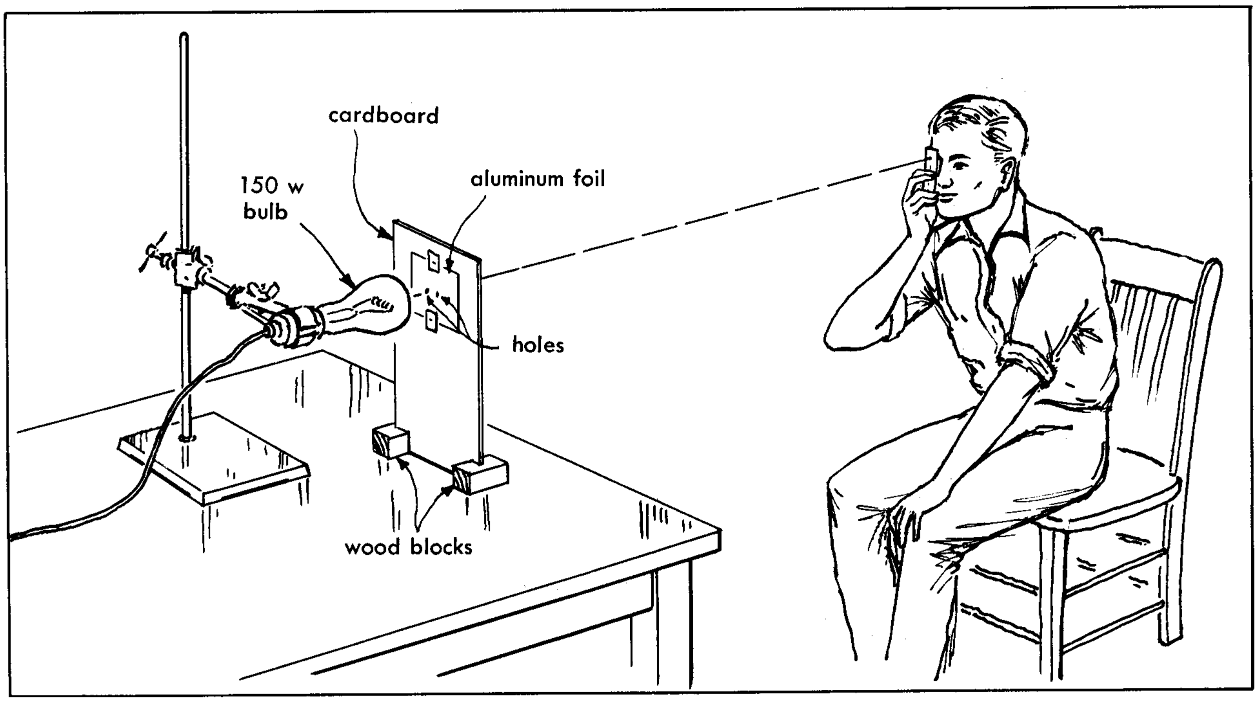


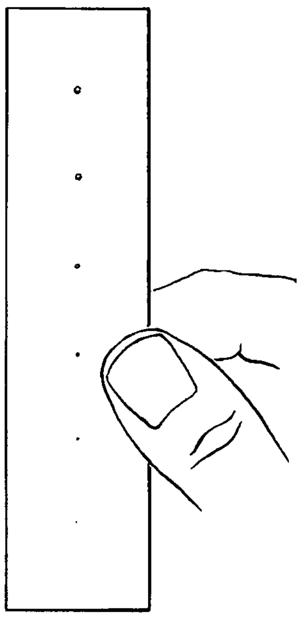
Figure 1
To make the sources, use a needle to punch two holes about ¾ cm apart in a piece of aluminum foil. Mount the foil directly in front of the filament of the 150-watt bulb.
The holes through which the sources are viewed are shown at the right. To make these holes, puncture a strip of foil with the tip of a needle, making the largest hole the thickness of the needle and the smallest just large enough to see light through.

two separate points of light with your eye? How large is the aperture through which you view the sources?

Look at the sources through one of the middle-sized apertures. Can you resolve the two sources? Now increase your distance from the sources and observe the change in their appearance. Find the distance where the sources are just resolved. At this distance look at the two sources through each of the different-sized apertures and sketch their appearances. Why does the resolution of the sources depend on their distance from the aperture? Why does it depend on the size of the aperture?

While looking at the sources through the aperture that just resolves them, have your partner hold first red and then blue cellophane in front of the sources. How does the wavelength of the light affect the resolution? How do you explain this effect?

How would the sources appear if they were larger but the distance between their inner edges were the same?



18 Measurement of Short Distances by Interference

A thin layer of air between two glass plates produces light effects similar to those seen on a soap bubble. To see this, place two freshly cleaned glass plates, about 20 cm long, on a black background. Darken the room and illuminate the plates with green or yellow light. If the glass plates are very flat, you will see a few irregular bands of light reflected from the glass. What causes these bands?

Press down on the plates with a pencil. Can you make one of the bright bands move and take the place of an adjacent band? If so, how much closer have you pushed the top plate to the bottom one at that point?

You can measure the thickness of a piece of very thin material by inserting it between the plates at one end (Fig. 1). Be sure that the material is smooth and that the plates are very clean. (Hold the plates together tightly with rubber bands close to the ends.)

How much does the separation of the plates change between two adjacent bright bands? Count the number of bright bands in a 2-cm span. (A magnifying lens may help.) What is the thickness of the material? Compare your result with that obtained with a micrometer caliper. Which is more accurate?

What limits the range of thickness that can be measured in this way?

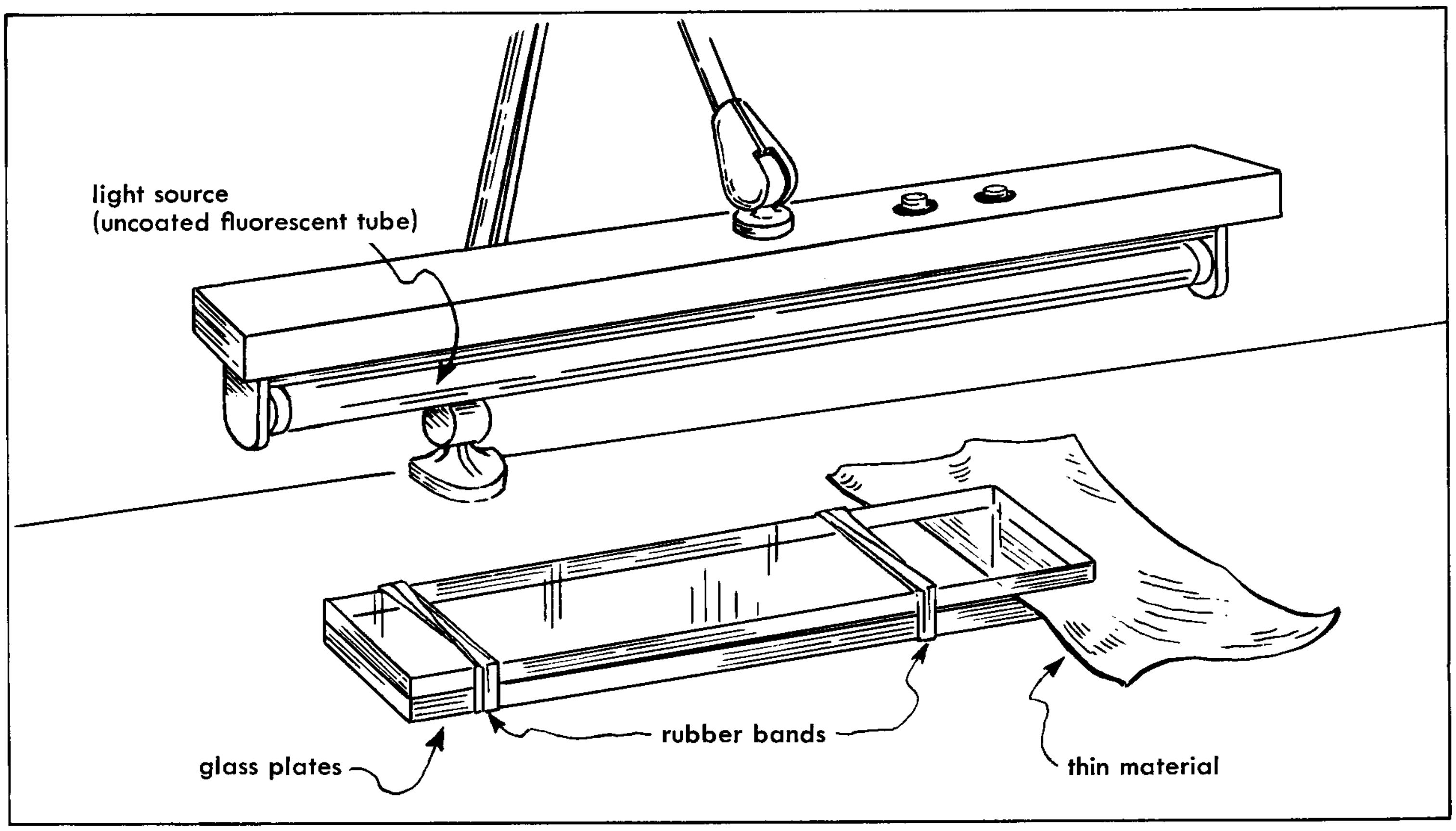

light source
(uncoated fluorescent tube)
glass plates
rubber bands
thin material

Figure 1

19 Motion: Velocity and Acceleration

Studying the motion of an object requires a record of the object's position at different times, preferably at regular time intervals. With such a record, you can study quite irregular motion—for example, the motion of your hand while you walk.

Set up the timer as shown in Fig. 1, grasp the end of the tape in your hand, and walk several steps, swinging your hand freely as you pull the tape, while your partner operates the timer.

If you choose the time interval between two consecutive marks as a unit of time, a "tick," what does the distance between any two adjacent marks repre-

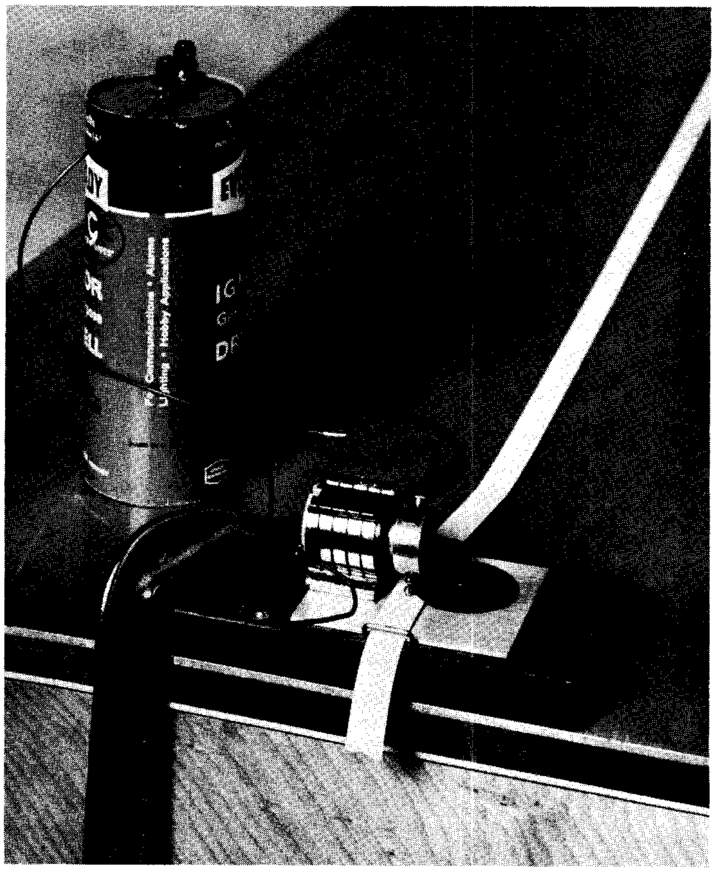


Figure 1

sent? From an inspection of your tape, can you find where your velocity was highest? Where it was lowest? Can you find where the acceleration was greatest? Where it was smallest?

Starting at any point near the beginning, plot a graph of position versus time. Since a tick may be too small a unit of time, it will be more convenient to use 4 ticks (which you can call a "tock").

You can also obtain an approximate velocity vs. time graph directly from the tape, by plotting the average velocity over a one-tock interval as a function of time. This average velocity is an approximation for the instantaneous velocity at the middle of the interval. For example, the average velocity between time $t = 1$ and $t = 2$ in Fig. 2 is 6 cm/tock. This is an approximation of the instantaneous velocity at $t = 1.5$.

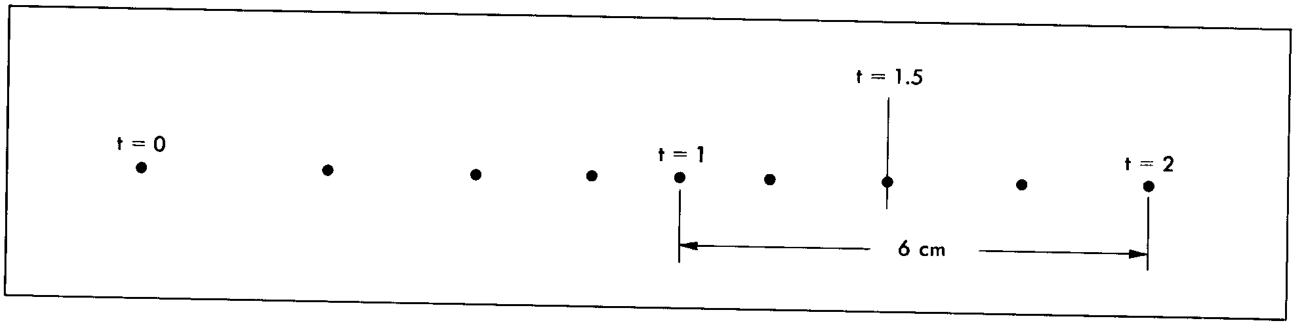


Figure 2

You can check a few points on this by finding the slope of position vs. time graph at corresponding points. How closely do they agree?

From your velocity vs. time graph, plot the position vs. time by measuring the area under the curve as a function of time. Compare the positions found this way with the positions measured directly on the tape.

From the plot of velocity vs. time, make a plot of acceleration vs. time. How good was your early guess as to the times of the greatest and the smallest accelerations?

20 Changes in Velocity with a Constant Force

You know qualitatively from everyday experience that you must apply a force to move an object from rest or to change its velocity while it is moving. With the apparatus shown in Fig. 1 you can investigate the quantitative relation between the velocity changes and the force.

The cart, loaded with bricks and running on roller-skate wheels, can be pulled forward with a constant force by hand. To make sure this force is constant, we apply it through rubber strands which are kept stretched at a

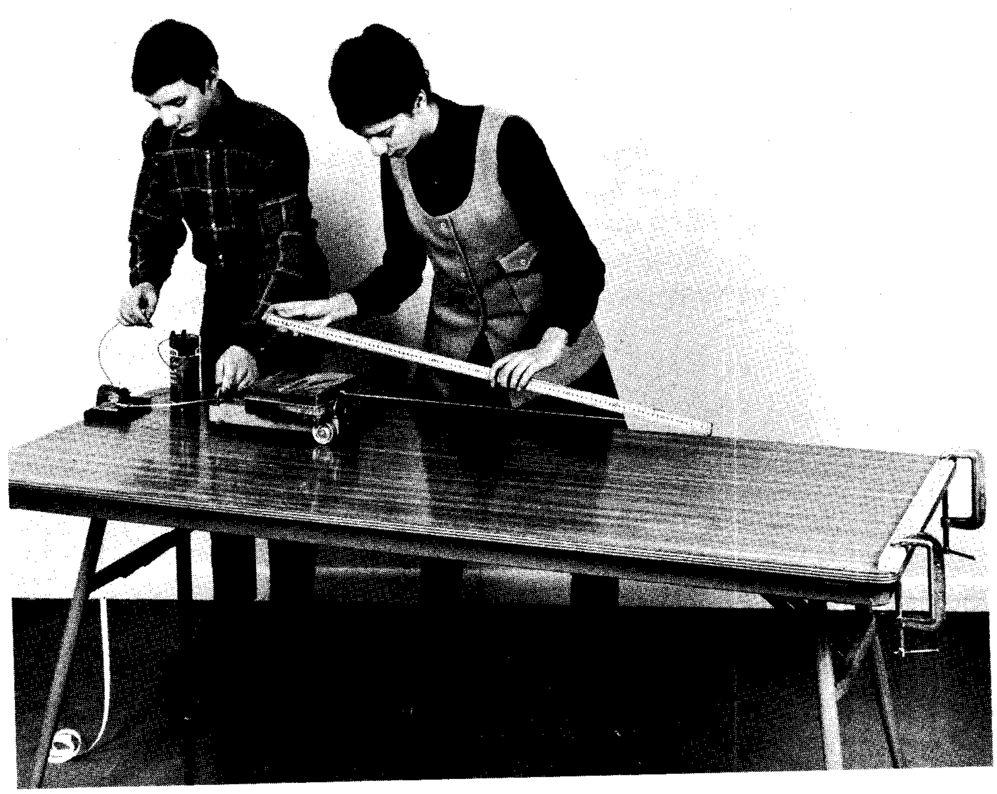


Figure 1

constant length as the cart is pulled along. As it moves, the cart pulls a strip of paper tape under the striker of an electric timer clamped to the table edge. From these tapes you can then find the velocity at different points on a run and can plot a curve of the velocity of the cart as a function of time.

The experiment is best performed on a smooth, level table. If necessary, level the table with wedges under the legs and check with a spirit level. Crumbly bricks may be wrapped in aluminum foil or wrapping paper to keep their grit from getting on the table.

Before making runs to find how the velocity changes with a constant force, you should be sure that the cart moves with a nearly constant velocity when you do not pull it. Load the cart with two bricks and make several tapes with the timer, giving the cart different initial pushes. Look carefully at the tapes. Is the velocity more nearly uniform when the cart moves slowly or when it moves rapidly?

Now you can study the effect of a constant pull on the motion of the cart. Attach one end of a rubber loop to the cart as shown in Fig. 2. Hook the other end of the rubber loop over the end of a meter stick. While your partner holds the cart, extend the meter stick forward alongside the cart until the rubber loop stretches to a given total length—say 80 cm. Your partner starts the timer and a few seconds later, on signal, releases the cart. You move

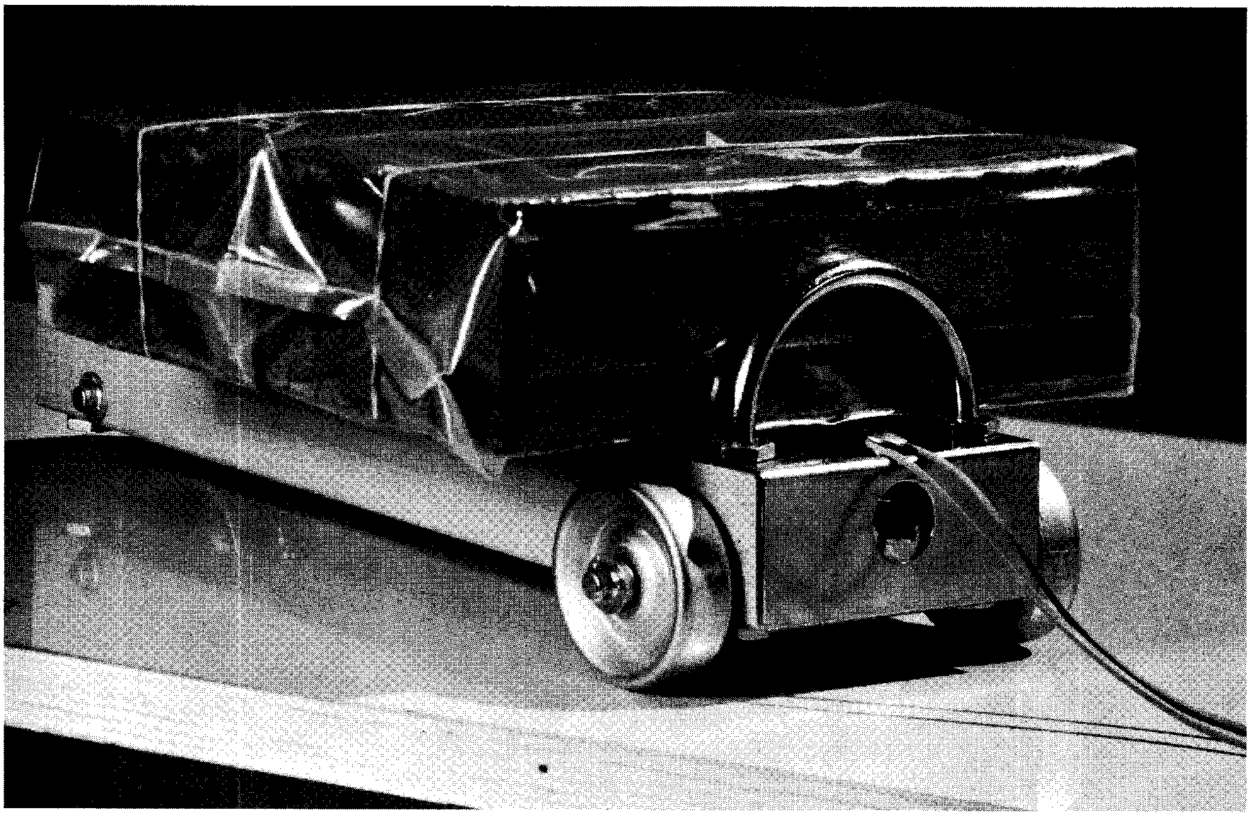


Figure 2

forward, pulling the cart while keeping the rubber strands stretched to the 80 cm mark. You will find it worthwhile to make a few practice runs.

Now attach the paper tape to the cart loaded with two bricks and run off a tape. If you could not keep the rubber stretched to a constant length toward the end of the run, discard the last part of the tape. From this tape, plot a graph of the velocity as a function of time (see Experiment 19). It is not necessary to use all the marks on the tape in calculating the velocity. Instead, use groups of ten marks for a convenient unit time interval, measuring the velocity in meters per ten "ticks." Analyze only that portion of the tape which represents the part of the run where you are reasonably sure the force you applied was constant.

Run off another tape, using four bricks on the cart and the same rubber loop. Plot the data from this tape on your original graph. What do you conclude about the acceleration produced by a constant force?

Is the force exerted the only force acting on the cart?

Was the acceleration greater or smaller when a larger mass was accelerated?

The Dependence of Acceleration on Force and Mass

21

The acceleration produced by a constant force was the subject of the preceding experiment. Now you can investigate quantitatively how different forces accelerate a given mass and how a given force accelerates different masses.

Acceleration Caused by Different Forces

Using one, two, three, and four rubber loops (Fig. 1, Experiment 20) to produce the accelerating force, make tape recordings of the motion of the cart when it is loaded with four bricks. Find the acceleration from the tapes and plot a graph of acceleration as a function of the force, that is, the number of loops.

Since you know from the last experiment that the acceleration is constant for a constant force, it is not necessary to calculate the acceleration for many different intervals in the same run. Find the acceleration from the change in velocity during two equal time intervals. It may be wise to include neither the start of the tape, where the data cannot be resolved, nor the last part of the motion, where it is difficult to keep the force constant.

What do you conclude from your graph? What can you say about the ratio of force to acceleration in this part of the experiment?

Assuming no friction in the apparatus, should the graph pass through the origin? Where, with respect to the origin, would you expect your graph to pass?

The Effect of Mass on the Acceleration Produced by a Constant Force

With one rubber loop find the acceleration of the cart when it is loaded with one, two, three, four, and five bricks. Plot a graph of the ratio of force to acceleration as a function of the number of bricks. What do you conclude from your graph?

From your graph, can you express the mass of the cart alone in terms of the mass of the bricks?

How could you find the mass of a chunk of lead or a heavy stone, using the apparatus? Try it.

22 Inertial and Gravitational Mass

The inertial balance, a simple device for measuring the inertial mass of different objects, is shown in Fig. 1. Put different quantities of matter on the platform and qualitatively observe the periods of vibration of these masses. Is the period greater or smaller for larger masses?

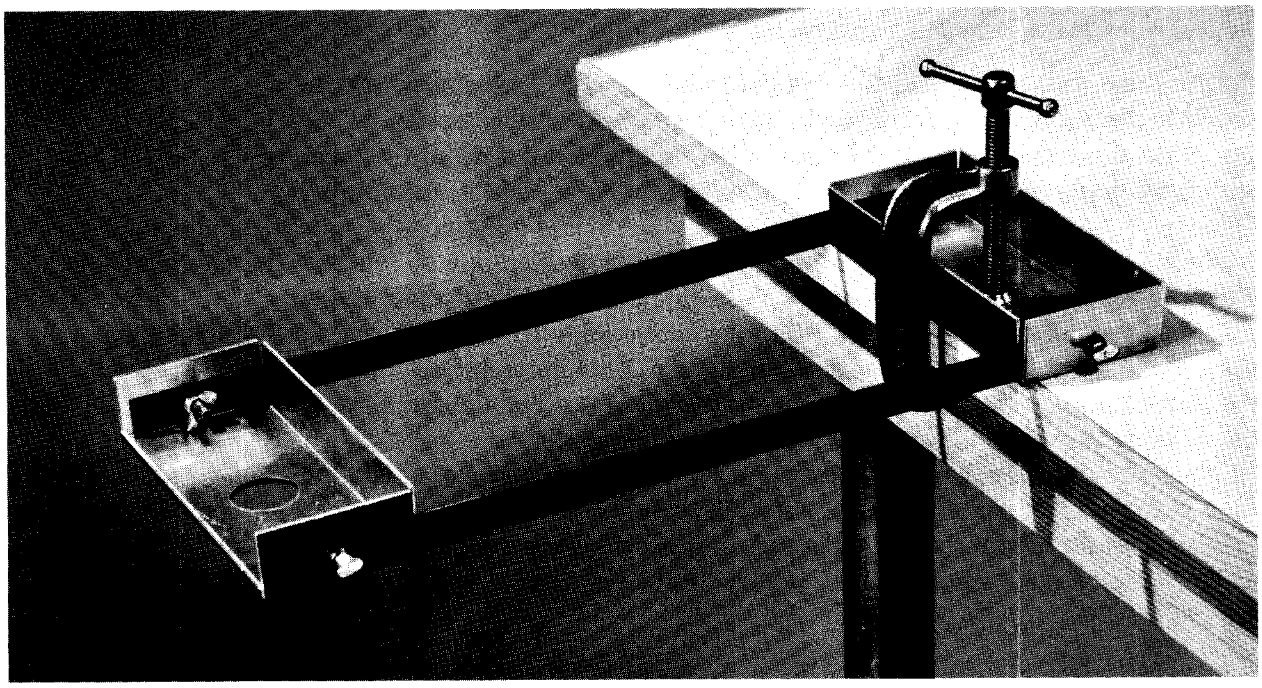


Figure 1

Find the quantitative relationship between the quantity of matter on the balance and the period of vibration by plotting a graph of the period as a function of the mass. You can do this in the following way:

First, measure the period of the balance alone by measuring the time for as many vibrations as you can conveniently count. Since the period of the balance is very short, it is difficult to count the vibrations visually. Hold a small piece of paper near one of the steel strips and count the audible snaps made by the paper when the blade just ticks it. It may be easier to count in groups of three or four vibrations.

Select six nearly identical objects or unit masses such as C clamps. Now measure the period of the balance loaded with each of the six C clamps (Fig. 2). How many vibrations should you time and for how many seconds should you time them to make sure that your error is no greater than about 2 percent?

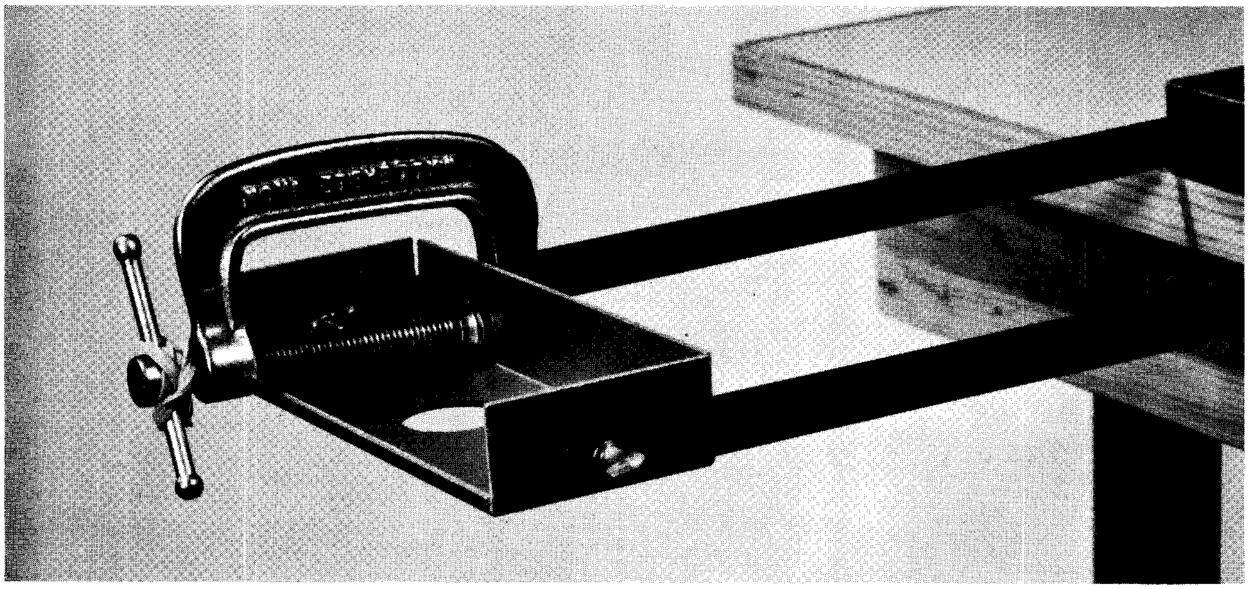


Figure 2

Now find the periods with one, two, three . . . unit masses on the balance and from these data plot the period as a function of the mass (number of clamps) on the balance.

Measure the period of an object of unknown mass, of different material and shape—a stone, for example. Using the clamps as unit masses, find the inertial mass of the stone. Ordinary weighing will give you the gravitational mass, in grams, of each of the clamps. To within what percent do they have the same gravitational mass? Try to predict the gravitational mass of the stone from your previous measurements. Check it by weighing the stone.

If you had found similar results with other objects, what would you conclude about gravitational and inertial mass? Are they equal? Proportional? Independent? Must the units of inertial mass be the same as those for gravitational mass? How would the results of this experiment differ if you did the experiment on the moon?

To check whether or not gravity plays a part in the operation of the inertial balance, load it with the iron slug. This can be done by inserting a wire through the center hole of the slug and setting the slug into the hole in the platform. The slug then rests on the platform. Measure the period of the loaded balance.

Now lift the slug slightly so that its mass no longer rests on the platform and hold it in this position by a long thread tied to a ringstand (Fig. 3). How do the periods compare in these two cases? Is gravity relevant here?

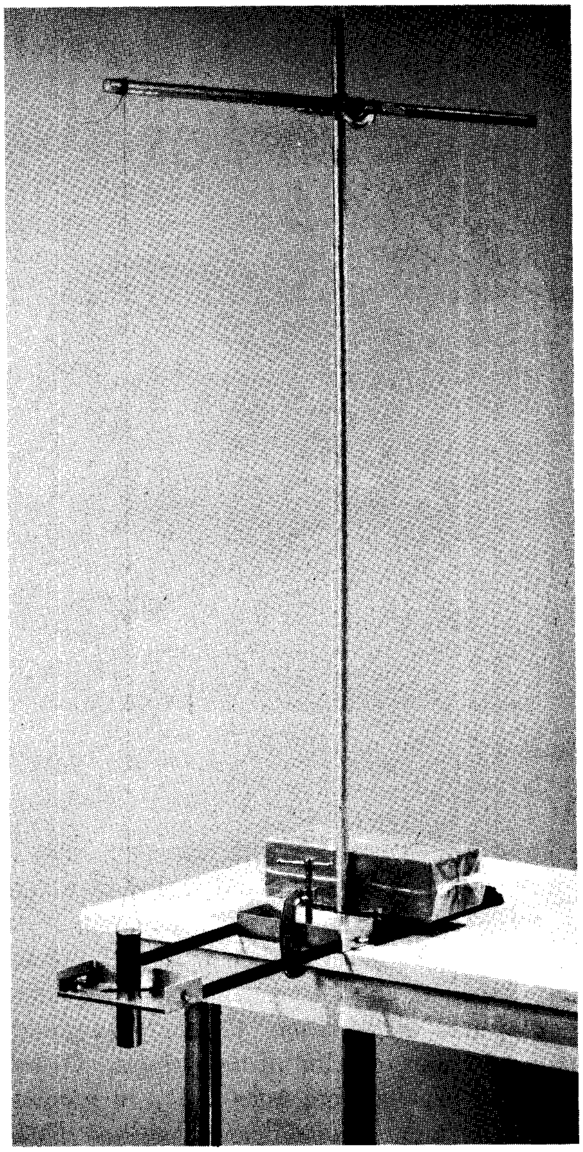


Figure 3

Forces on a Ball in Flight

23

Figure 1 is a multi-flash photograph of projectile motion. It was made by throwing a small ball into the air at an angle of 27° with the horizontal. The time interval between successive exposures was 1/30 sec and the ball moved from left to right in the picture. The ball's trajectory looks like those described in Section 12–4 of the text.

Examine the photograph. Is the horizontal velocity of the ball constant? What can you conclude about the resultant force acting on the ball if the horizontal velocity is not constant?

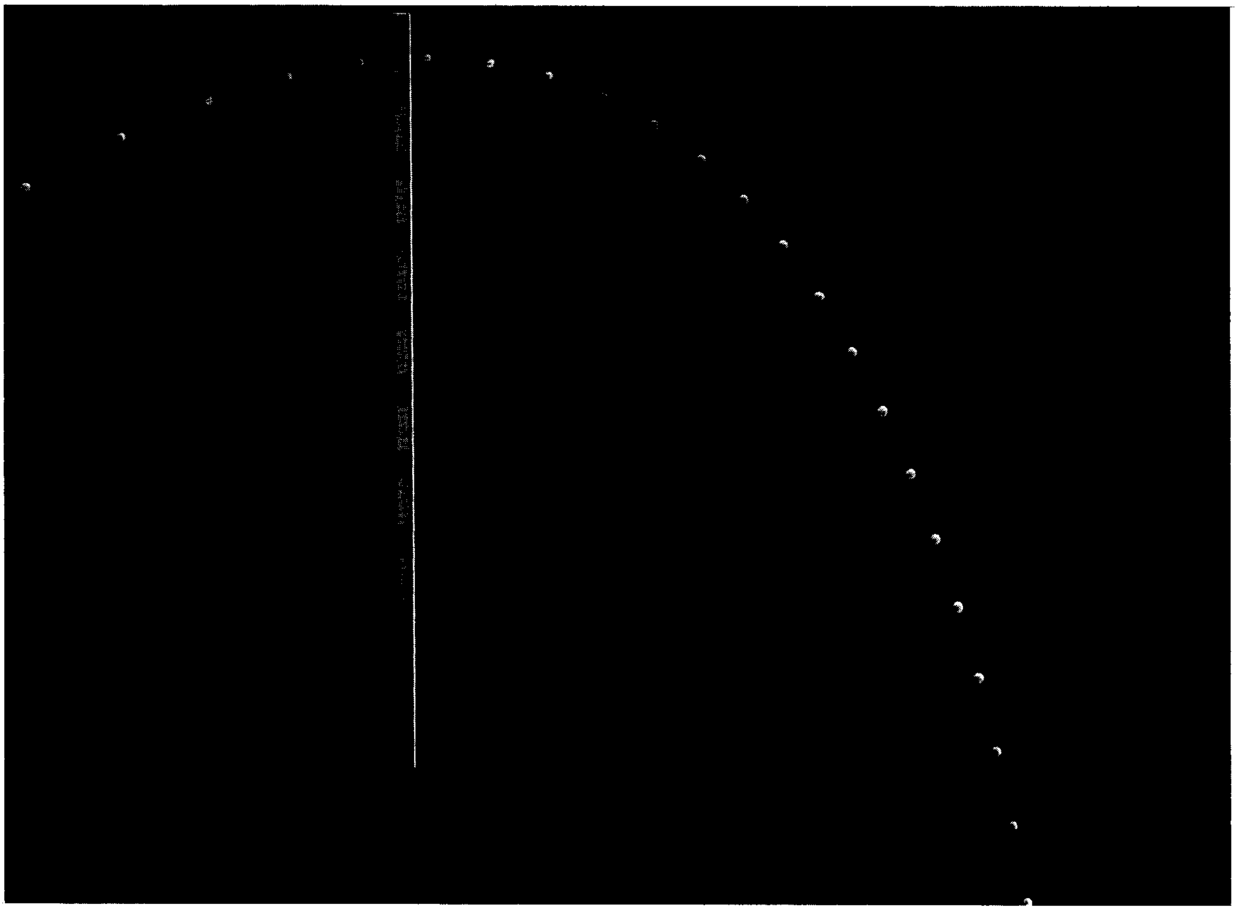


Figure 1. Scale: 1 to 10

If we analyze the photograph in detail and find the changes in velocity caused by the resultant force, we shall learn more about the forces acting on the ball than we can from a casual examination of the photograph.

Analyze the velocity changes which occur during successive 0.1 sec time intervals (three intervals on the photograph) in the following way: Clip transparent centimeter graph paper or tracing paper on top of the photograph and mark the center of each image. Draw straight lines connecting every third point. These lines represent the displacement of the ball during each 0.1 sec and are therefore a measure of the average velocities during these equal time intervals. You can find the velocity changes in each of these intervals by the construction shown in Fig. 2a, where $\vec{v}_1$ is redrawn as a dashed line.

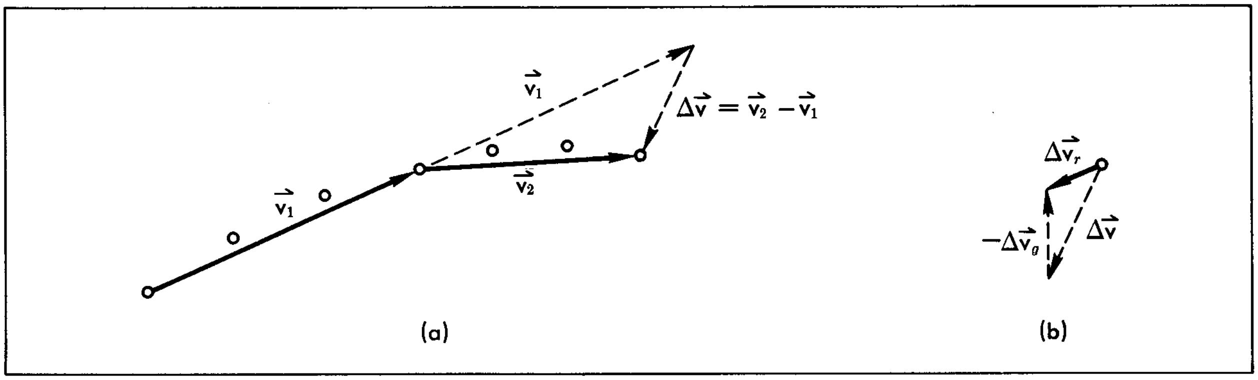


Figure 2

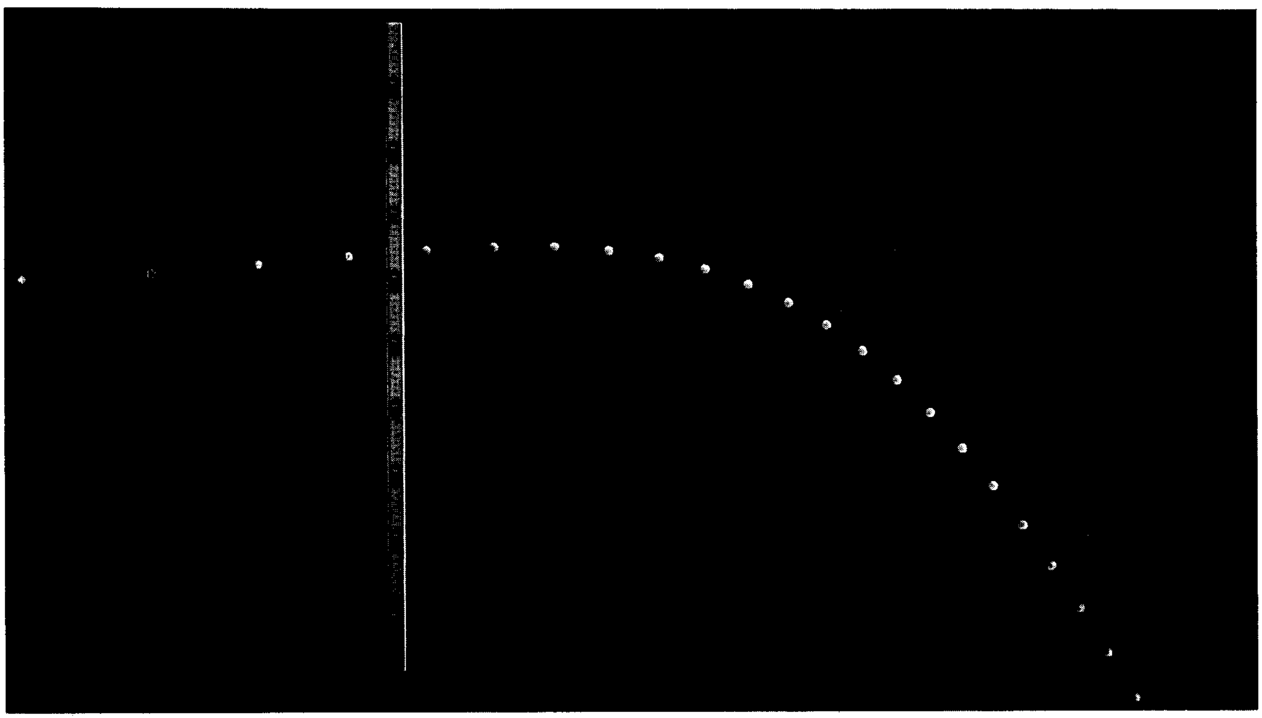


Figure 3. Scale: Approx. 1 to 11.5. The meter stick is vertical.

Is the direction of the velocity change the same in each interval? Are the magnitudes of the velocity changes the same? What do you conclude about the direction of the resultant force on the ball?

What change in velocity of the ball in each 0.1 sec was caused by the force of gravity? In what direction did it act? Express this velocity change $\Delta\vec{v}_g$ in meters per *tenth* of a second and subtract it from each of the total velocity changes $\Delta\vec{v}$ on your diagram (Fig. 2b). The velocity change due to gravity must also be reduced to the scale of the photograph before you subtract it on your diagram. By using a ruler, you can see that the photograph is one tenth of its real size.

Do the residual velocity changes $\Delta\vec{v}_r$ all have the same magnitude? In what direction are they? Describe, qualitatively, the properties of the force that caused them. What do you think was responsible for the force?

What can you conclude about the mass of the projectile?

Plot on your diagram the path the ball in Fig. 1 would have followed if gravity had been the only force acting on it.

How do you explain the paths followed by the projectile in Fig. 3 and 4?

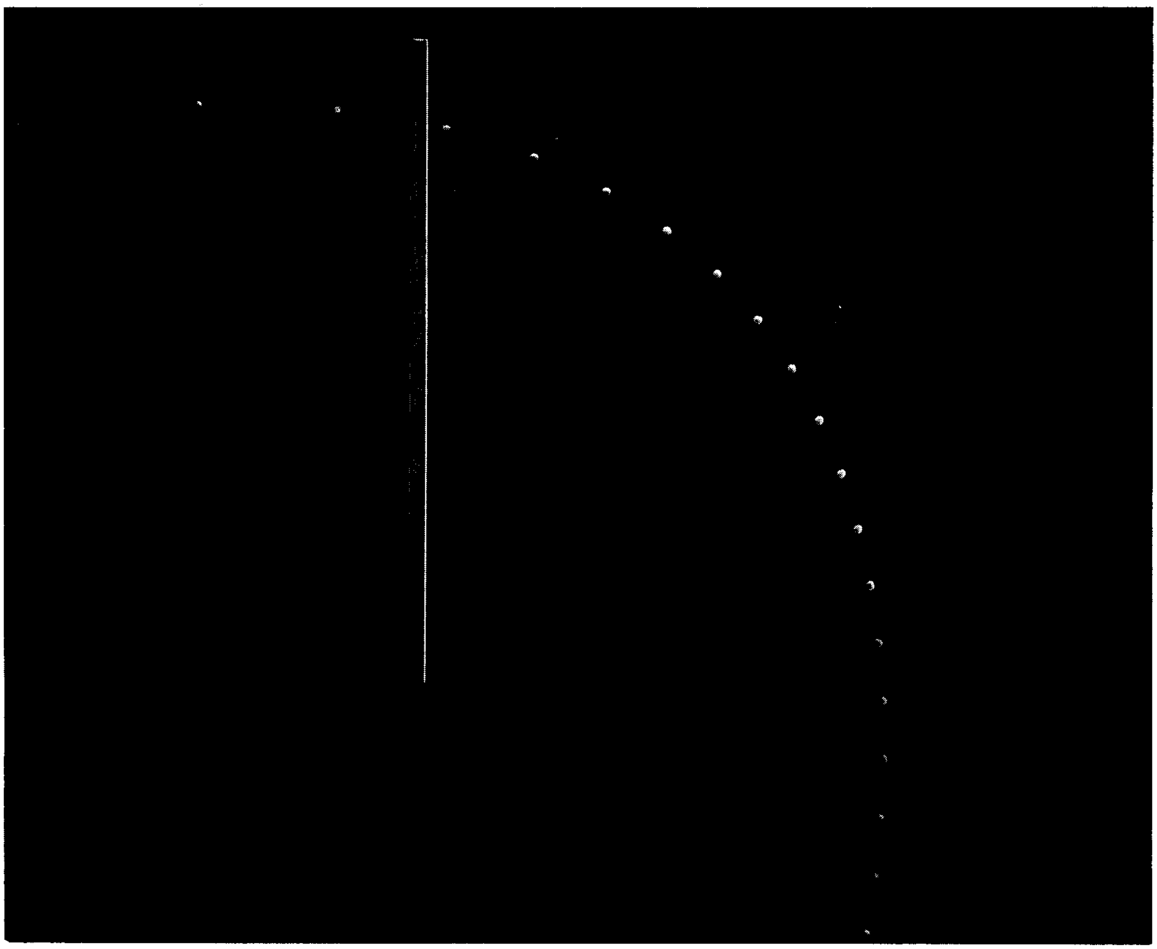


Figure 4. Scale: Approx. 1 to 11.5.

24 Centripetal Force

Motion in a circle at constant speed is an accelerated motion; although the magnitude of the velocity stays the same, the direction of the velocity vector is continuously changing. We know from Newton's law that a force is needed to maintain this acceleration. How is this force related to the object's speed, its mass, and the radius of the circle?

To answer these questions we shall use the simple apparatus shown in Fig. 1, which allows us to measure the force while observing the motion. When the glass tube is swung in a small circle above your head, the rubber stopper moves around in a horizontal circle at the end of a string which is threaded through the tube and fastened to some washers hanging below. The force of gravity on these washers, acting along the thread, provides the horizontal force needed to keep the stopper moving in a circle. This horizontal force is called the centripetal force.

With only one washer on the end of the string to keep the stopper from getting away, whirl the stopper over your head while holding onto the string below the tube. Do you have to increase the pull on the string when you increase the speed of the stopper? What happens if you let go of the string?

Now quantitatively investigate the dependence of the accelerating force on the speed, the mass, and the radius. First find out how the force depends on the speed, keeping the mass and the radius constant.

Pull enough string through the tube so that the stopper will whirl in a circle of about 100 cm radius. Attach an alligator clip to the string just below the tube to serve as a marker so that you can keep the radius constant while whirling the stopper. Hang six or more washers on the end of the string.

To find the rate of revolution of the stopper, have a partner measure the time while you swing the stopper around and count the number of revolutions. From the time and number of revolutions calculate the period. Repeat the experiment with larger numbers of washers.

Plot the period of the motion as a function of the number of washers. Can you think of a more useful way to plot your data? Try plotting the frequency instead of the period. Try f^2. What is the dependence of the centripetal force on the frequency when the revolving mass and the radius are kept constant?

To investigate the dependence of the centripetal force on the revolving mass, you could whirl two stoppers on the end of the string. What would you expect to find? On what do you base your prediction?

It is more difficult to investigate experimentally the dependence of the centripetal force on the radius when the period and mass remain constant. Can you suggest a way of doing this? What is the dependence of the centripetal force on the mass, the radius, and the period?

You will notice that as you swung the stopper around, the part of the string from the tube to the stopper was not quite horizontal. The gravitational force on the stopper pulled it down. Can you see why this effect of the gravitational force does not change the relation between the force (measured in number of washers), the length of the string from the tube to the stopper, and the period of revolution?

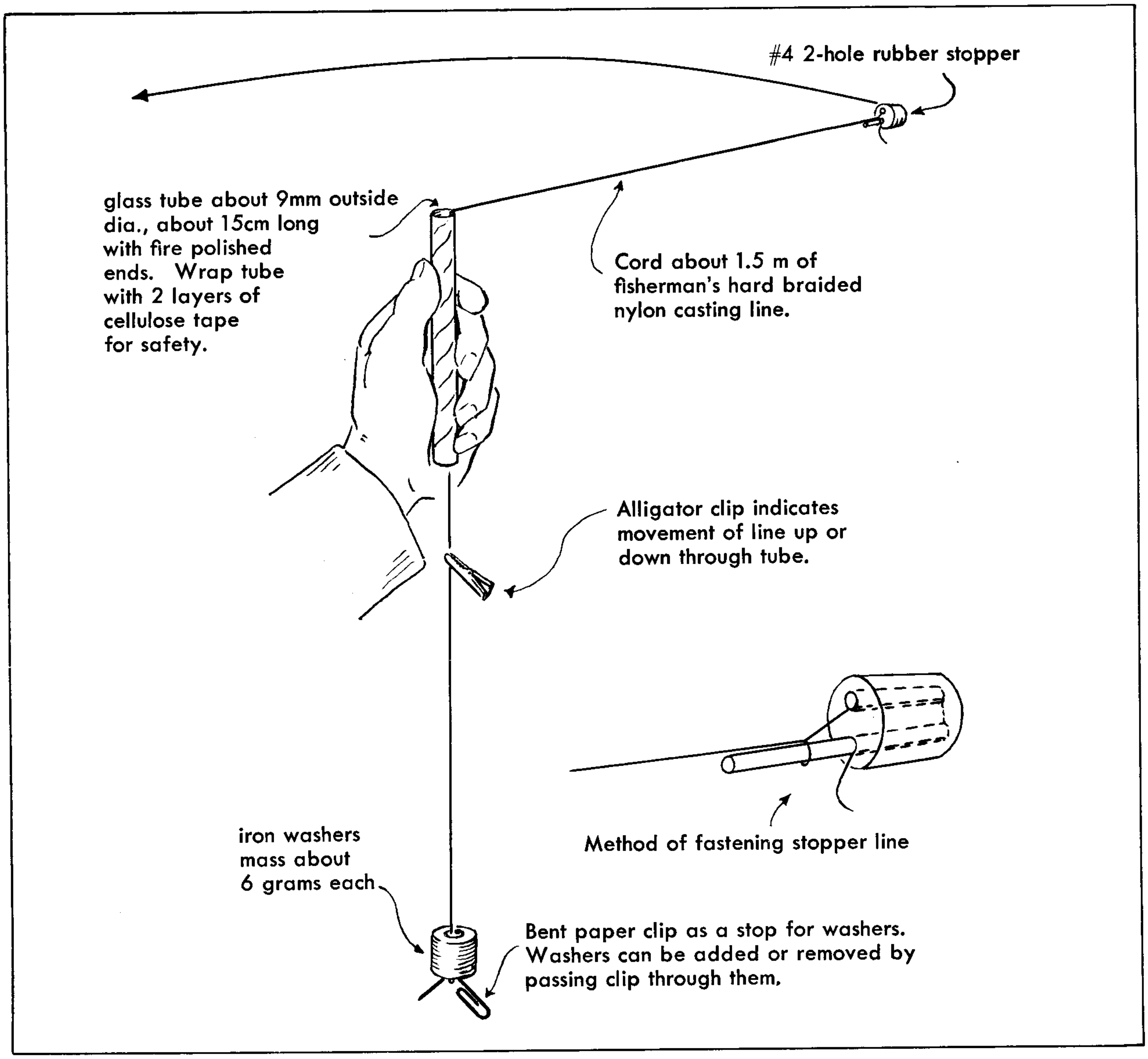


Figure 1

25 Simple Harmonic Motion

An object performs simple harmonic motion if acted upon by a restoring force whose magnitude is proportional to the displacement. Find out if an object hung on the end of a particular spring will vibrate in simple harmonic motion, by making a graph of the extension of the spring as a function of the stretching force applied to it. (This graph can be used later in Experiment 32 if you use the same spring in that experiment.)

Predict the period of vibration of, for example, a 500-gm mass hung from the spring, and check your prediction. What is the effect of the amplitude of vibration on the period?

Determine the mass of an object such as a C clamp by measuring its period of vibration when suspended from the spring. Have you found the inertial mass or gravitational mass by this measurement? You can check your measurement by weighing the clamp on a balance.

Determine the force constant, the ratio of force to displacement, of a second spring from the period of the motion of a known mass.

What do you predict for the period of a known mass suspended from the second spring if the upper end of the second spring is suspended from the lower end of the first spring?

What assumption have you made in all of your predictions about the mass of the spring on the period?

If you have time to spare, suspend a 1-kg mass from the first spring. How can you use the laws governing simple harmonic motion to predict its maximum velocity, if you displace the mass a known distance from its equilibrium position and release it? (Make sure that the coils of the spring do not close completely at the highest position of the mass; if they do, use a smaller initial displacement.) Can you check your prediction with tape and a calibrated timer?

Momentum Changes in an Explosion

26

Two carts are pushed apart from rest as the result of a sudden force—an "explosion"—acting between them. How do the momenta of the carts change?

To apply the sudden force we use a spring which we compress and suddenly release (Fig. 1). Release the spring with the cart at rest. What do you

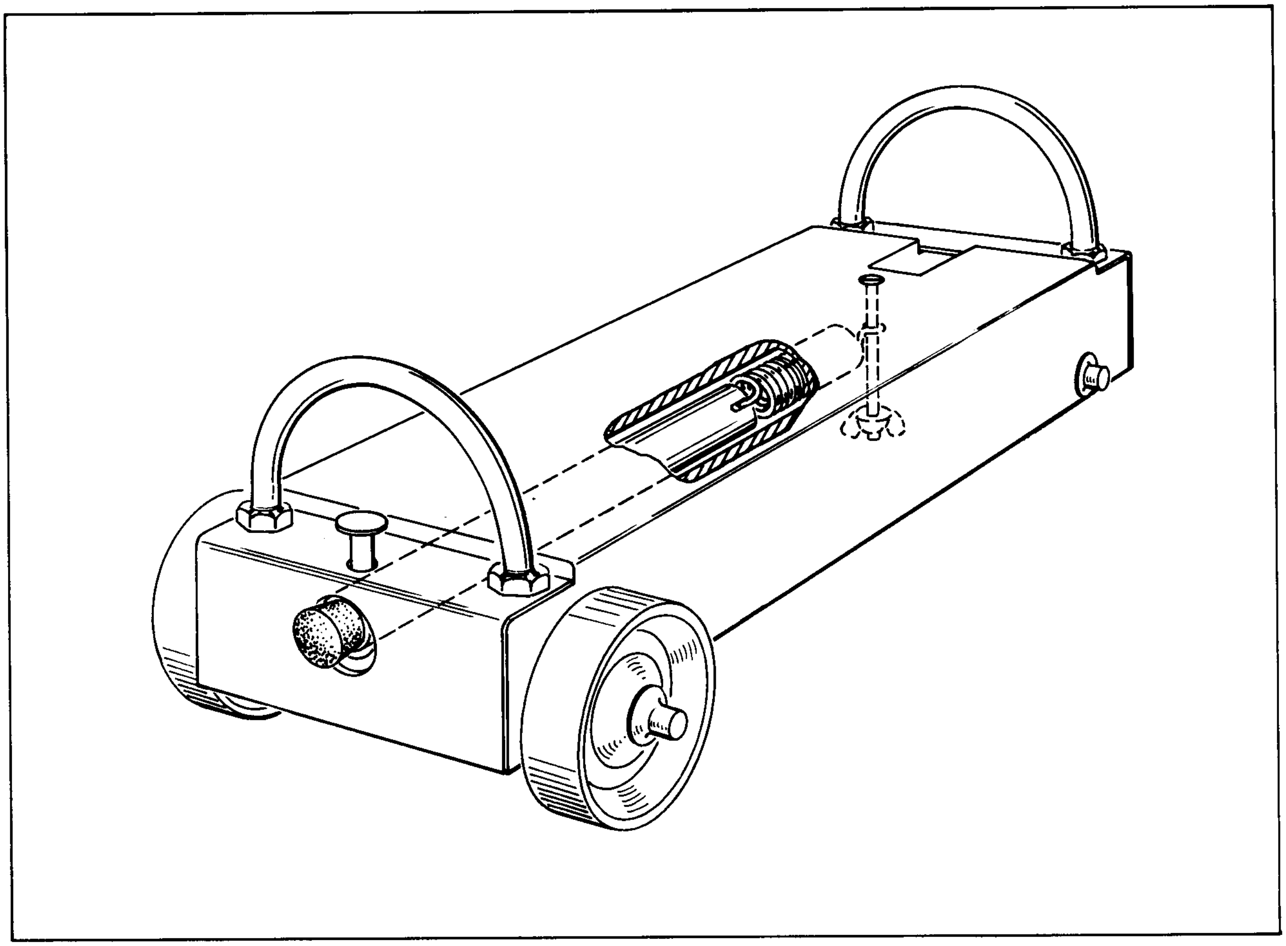


Figure 1
To load the exploder, push the tube into the cart and lodge it behind the metal plate. To release, tap the pin at the front.

observe? Try this with different loads on the cart. What do you conclude about the horizontal component of the momentum of the cart before and after the explosion?

Place a second cart next to the first one so that the spring will push against the second cart when released. What happens now as you release the spring? Do this experiment with various loads on the carts. Qualitatively, what would you say about the velocities of the two carts as you load them with different masses? How do you think the momenta of the two carts compare after the "explosion"?

To make this experiment quantitative we need to measure the velocities and the masses of the two carts. But we do not have to know their velocities in meters per second; any unit will do. It is possible to find their velocities in terms of the distances both carts move during the same time interval. Suppose we release the carts just halfway between two wooden bumpers and they go at the same speed. We shall hear just one sound as they hit the bumpers at the same time. If one goes faster than the other, it will hit earlier and we will hear two distinct sounds instead of one. We can, however, move the starting point so the faster cart has to travel a longer distance before hitting the bumper. After several trials we can find a position from which both carts will take the same time to travel to the bumpers. The distances traveled by the carts from rest positions are shown as x_1 and x_2 in Fig. 2. The carts travel these distances in the same time interval t and, if they move at constant velocity, we can write for their velocities:

$$v_1 = \frac{x_1}{t}; v_2 = \frac{x_2}{t}$$

and

$$\frac{v_1}{v_2} = \frac{x_1}{x_2}.$$

The velocities, therefore, are proportional to the distances moved in the same time interval.

Using this method of moving the starting point to give equal times, determine the ratio of the momenta of your carts after explosion. What is the change in momentum of each cart as a result of the explosion? Try this with different combinations of masses on the cart. Can you draw any conclusions concerning the total momentum of the system after the explosion compared with the total momentum before the explosion?

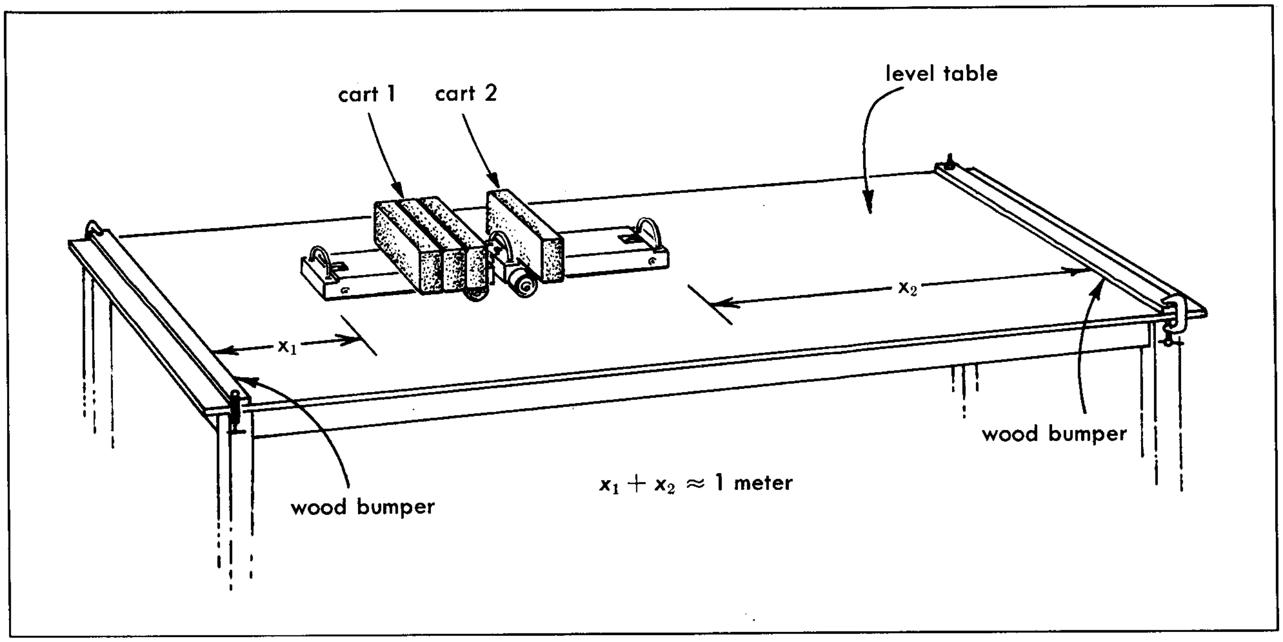

cart 1
cart 2
level table
x1
x2
wood bumper
wood bumper
x1 + x2 ≈ 1 meter

Figure 2

27 The Cart and the Brick

What happens when a suspended brick is dropped on a moving cart as the cart passes beneath the brick? Suspend a brick so that the cart can just pass beneath without touching it (Fig. 1). The hanging brick should be horizontal and motionless. Move the cart back, give it a push, and release the brick as the cart passes beneath it. What happens? Try the experiment again with the cart loaded with different numbers of bricks. What is the effect of increasing the mass of the cart by loading it with bricks?

To make accurate measurements, record the motion for both loaded and unloaded carts. Since you wish to have the motion as uniform as possible before and after the brick collides with the cart, start the cart with a reasonably high speed.

From the tapes and the masses of the cart and the brick, compute the change in momentum of the cart and the change in horizontal momentum of the brick. You can compute the momenta in units of kilogram-meters per "tick." How do they compare? What is the total horizontal momentum of cart and brick before and after they interact? Is momentum conserved?

What is the horizontal impulse applied to the falling brick? Try to estimate the length of time of the interaction by examining your tapes. Can you make a rough estimate of the horizontal force applied to the falling brick? How does this compare with the force the brick applied to the cart?

What happened to the vertical momentum of the brick? Would it make any difference if the brick were dropped from different heights as long as you didn't break the cart or the table?

What would happen if, instead of dropping the brick on the cart, you suspended a funnel full of sand above the table and let the sand run into a box on the cart as it passed beneath the funnel? What would happen to the velocity of the cart if, instead of letting the sand run into the cart, you let the sand run out of it?

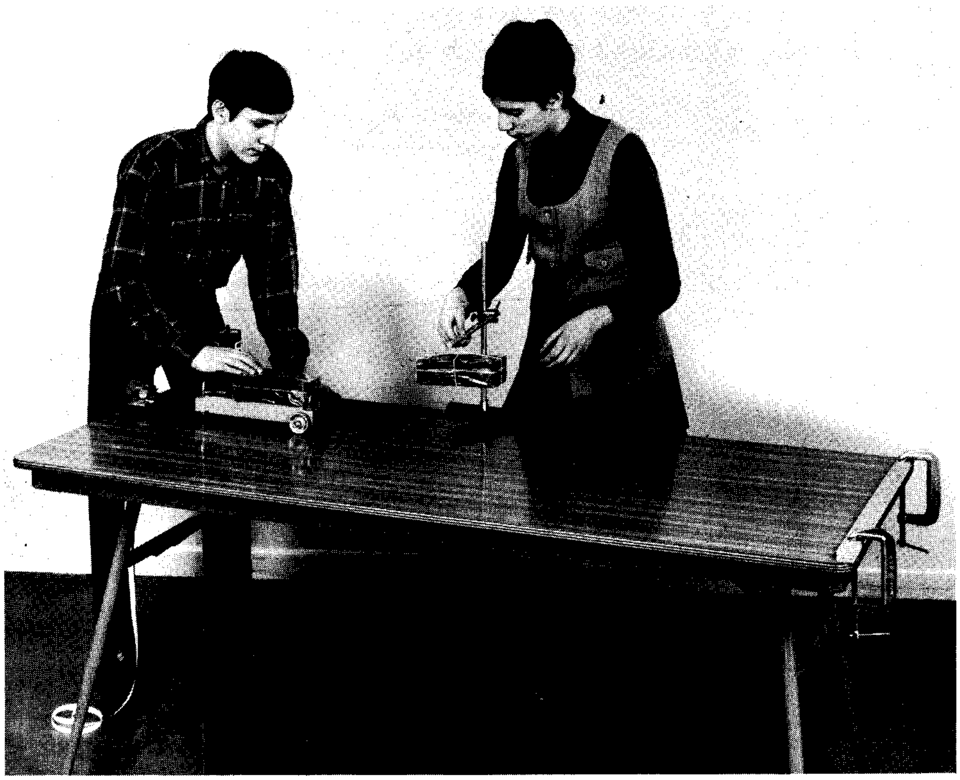


Figure 1

28 A Collision in Two Dimensions

Previously we investigated the momenta of colliding bodies moving along a single straight line. What happens when two bodies go off in different directions after colliding? To find out, we shall roll one steel ball down an incline so that it makes a glancing collision with another steel ball of the same size,

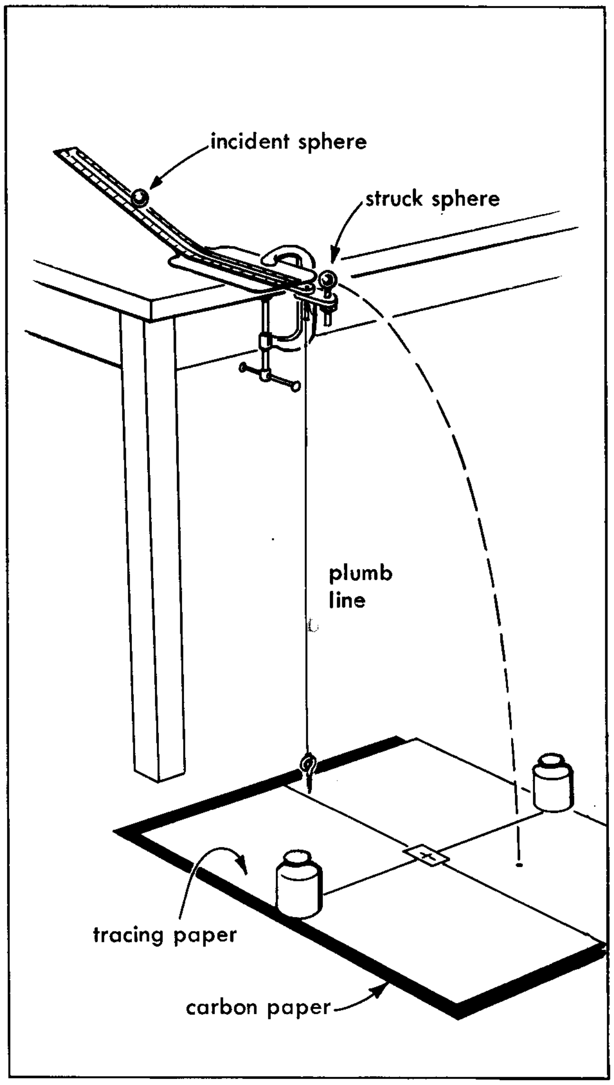


Figure 1

knocking it off a support near the edge of the table (Fig. 1). We shall then find the momentum of each from their masses and velocities.

To find the velocities of the spheres, we shall use what we have learned about projectile motion (see text, Section 12–3). As long as air resistance can be neglected, objects projected with different horizontal velocities from the edge of a table take the same time to fall to the floor. The horizontal component of their velocity remains unchanged and therefore the distance they go horizontally is proportional to their horizontal velocity. We can use this fact to measure the velocities of the spheres after they have collided. All we have to do is to compare the horizontal displacements of the balls.

To give an initial velocity to one of the spheres, roll it down the grooved ruler (Fig. 2). The target sphere rests in the slight depression on the top of the set screw. Adjust the height of the set screw so that it will support a steel ball at the same height as an identical steel ball placed at the bottom of the incline.

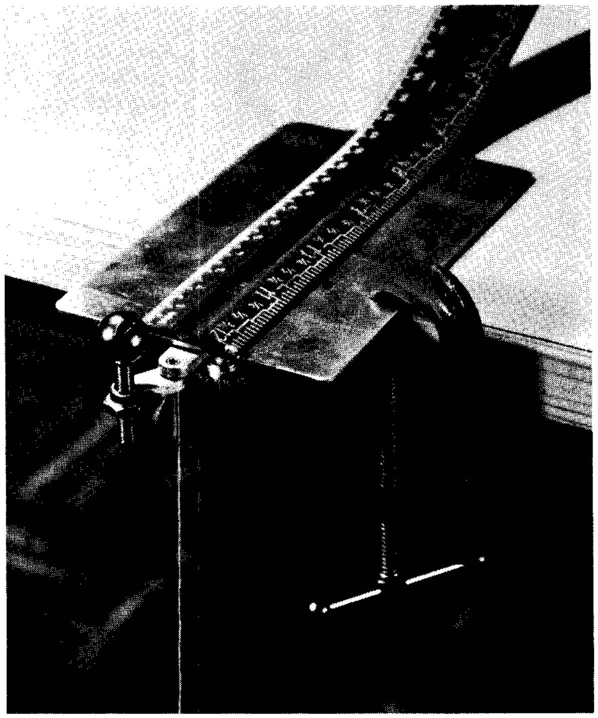


Figure 2

Tape four sheets of onionskin or tracing paper together to make a single large sheet. Be sure the sheets do not overlap. Do the same with four sheets of carbon paper. Adjust the carbon paper, carbon side up, on the floor with the tracing paper on top of it; the plumb bob should hang directly over the middle of the shorter side (Fig. 1). Mark this point on the paper and put weights on the paper to hold it in place. Release a steel ball 25 cm from the lower end of the ruler ten or fifteen times and circle the distribution of points on the paper. To what degree is the initial velocity always the same?

With a steel ball balanced on the set screw, try several collisions, releasing the incident ball from the same point as before on the ruler. To change the point of collision, turn the arm supporting the target ball through a small angle. A numbered circle around each impact on the paper will help you identify the different marks on the paper.

Draw on the paper the vectors that represent the velocities of the balls after collision. The position of the target sphere at the instant of impact can be determined with the help of Fig. 3.

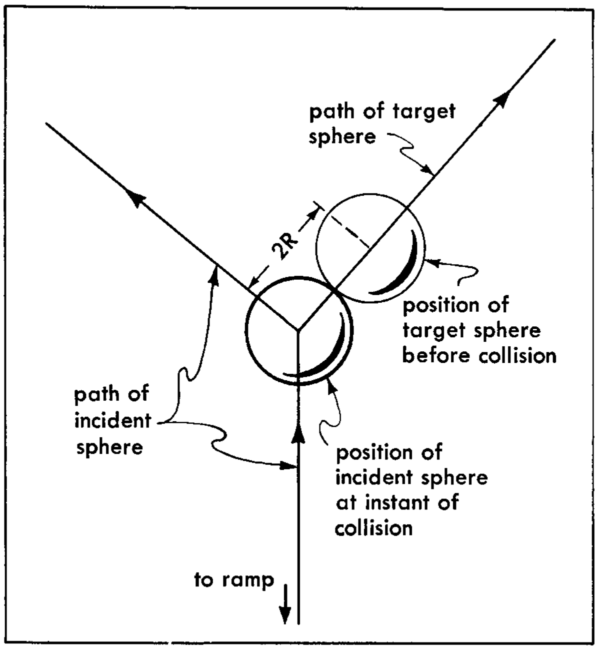


Figure 3

Since the masses of the balls are equal, the velocity vectors represent the momenta of the balls. Add the two momentum vectors graphically on your paper, placing the tail of the momentum vector of the target ball at the head of the momentum vector of the incident ball.

How does the vector sum of the two final momenta compare with the initial momentum of the incident ball? Is momentum conserved in these interactions? How does the arithmetic sum of the two magnitudes of the momenta after collision compare with the magnitude of the initial momentum of the incident ball?

Repeat the experiment using two spheres of unequal mass but of the same size. Which one should you use as the incident sphere? How does the vector sum of the final velocities compare with the initial velocity? How can you convert the velocity vectors to momentum vectors now that the masses of the two spheres are not equal? How does the vector sum of the final momenta compare with the initial momentum?

Compare the vector components of the final momenta of the two balls in a direction at right angles to the initial momentum. What do you find?

Elastic Collisions

29

When you studied collisions in two dimensions (Experiment 28), you were concerned only with comparing momenta before and after the collision. The records of these collisions, however, can serve also for the comparison of the kinetic energy of the balls before and after the collision.

Consider first the two steel balls of equal mass. Their kinetic energy before the collision is $\frac{1}{2}mv_1^2$; after the collision it is $\frac{1}{2}mv'^2_1 + \frac{1}{2}mv_2^2$. If kinetic energy is conserved in the collision, i.e., if the collision is elastic, then

$$v_1^2 = v'^2_1 + v'^2_2.$$

What does this equation say about the angle between the velocity vectors v'_1 and v'_2? Measure these angles for the runs you made in the preceding experiment. What do you conclude about the elasticity of the collisions?

When the colliding balls have different masses, the elasticity of the collision can no longer be determined by inspection or just the measurement of an angle. To reduce the amount of calculation needed to check if the relation $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2$ holds, you can first divide both sides by $\frac{1}{2}m_1$. From your data on the collisions of the steel ball and the glass ball, what do you conclude about the elasticity of the collisions?

30 Simulated Nuclear Collisions

Nuclear collisions are often studied in photographic emulsions and in bubble chambers. In these instruments, charged particles moving at high speed ionize atoms along their paths and leave visible tracks. The energy to ionize the atoms comes from the kinetic energy of the charged particles, which therefore slow down. The distance a particle travels in the chamber before it comes to rest is called its range. The range depends on the particle's kinetic energy as it enters the chamber. By shooting particles of known energy into the chamber, we can establish the relation between range and energy. We can then use this relation to find the energies of particles by observing their ranges. In this way we can find the energies of particles emerging from a nucleus as the result of a collision. If the masses of the particles are known, we can find their momenta.

There is strong evidence that momentum is conserved in nuclear collisions. When we observe a collision in which momentum appears not to be conserved, we conclude that at least one uncharged particle, which left no track, carried the missing momentum.

In this experiment you will study a situation analogous to a nuclear collision; the particles will be nickels and the emulsion or bubble chamber will be a sheet of paper which the nickels slide across until brought to rest by friction. The distance a nickel slides across the paper (its range) depends on its kinetic energy. To find the range-energy relation for a nickel, we can launch it down an incline, giving it different energies by starting it from different heights (Fig. 1). We then measure how far the nickel slides before coming to rest for each energy we give it. From the mass and the range-energy relation, we can find the velocity and momentum of the nickel when it comes out of a collision.

Before simulating a nuclear collision, we must find the range-energy relation for nickels. Select three nickels that slide easily down the ramp and have nearly the same range when sliding with the same face down. Find the distances these nickels slide on the paper for different release heights. Make several runs at each height and record the average range for each height. How is the kinetic energy at the bottom of the incline related to the release height? (Friction on the steep incline may be ignored.) A graph of the kinetic energy as a function of the range is the range-energy relation.

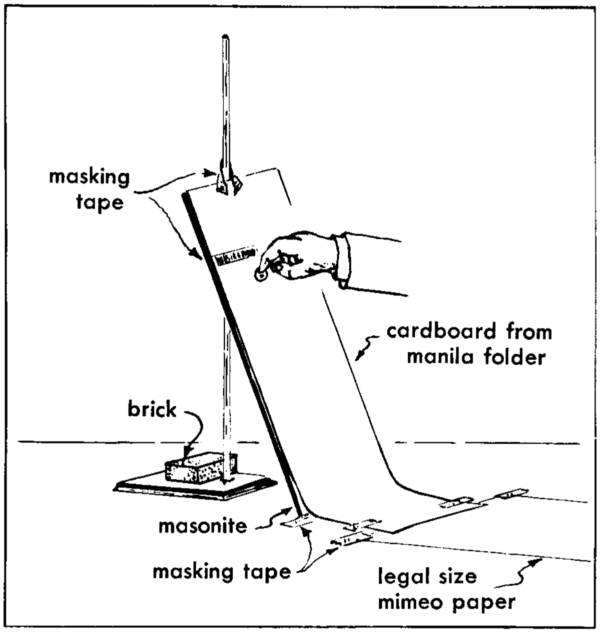


Figure 1

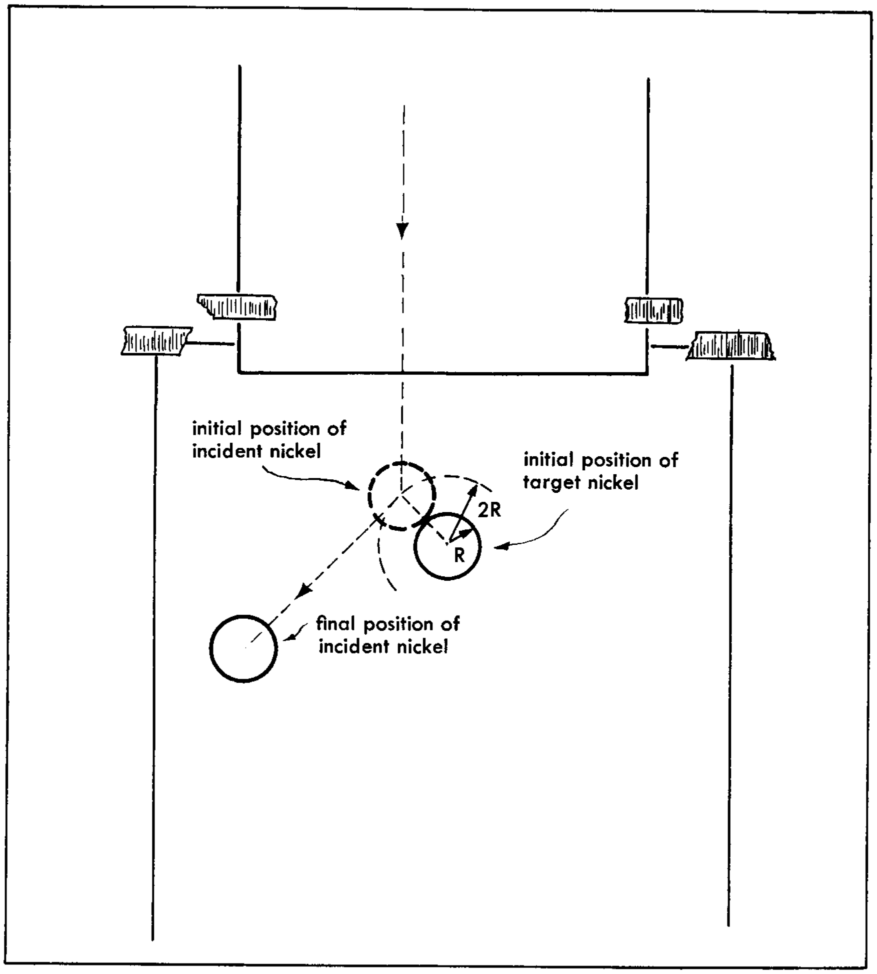


Figure 2

You are now ready to simulate a nuclear collision by placing a nickel (the nucleus to be hit) on the paper and sliding another nickel down the incline, giving it a known kinetic energy. (The incline corresponds to an accelerator giving an atomic particle a known kinetic energy.) The target nickel should be about 10 cm from the bottom of the incline to keep the incident nickel from bouncing over it.

The next part of the experiment should be done by one partner while the others are not present. Slide a nickel down the incline at the target nickel. Record the release height on the incline, the final position of the incident nickel, and the initial position of the target nickel. Secretly record the final position of the target nickel by coordinate measurements along two edges of the paper. The other laboratory partners now find the momentum and final position (i.e., the kinetic energy) of the target, which corresponds to an uncharged atomic particle leaving no visible track. To find the position of the incident nickel at the instant of collision, see Fig. 2.

What fundamental law have you assumed in finding the unknown momentum? What fraction of the kinetic energy of the incident nickel is lost in this collision?

Repeat the experiment using two nickels placed next to each other as the target. Find the "unknown" momentum and final position of one of the target nickels.

Inelastic Collisions 31

We have seen in Section 15–6 that a collision between two bodies will be elastic if the force between them depends only on their separation. If during the collision one of the bodies is permanently deformed (even slightly!), then the force is most likely larger when the two bodies hit each other than when they recede from each other. We can arrange such a situation by putting a piece of adhesive tape on the target steel ball in Fig. 1, Experiment 28.

Before making any runs, what do you predict (qualitatively) about (1) the total kinetic energy after the collision, (2) the total momentum after the collision, and (3) the angles between the momenta of the balls after the collision, and (3) the angles between the momenta of the balls after the collision as compared with the angles in Experiment 29?

Check your predictions by experiment, following the same guidelines as in Experiment 28.

How will putting some tape on the target glass ball affect the outcome of collisions? Try it and compare the results with those of Experiments 28 and 29.

32 Changes in Potential Energy

Hang a spring from a ringstand and attach a mass of about one kilogram to it. Lift the mass a few centimeters above its rest position and let it fall. At the top and bottom of its motion it is at rest. When the mass is at the bottom of its motion, its energy is stored in the spring. At the top of its motion its energy is stored in the gravitational field. Compare the change in gravitational energy with the change in potential energy stored in the spring.

You can find the change in potential energy of the spring when it is stretched a distance Δx from x_1 to x_2 by calculating the work done in stretching it from x_1 to x_2 (Fig. 1). The change in gravitational potential energy when the mass falls this same distance Δx can be found by calculating the work done in lifting the mass through the distance Δx. You can then compare the gravitational energy lost as the mass falls from rest to its lowest point with the spring energy gained as the spring stretches.

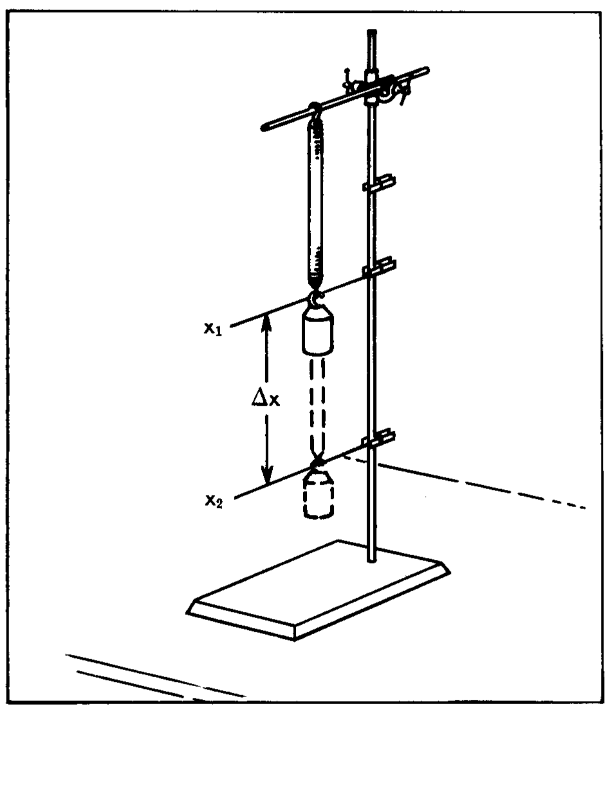


Figure 1

To find the potential energy of the spring, we first find how the extension x of the spring is related to the force that stretches it. (If you are using the same spring as in Experiment 25, you may use the information you obtained then.) Hang known masses up to a maximum of about 1.5 kg on the end of the spring and find the extension x in meters as a function of the force F in newtons. Plot a graph of x as a function of F. Is F proportional to x for this spring in the range of your measurements?

If your graph is a straight line, find the spring constant $k = \frac{F}{x}$ from the slope and write down the potential energy function of the spring; that is, the equation for the energy stored in the spring as a function of the extension of the spring. How can you find the potential energy stored in the spring for a given extension if your graph is not a straight line?

Now hang a one-kilogram mass on the spring and support it with your hand so the spring extends about 20 cm more than its natural length when hanging without the mass. Use clothespins clamped to the ringstand to mark the lower end of the unloaded spring and the point from which you drop the mass. Release the mass and note how far it falls. Place a clothespin on the stand to mark the lowest point of the fall. Release the mass several times until you have accurately located the lowest point of the vibration.

Calculate the loss in gravitational potential energy and the gain in potential energy of the spring when the mass falls. How do they compare? Repeat the above experiment, releasing the mass from a point about 25 cm from the lower end of the unloaded spring. Repeat the experiment with a 0.5 kg mass and calculate the change in gravitational potential energy and spring potential energy when the mass falls from a point about 10 cm below the end of the unloaded spring.

Is energy conserved in these interactions between the masses and the spring? Are they elastic interactions?

What is the sum of the two potential energies when the kilogram mass has reached the halfway mark in its fall? How does this compare with the initial energy of the mass? How do you explain this? How could you check your explanation?

If you have time, plot a graph of the sum of the two potential energies as a function of the spring extension. What can you learn from this graph?

33 Completely Inelastic Collisions

In some collisions like those you investigated in Experiment 31, Inelastic Collisions, momentum is conserved but the total kinetic energy after the collision is less than before the collision. What fraction of their total kinetic energy can two balls lose and still conserve momentum?

To find out, you can use the apparatus you used in Experiment 30. However, instead of a steel target ball you will use a large plastic ball that has a tapered hole drilled in it. The incident ball rolls into this hole when it leaves the incline and becomes wedged in the plastic ball. There are two small pieces of sponge rubber glued just inside the hole, (Fig. 1). These must be lined up vertically, as shown in the figure, or the incident ball will not stick in the hole.

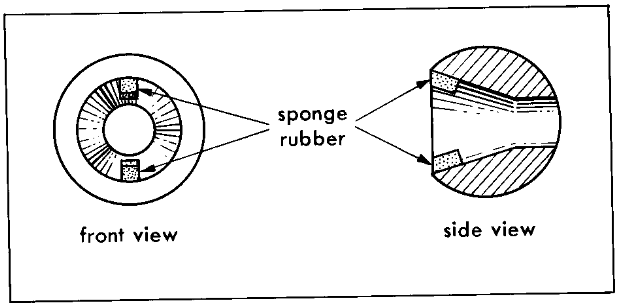


Figure 1

Adjust the position of the screw on which the target ball rests so that when the target ball is in position its center is level with the center of the incident steel ball as it leaves the incline (Fig. 2).

Before proceeding with the experiment, weigh the incident and target balls and predict, on the basis of the conservation of momentum, what fraction of the kinetic energy is lost in the collision. What happens to it? Check your prediction.

Make a sketch showing qualitatively the velocity in the center-of-mass frame of reference of the steel ball and the plastic ball just before and just after the collision. Could there be a larger loss of kinetic energy than in a collision where the balls stick together?

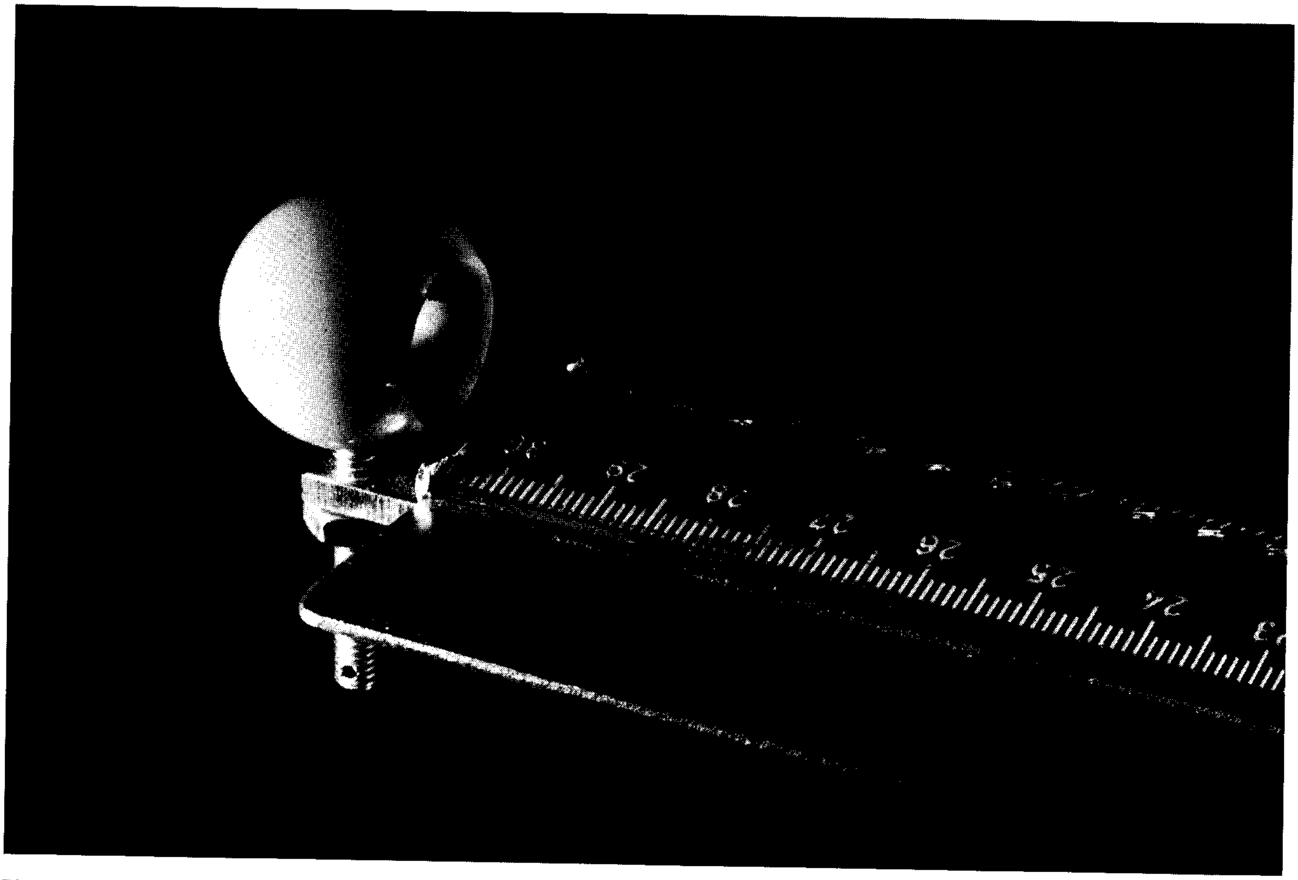


Figure 2

Assuming that it takes 0.4 joule to raise the temperature of 1.0 gram of steel by 1.0°C, and 1.6 joules to raise 1.0 gram of plastic by 1.0°C, estimate the rise in temperature of the two balls as a result of the collision. You can estimate the velocity after the collision from the vertical distance the balls fall and the horizontal distance they move after the collision.

34 Electrified Objects

Much of the qualitative behavior of electric charges was discovered during the eighteenth century. Common materials like glass were rubbed with different kinds of cloth to produce electric charges. You can discover for yourself the behavior of electric charges by rubbing easily charged plastic strips with paper or cloth.

Hang a strip of cellulose acetate and a strip of vinylite by short lengths of masking tape from a crossbar of a ringstand so they can swing freely without twisting. Briskly rub the vinylite strip and the acetate strip with a dry piece of paper. Do not touch the rubbed surfaces. Rub another vinylite strip with paper and bring it near each of the suspended strips. What do you conclude from the results?

Now rub another strip of acetate with paper and bring it near the hanging strips. What do you infer?

Have you found one, two, or three kinds of charge? Assign names to each kind of charge you have found and use these names throughout the rest of the experiment.

Rub a comb, plastic ruler, or other substance that charges easily on your clothes and observe its effect on the two suspended pieces of plastic. Which kind of charge does the substance have?

What general conclusions about the electrification of bodies can you make as a result of your observations in this experiment?

What would be the result of changing the names you have given to the charges you observed?

What happens when you hold a charged strip close to a tiny piece of uncharged paper or thread?

Electrostatic Induction 35

You know from everyday experience that electric charges do not flow easily in materials such as glass, ceramics, and plastics. These are called insulators. Other materials, mostly metals, in which electric charges move easily, are called conductors. In this experiment you will investigate the consequences of the free motion of charges in a conductor.

Place two metal rods end to end on glass beakers so they touch, and bring a charged piece of plastic close to one end of the rods (Fig. 1). (Do not get the

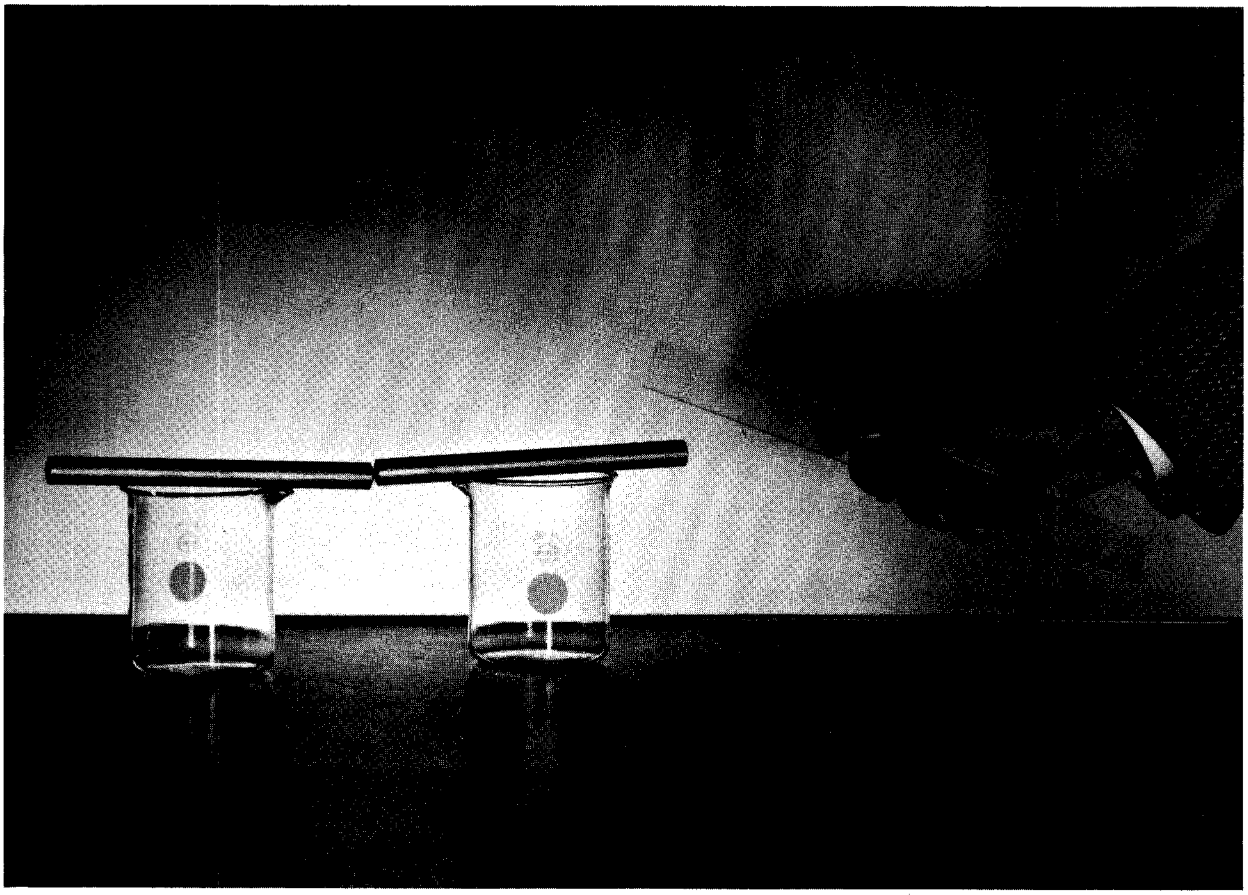


Figure 1

plastic so close that a spark jumps between the plastic and rod.) With the charged plastic close to the rods, separate the rods by moving one of the beakers without touching the rods. Remove the plastic and transfer some of its charge to a small piece of foil hanging by a thread from the crossbar of a ringstand. Move one rod and then the other close to the foil. How do you explain the results?

Now bring the rods into contact again and then bring them near the charged foil. How does the charged foil behave when it is near the rods?

Bring the charged plastic again close to one end of a single rod and touch the other end of the rod briefly with your finger. Remove the plastic and test for the presence of charge on the rod, using the charged foil. Is the charge on the rod the same as or opposite to the charge on the plastic?

The metal foil you have been using gives an indication of the presence and sign of a charge but is not good for measuring the quantity of charge. An electroscope is a better instrument than a piece of foil for measuring charge. Repeat the last part of the experiment, using an electroscope in place of both the rods and the foil.

The Force Between Two Charged Spheres

36

The force between electrically charged bodies depends on their separation and on the magnitude of their charges. The nature of the dependence can be measured in several ways. One simple method, which will be used in this experiment, measures the force on a charged body by balancing it against a known force—the force of gravity. We can suspend a small charged sphere with an insulating thread and bring another charged sphere close to it. From the deflection of the suspended sphere from the vertical, we can measure the electric force on it in terms of its weight.

A light, conducting ball A at the bottom of a "V" of very fine nylon thread is shown in Fig. 1. The ball can swing in only one vertical plane. Read the position on the scale of one edge of the hanging ball. The reading should be taken when the mirror image of the sphere is hidden behind the real sphere. This ensures that your line of sight is perpendicular to the scale when you take a reading (Fig. 2). You can charge the ball by conduction with a charged penny (Fig. 3), and bring near it a like-charged ball B on an insulated support.

Take readings of the positions of the balls as B is moved closer to A. Be sure to use the same side of each ball every time you read its position. Some charge may leak away slowly across the surface of the thread and the insulating support, thereby introducing an error. How can you test for leakage? When should you test for it, during the run or at the end?

When the suspended ball is at rest, the net force acting on it is zero. That is, the vector sum of the tension in the thread $\vec{\mathbf{T}}$ and the weight of the ball $m\vec{\mathbf{g}}$ is equal and opposite to the electric force $\vec{\mathbf{F}}$. From Fig. 4 it can be seen that, for small angles, the ratio of the magnitude of the electric force to the magnitude of the weight $\frac{F}{mg}$ is equal to $\frac{d}{L}$, the ratio of the horizontal displacement of the suspended ball to the length of the suspension. Hence

$$F = \frac{mg}{L}d = (\text{constant}) \cdot d.$$

Since we are not concerned here with particular units of force, we can measure the force in terms of d. We can, therefore, study the dependence of F on r by plotting d as a function of r.

Plot a graph of the force as a function of the separation of the two balls.

How is the force at a separation r related to the force at a separation of $\frac{1}{2}r$? $\frac{2}{3}r$? What kind of dependence does this suggest? Plot a graph to check it. If your graph deviates from your prediction, how can you account for the deviation?

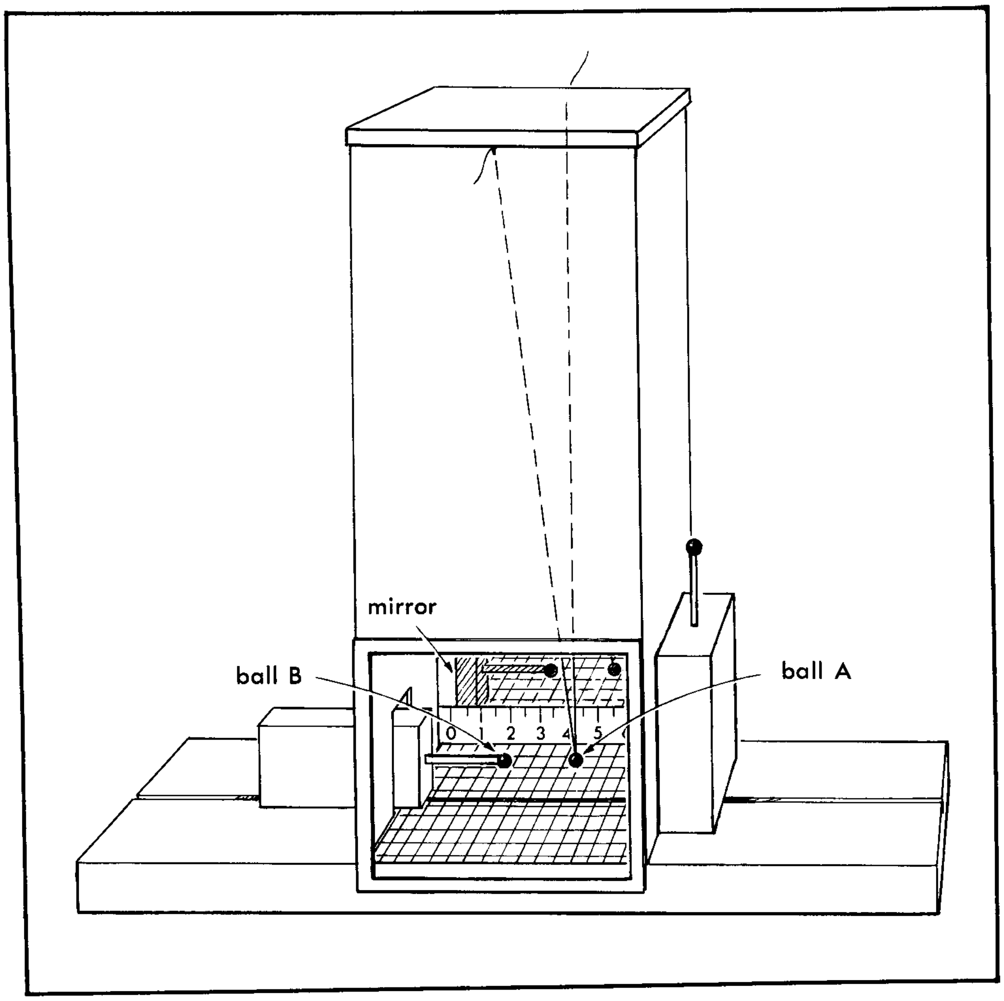


Figure 1

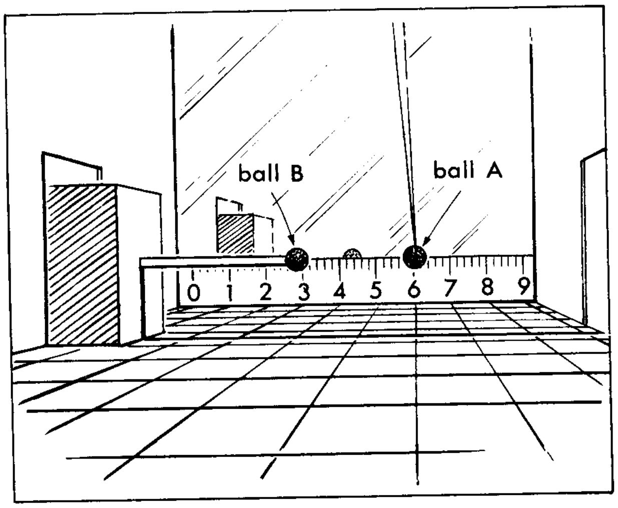


Figure 2
The scale and mirror of the apparatus shown in Fig. 1. The point of view is perpendicular to the scale at the position of ball *A*, whose image is hidden. To find the position of ball *B* on the scale, it is necessary to move your head slightly to the left until the image of ball *B* is hidden by the ball itself.

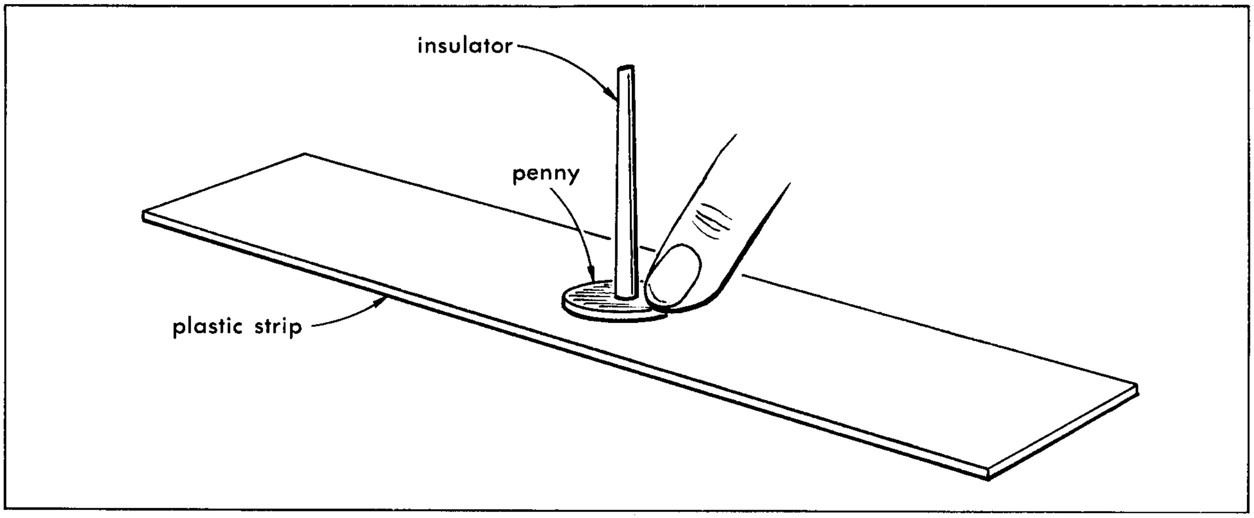


Figure 3
A penny with an insulating handle can be used to charge the spheres. Rub an acetate or vinyl strip with silk and place the strip on the table. Put the penny flat on the strip, handle up, and momentarily touch the top of the penny with your finger. When the penny is picked up by the insulating handle, it will be charged. The charge can then be transferred to the conducting balls by contact.

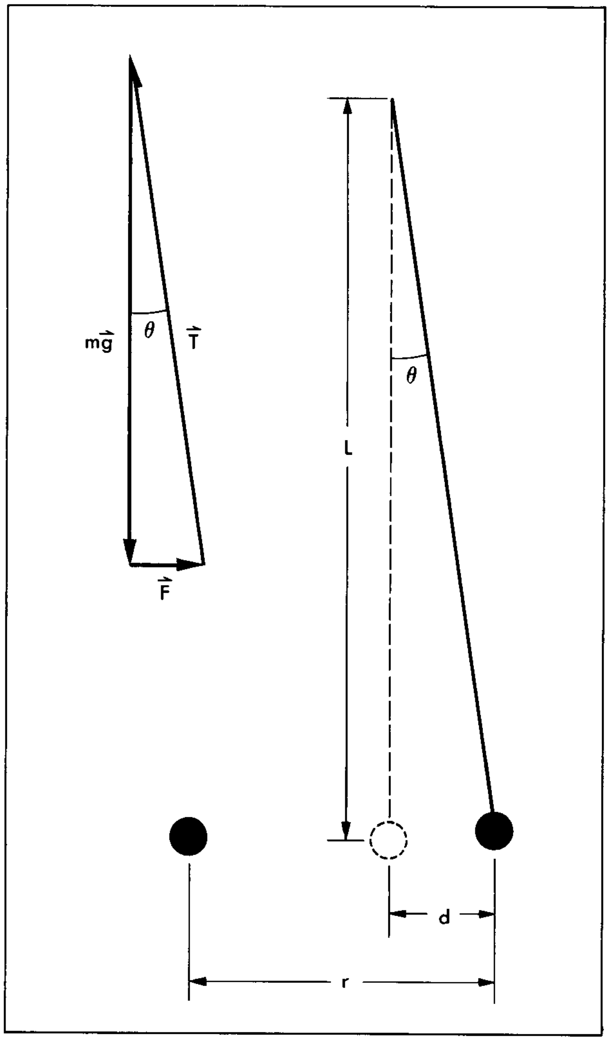


Figure 4

37 Driving Force and Terminal Velocity

To answer the question "Is there a smallest unit of electric charge?" we must be able to work with and measure extremely small charges. We detect electric charges through the electric forces exerted on charged bodies. To detect very small charges, therefore, we must be able to handle very small forces. The weight and other forces acting on bodies of ordinary size are so large that electric forces are insignificant unless the charge is great; therefore, very small objects are essential. Useful objects for this purpose are the small plastic spheres made for calibrating electron microscopes. Figure 1 shows a few of them. The spheres are rarely neutral; most of them carry a small electric charge. We shall attempt to measure the charge by measuring the electric forces acting on them.

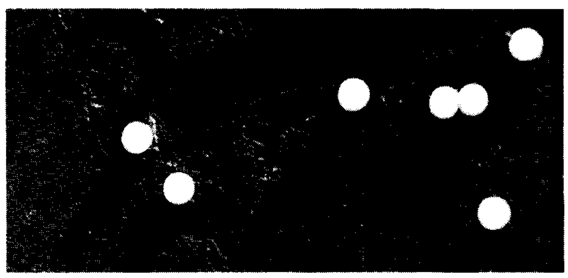


Figure 1
An electron-microscope photograph of a few latex spheres of diameter 1.8 microns. Those used in the experiment are a little smaller.

The apparatus is shown in Fig. 2. The only critical adjustment involves positioning the light source so that the image of the bulb filament is formed right at the center of the plates. To adjust the light source, you can hold a piece of paper vertically over the center of the plates, tilt the light source so that it shines on the paper, and slide the light-source tube back and forth until a clear image is formed on the paper. The filament of the light source should be vertical.

Plug the wires from the plates into the connectors next to the switch. CAUTION: The voltage is dangerously high; do not turn on the power supply until you have finished making the connections.

The switch controls the charge on the plates. In the center position there is no charge; with the switch up, one plate is positive, the other negative; and with the switch down, the polarities are reversed.

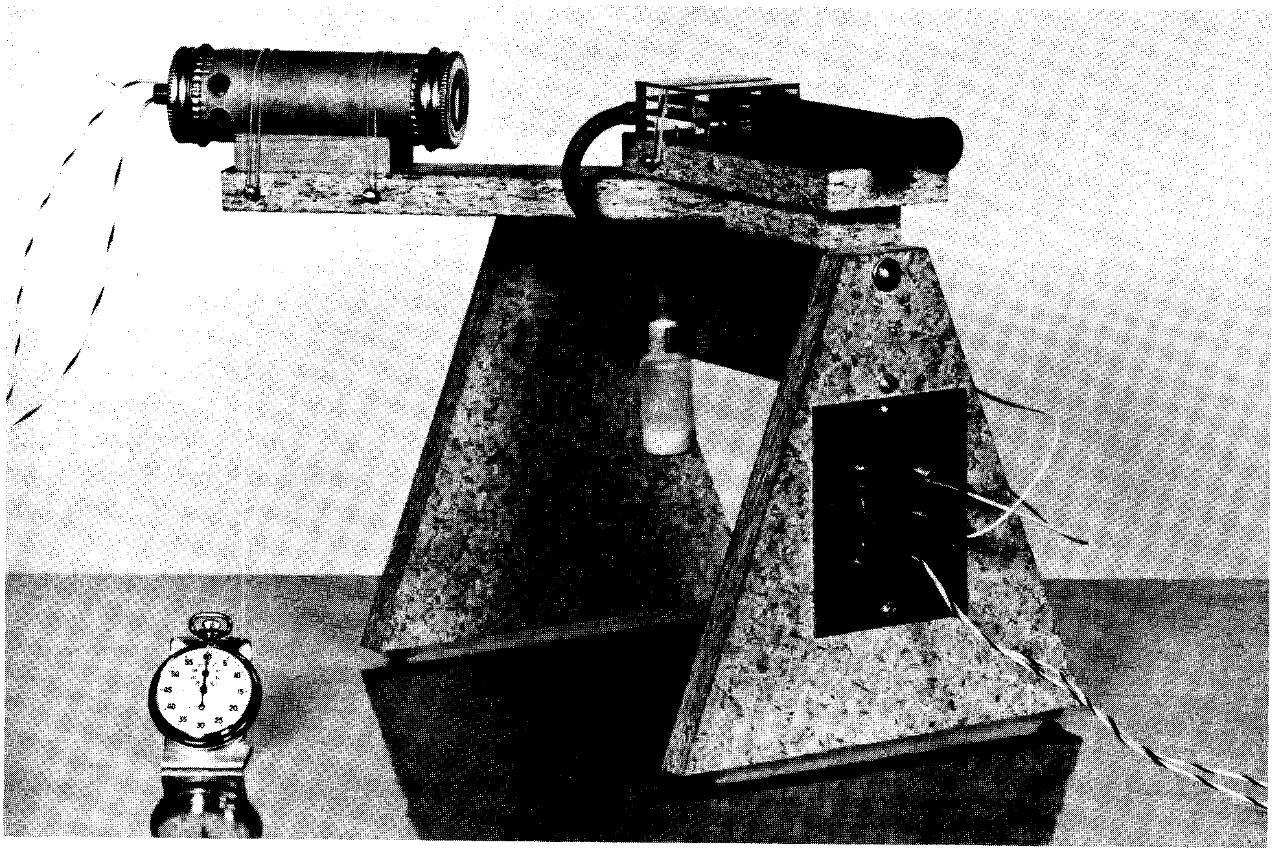


Figure 2
The small plastic bottle contains a suspension of the spheres in water. When you squeeze the bottle, a fine spray of water with many spheres is blown into the region between the plates. The water rapidly evaporates, leaving a cloud of spheres visible through the microscope as bright points.

With the light turned on, squeeze the bulb to bring in a cloud of spheres, leaving the switch centered. What do the spheres do? (Note that everything appears inverted when you look through the microscope.) Are they all moving in the same direction? Are they accelerating as they move across the field? Select a sphere and measure its speed in two parts of the field of view. What forces are acting on it?

How does charging the plates affect the motion of the spheres? What happens when you reverse the direction of the electric field? Measure again the velocity of a sphere in two parts of the field of view to see whether it is accelerating when an electric force is being applied. An explanation for the observed motion is that the air resistance on these tiny spheres increases rapidly and they very soon move at a constant terminal velocity with the force of air resistance equal and opposite to the driving force (which may be gravity alone or gravity plus an electric force). In this experiment we wish to find the relation between the terminal velocity and the force driving the sphere.

You will need three measurements on each of about a dozen spheres. A set for one sphere consists of: a velocity in free-fall under gravity alone with no charge on plates; a velocity V^+ where the electrical force is in the same direction as the gravitational force and the magnitudes of the forces add $(F_g + F_e)$; and a velocity V^- where the electrical force is opposite to the gravitational force $(F_g - F_e)$. The diagram of Fig. 3 shows the forces involved.

For each sphere you will therefore have three velocities: one for which the driving force is gravity alone; one, V^+, with an electric force added; and one, V^-, with the same electric force subtracted.

Measure the velocities by timing the motion over, say, ten spaces.

Plot the data for any one sphere on a graph like that in Fig. 4. How does the velocity V^+, observed when the driving force is $F_g + F_e$, compare with the velocity when gravity alone is the driving force? How does the velocity V^-, observed when the driving force is $F_g - F_e$ for the same sphere, compare with the velocity when gravity alone is the driving force? What is the shape of the graph of velocity as a function of driving force for one sphere? What is the shape of the graph of velocity versus force for your other spheres? What can you conclude about the relation between velocity and driving force?

Why, when the electric force is zero, do some spheres differ from others in velocity?

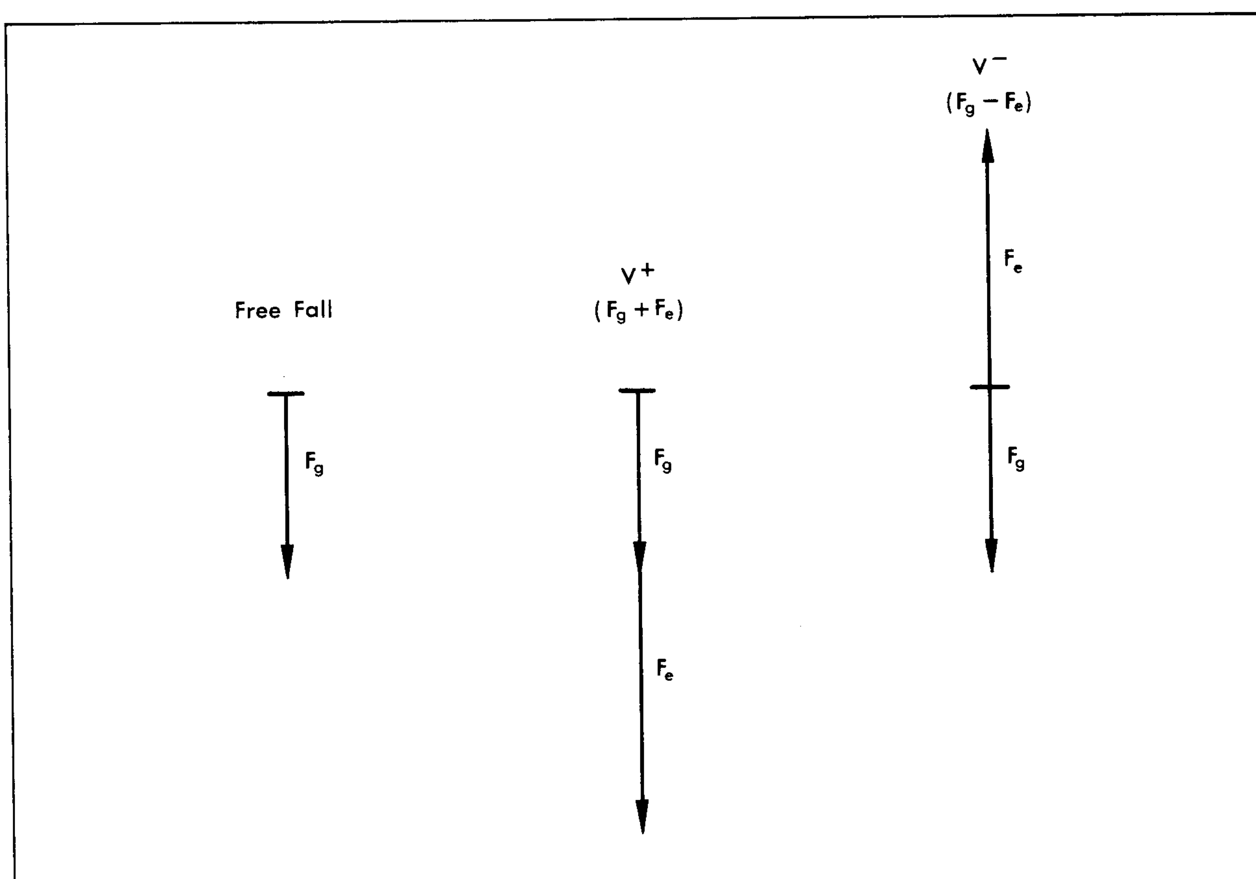


Figure 3

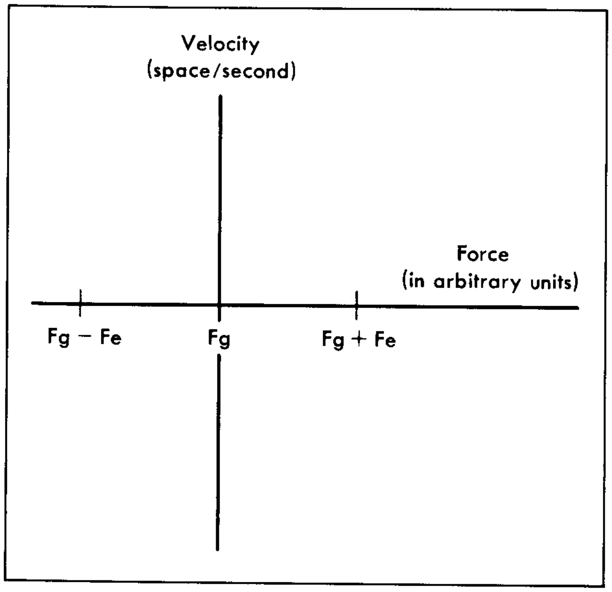

Velocity
(space/second)
Force
(in arbitrary units)
Fg − Fe
Fg
Fg + Fe

Figure 4

38 The Millikan Experiment

In the previous experiment we established that the terminal velocity of the small plastic spheres is directly proportional to the driving force acting on them. We shall now use this result to investigate whether the different charges appearing on different spheres are multiples of a unit charge. If we are careful, we can choose spheres of the same mass and thus ensure that the force of gravity is the same on each. Furthermore, since we keep the charge on the plates constant, the electric forces will be proportional to the charges on the spheres, and the differences in observed velocity will be proportional to differences in charge on the spheres.

It is important to look for a sphere each time which has a small charge, since, if there is a unit charge, the relative difference in charge will be greatest for small charges. You can make the selection in the following way:

Set up the apparatus as in the previous experiment. CAUTION: Do not forget that the voltage on the plates is dangerous. Squeeze the bulb to bring in a batch of spheres; then throw the switch to charge the plates and sweep out the fast-moving spheres that have large charges. Reverse the field several times so as not to lose the slow-moving spheres.

Place the switch in the neutral position and keep it there until most of the fast-moving, clumped spheres have dropped out of the field of view. If you still have a large number of spheres in view, repeat the two steps described above.

To insure that the sphere you have chosen is a single sphere, time its fall using only the gravitational force (with the electric field off). If the falling time is comparable to the falling time you found for a single sphere in the previous experiment, you may use it. If not, reject it and find another sphere.

With the switch you can get two combinations of forces: either $F_g + F_e$ (electric force in the same direction as gravity) or $F_g - F_e$ (electric force in the direction opposite to gravity). For each sphere you have a velocity V^+ proportional to $F_g + F_e$ and a second velocity V^- proportional to $F_g - F_e$. If you subtract V^- from V^+, to what force is the resultant velocity proportional? Could you use these differences of velocity as a good measure of the charge?

Measure V^+ and V^- for the sphere you have chosen. Repeat the experiment and measure V^+ and V^- for at least 12 other spheres, trying to find spheres which fall at different rates with the field on.

There are various ways in which you could analyze these data to look for evidence of the discreteness of electric charge. One simple way is to plot $V^+ - V^-$, equally spaced along the horizontal axis, in increasing order on a bar graph. (Recall that V^+ and V^- are vector quantities.) What evidence do you see for the existence of a natural unit of charge? What is the smallest number of charges you measured?

39 The Charge on a Capacitor

A capacitor is made by placing two metal plates side by side with a thin insulating spacer between them. If the capacitor is connected to a battery, charge will move onto the plates. In this experiment you will measure the amount of charge transferred by a battery to the plates and also find the relationship between the charge that accumulates on the plates and the potential difference applied to the capacitor.

Each of the two capacitors used in the experiment consists of two metal foils, separated by a paper insulator, that are rolled up into a cylinder. Try charging the capacitor with a battery, disconnecting the battery, and then discharging the capacitor through a milliammeter. How do you account for what happens?

Connect all the apparatus, except the milliammeter, as shown in Fig. 1. With the dry cell disconnected from the motor and one charging dry cell connected, turn the motor slowly with your hand. What happens to the capacitor during a full rotation of the shaft of the motor?

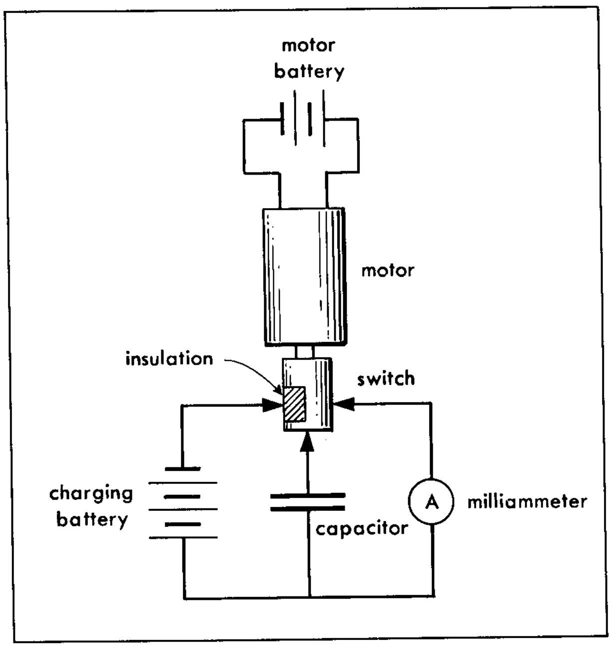


Figure 1

Now connect the motor to two cells in series and measure the frequency of the motor with a stroboscope. What is the current through the milliammeter with the motor turning steadily and one charging battery connected? Why does the meter now show a steady current? How many times per second was the capacitor charged and discharged? How many coulombs of charge were placed on the capacitor during each charging?

Repeat the experiment using two and then three charging dry cells in series.

Does the capacitor collect its maximum charge during the short time interval it is connected to the charging battery? You can investigate this by repeating the experiment with the motor connected to one cell only, thereby increasing the charging time. How much charge is now placed on the capacitor during each charge using one, two, and three charging batteries? Is it reasonable to assume that the capacitor collects its maximum charge during each charging?

What is the ratio of the charge on the capacitor to the potential difference? (This ratio q/V is called the capacity, and a capacity of one coulomb per volt is called a farad.)

It seems reasonable to assume that the capacitor will charge more slowly if a resistor is connected in series in the charging circuit. You can investigate this by charging and discharging the larger capacitor with the motor and with resistors of different sizes (100 ohm, 200 ohm, 10^3 ohm, and 10^4 ohm) connected in series in the charging circuit. In each case run the motor with one battery and then two, and compare the results. What do you conclude?

Connect two capacitors in parallel and measure the capacity. How does this compare with their separate capacities? Try two in series.

40 Energy Transferred by an Electric Motor

why do?

An electric motor is a device which converts electric energy into mechanical energy. If the EMF of a battery supplying energy to a motor is $\mathcal{E}$ and a charge q flows through the battery, the energy supplied by the battery is $q\ \mathcal{E}$. If the motor is used to lift a mass m a height h, the increase in the potential energy, or the useful work done by the motor, is mgh. Is the work done by the motor equal to the energy supplied by the battery?

The apparatus for this experiment is shown in Fig. 1. Be sure to mount the motor high enough so that the load of washers can be lifted about 2 meters. If the end of the dowel where the thread is tied is made slightly lower than the other end, the thread will wind up neatly. The details of the electric circuit are shown in Fig. 2.

With six washers on the end of the thread, you can vary the resistance until the washers are pulled up at about 0.5 m/sec. (The motor may not run steadily at lower speeds.)

What is the EMF of the battery? What is the current in the circuit while the washers are rising? How long does it take to lift them 1.5 m? How much

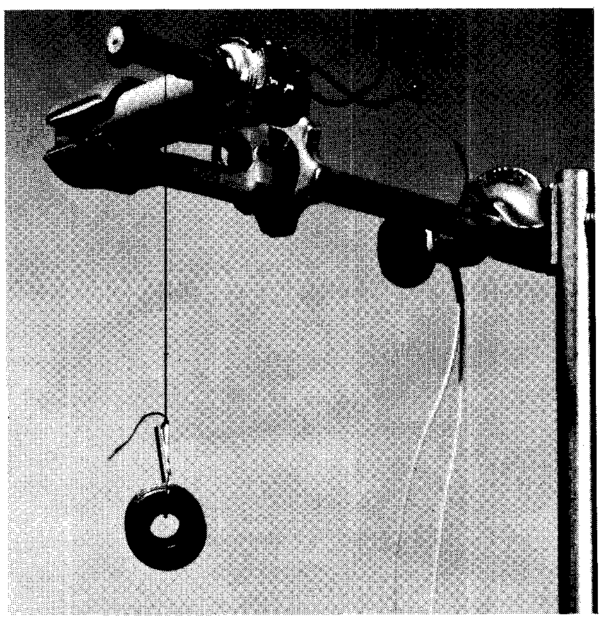


Figure 1

energy is supplied by the battery to lift the washers 1.5 m? How much does the potential energy of the washers change? How does the energy supplied by the battery compare with the change in the energy of the washers?

Repeat the experiment at several higher speeds. How does the current change?

Draw a graph of the current as a function of speed and a graph of the ratio of energy from the motor to energy from the battery (the efficiency of the system) as a function of speed.

Now repeat the experiment again, starting with six washers and a speed of about 0.4 m/sec, but this time keep the resistance fixed and remove the washers one by one until two washers are left.

Draw a graph of the efficiency of the system as a function of the number of washers. What do you conclude about the condition for highest efficiency?

Consider the system made up of the battery, resistance wire, and motor. Where is the energy going that is not converted into potential energy? (Neglect the internal details of the motor.)

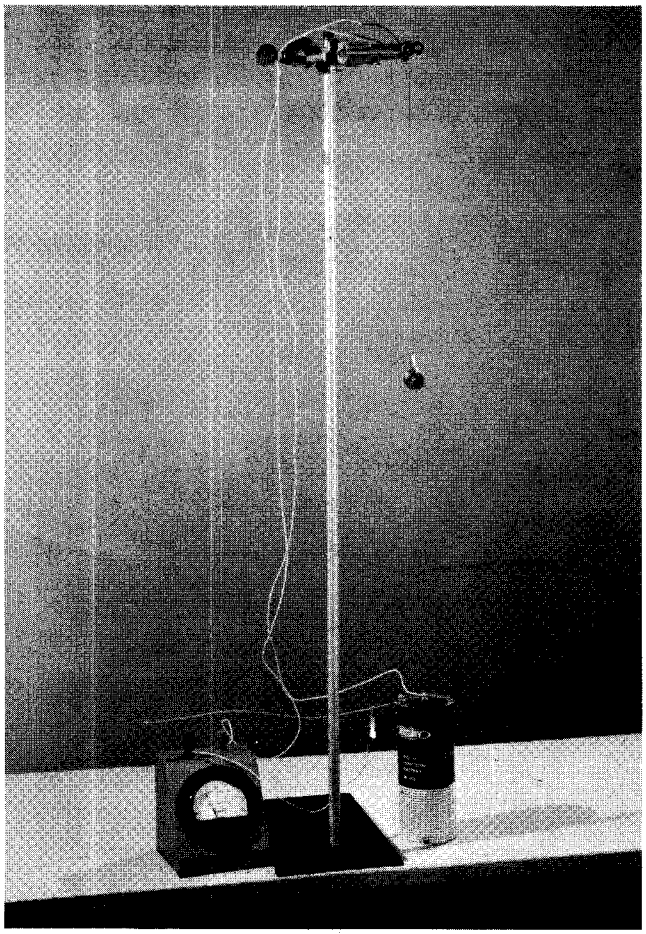


Figure 2

41 The Magnetic Field of a Current

Place a magnetic compass over a long piece of wire and momentarily touch the two ends of the wire to the terminals of a dry cell. The compass needle moves. Apparently the current in the wire creates a magnetic field which deflects the compass. How can we find the dependence of the direction and magnitude of a magnetic field on the current that produces it? A compass will indicate the direction, since it aligns itself in the field, and the magnitude of the field can be measured by comparing it with the constant field of the earth.

First investigate the direction of the magnetic field in the center of a coil of wire in the following way: Wind the wire into a coil of several turns on a frame as shown in Fig. 1. Place the compass in the center of the coil and note the

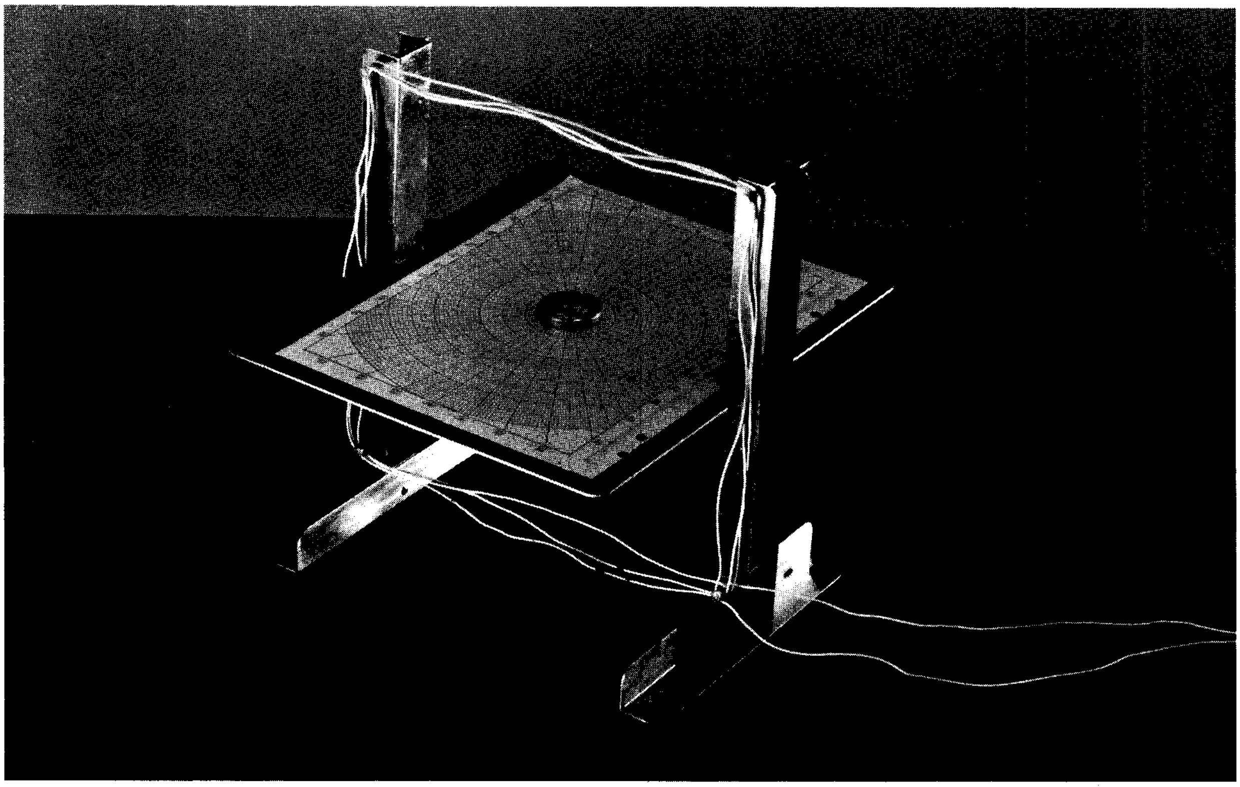


Figure 1

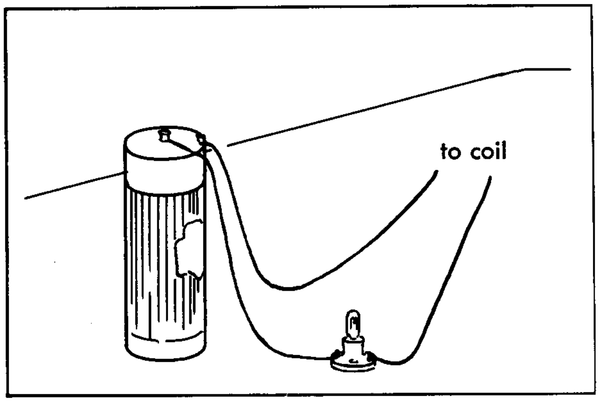


Figure 2

direction of the needle when there is no electric current in the coil. Now connect the coil through a flashlight bulb to a dry cell as shown in Fig. 2 and note the direction of the needle. The bulb keeps the current small.

Connect the coil directly to the terminals of the dry cell and observe the direction of the needle. (Do not leave the cell connected longer than necessary, because the large current flowing through the wire runs the cells down rapidly.) Turn the coil through a horizontal angle of about 30° and again note the direction of the field when a large current flows through the coil. What do you conclude about the direction of the field in the center of the coil due to the current?

Reverse the direction of the current and repeat the experiment. How does reversing the current affect the direction of the field?

With only one turn of the long wire on the frame, align the frame with respect to the earth's magnetic field so that the compass needle, placed at the center of the coil, lies in the plane of the coil. Again connect the ends of the wire to the dry cell through the flashlight bulb. The magnetic field produced by the current will be of about the same order of magnitude at the center of the coil as the horizontal component of the earth's field. Be sure that the wires from the coil to the cell are kept away from the loop so that the magnetic field from the current flowing in them will not contribute measurably to the field at the center of the coil. Measure the angular deflection of the compass needle. Reverse the direction of the current and again read the needle deflection.

Draw a vector diagram to find the strength of the magnetic field in terms of the earth's field.

Double the current flowing around the loop by adding another turn of wire to the coil and measure the compass deflection for both directions of current flow. Keep increasing the current in steps by adding turns of wire. When you have finished taking data, determine the field strength for each case by means of vector diagrams or trigonometry. What do you conclude about the magnitude of the magnetic field as a function of the current?

What will happen if you wind the coil with some turns going in one direction and others going in the opposite direction? What do you predict will be the magnitude of the resulting field? Measure the field to check your prediction.

Could you have measured the field resulting from the current if, initially, the needle was not parallel to the plane of the coil? Will the strength of the magnetic field of the compass needle influence the results of this experiment?

An alternate method of varying the current flowing around the loop is to vary the resistance of the circuit by means of a variable resistor connected with the coil. The circuit connections are shown in Fig. 3. If you have time, use this method. Are the field strengths measured for different ammeter readings consistent with the conclusions you have reached in the earlier part of the experiment about the field strength as a function of current?

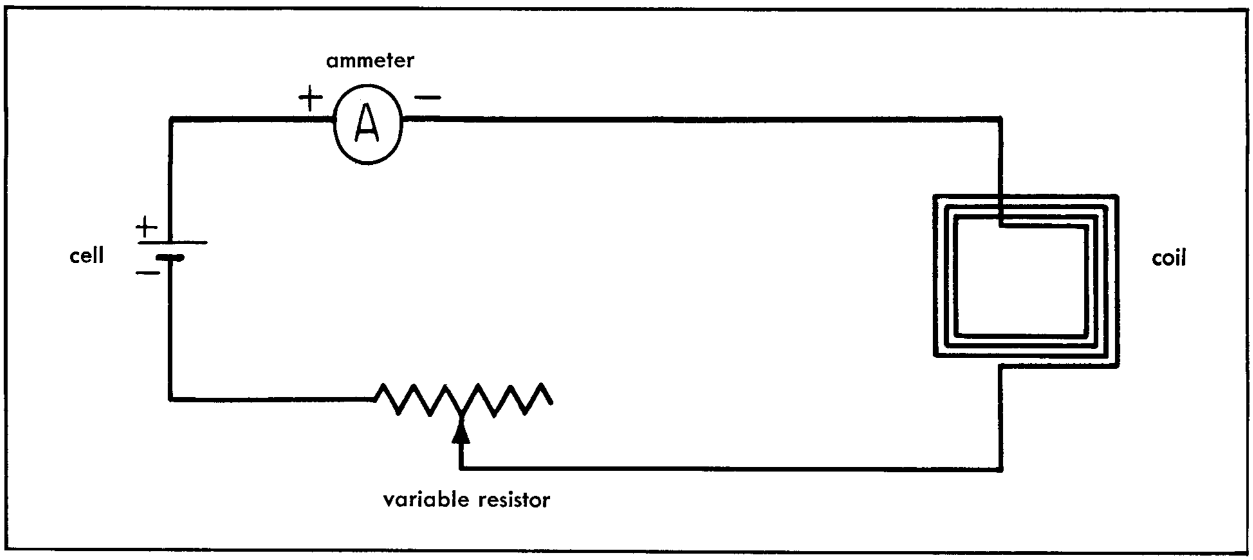


Figure 3

The Measurement of a Magnetic Field in Fundamental Units

42

In previous experiments we measured magnetic field strength in terms of the horizontal component of the earth's magnetic field. In this experiment we shall measure magnetic fields in more fundamental units, using the fact that a magnetic field exerts a force on a current-carrying wire. If we measure the force F in newtons, the current I in amperes, and the length of the wire L in meters, the strength of the field B in $\dfrac{\text{newtons}}{\text{ampere-meter}}$ is given by

$$B = \frac{F}{IL}$$

provided the wire is perpendicular to the direction of the field.

Figure 1 shows a sensitive balance that we can use to measure the force on a short length of current-carrying wire in a magnetic field. If the balance is so aligned that the end of the U-shaped metal loop (A in Fig. 1) is perpendicular

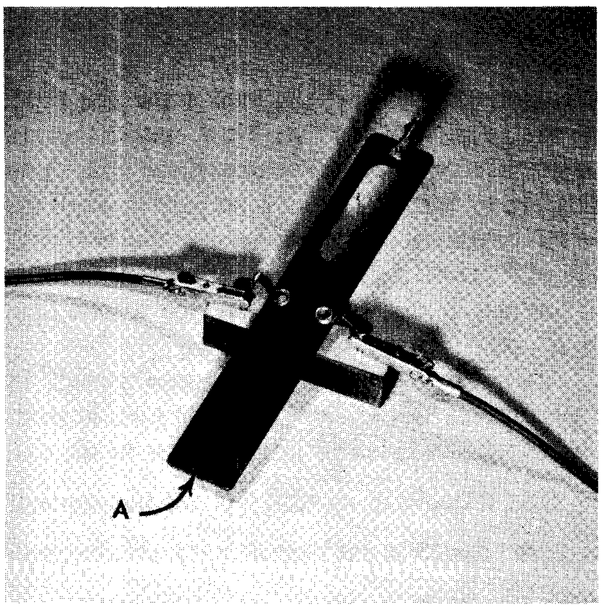


Figure 1

to the field while the sides are parallel to it, only the end will be subject to a force from the field. We can measure the force on the end of the loop by balancing it with a known weight hung from the other end of the balance.

In this experiment we shall determine the magnitude of the magnetic field in the center of a long coil (a solenoid) of current-carrying wire. Connect the loop, coil, variable resistors, and ammeters to a source of current as shown in Fig. 2. Be sure both the pointed tips of the loop and the tops of the supports are clean and shiny so that good electrical contact will be made.

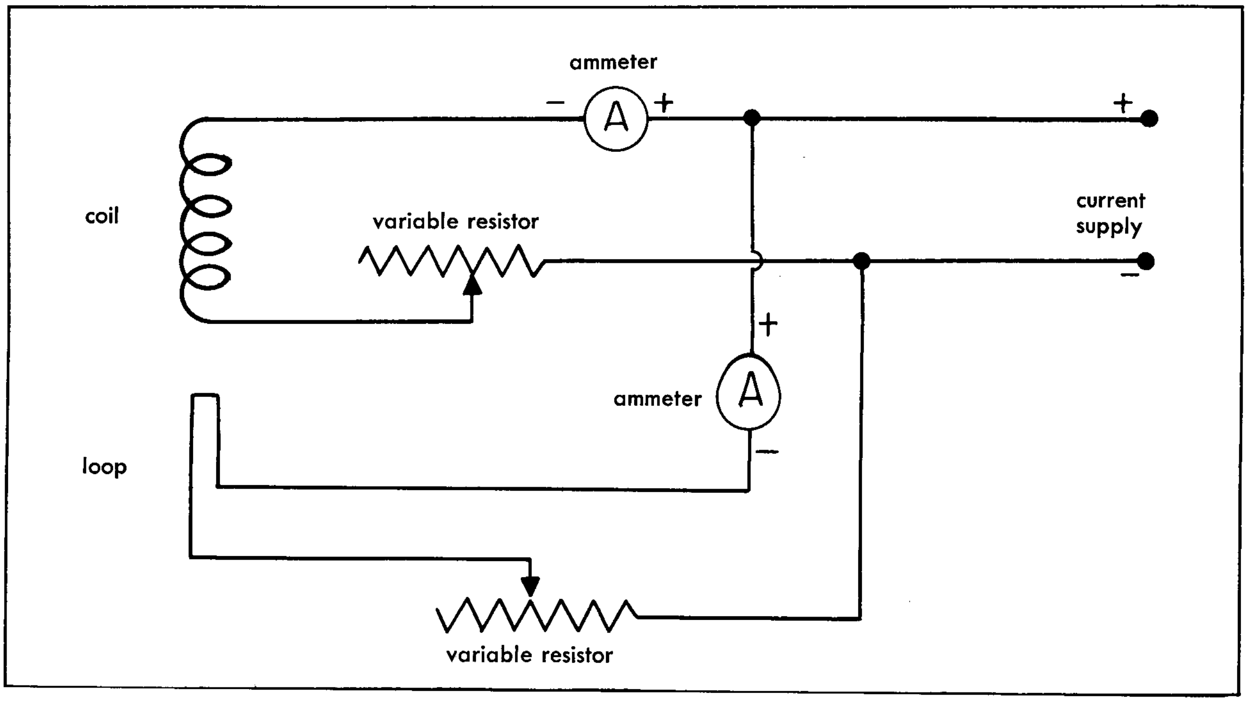


Figure 2

With no current flowing in the apparatus, balance the loop in the coil (Fig. 3). Level the loop by adjusting the position of the nut. Now, with a current of about 4 amperes, establish a magnetic field in the center of the coil. You can then measure this field by passing a current of about 1 ampere through the loop and finding the force needed to balance it. Roughly balance the loop with a short piece of string and then level it exactly by adjusting the current through it. (If the current in the loop fluctuates wildly when the balance is swinging, the contacts are corroded or rough.)

Find the weight of string needed for balance with other values of current in the loop. (The current in the loop should not exceed 5 amperes or the contacts will corrode.) What is the strength of the field in the center of the coil in $\frac{\text{newtons}}{\text{ampere-meter}}$? What is the field strength in newton-seconds per elementary charge per meter? (1 ampere $= 6.25 \times 10^{18}$ elementary charges/second.)

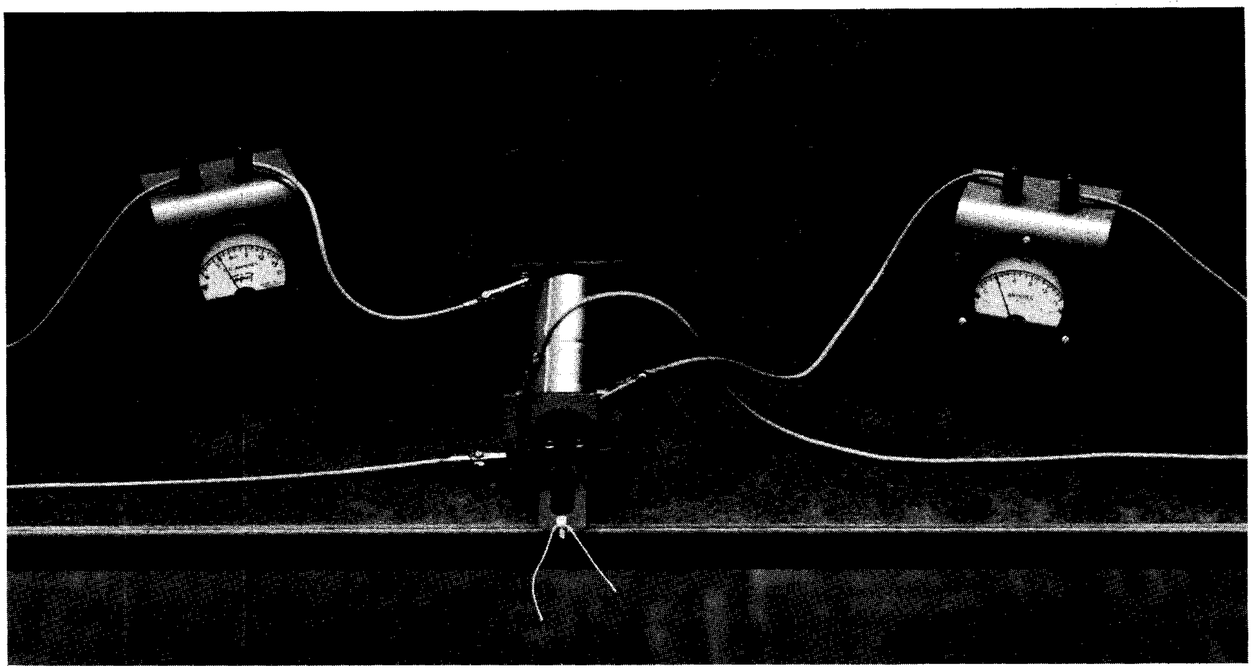


Figure 3

Measure the field in the coil resulting from several other values of current in the coil. (Five amperes is the maximum current the coil can carry without overheating.)

Do your measurements show that the field inside the coil is proportional to the current flowing through it? Draw a graph of the field strength as a function of the current in the coil. If all the coils used by your class are the same, pool all the class data in a histogram and from that determine the best value for the slope of the graph.

Could you use this apparatus to measure the field near a small permanent magnet? Can you use it to measure the field of the earth directly?

Why don't you use iron for the loop of the balance?

43 The Mass of the Electron

An electron, initially at rest, accelerates in an electric field and acquires kinetic energy equal to the product of its charge and the potential difference through which it moves; $\frac{mv^2}{2} = qV$. If the electron with velocity v then moves through a uniform magnetic field perpendicular to its direction of motion, the field exerts a centripetal force perpendicular to the electron's motion and the direction of the field. This force depends on the magnetic field strength B, the charge of the electron, and its speed; $F = Bqv$. The electron will follow a circular path of radius R given by

$$F = \frac{mv^2}{R}.$$

Equating the two expressions for the magnetic force, $F = Bqv$ and $F = \frac{mv^2}{R}$, gives

$$v = \frac{BqR}{m}$$

or

$$v^2 = \frac{B^2q^2R^2}{m^2}.$$

Substituting this expression for v^2 in the equation $\frac{mv^2}{2} = qV$ gives

$$m = \frac{B^2qR^2}{2V}.$$

Instead of using a tube like that described in the text for accelerating and deflecting electrons, we shall use a common commercial vacuum tube used in tuning a radio. Fig. 1 shows the construction of this tube. The electrons emitted by the cathode are accelerated by the potential difference between the cathode and the anode. They move radially outward in a fanlike beam, reaching nearly their maximum velocity by the time they emerge from beneath the black metal cap covering the center of the tube. Their speed is approximately constant over the remainder of their path to the anode.

The anode is coated with a fluorescent material which emits light when electrons strike it. Since it is conical in shape, we can see the path the electrons follow as they move outward from the cathode; when we look straight down from above, the conical anode slices the electron beam diagonally, showing the position of the electrons at different distances from the cathode. Two deflecting

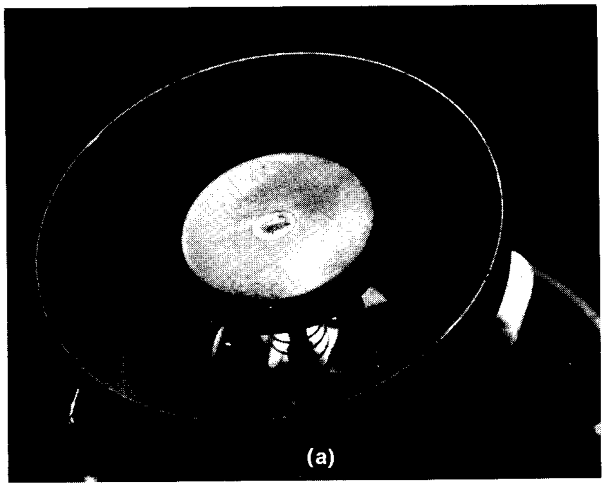


Figure 1 (a)
An electron tube or tuning eye with glass envelope removed.

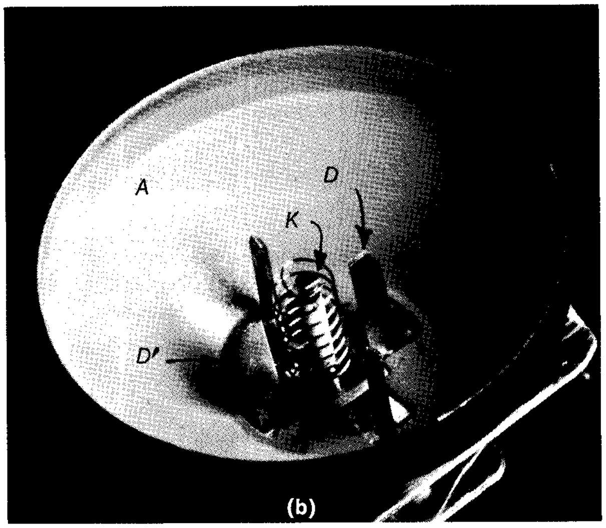


Figure 1 (b)
The metal center cap shown in (a) has been cut away from its wire supports and removed, revealing the important parts of the tube structure. *K* is the electron-emitting cathode. *D* and *D′* are the deflecting electrodes that form the shadow, and *A* is the anode coated with a fluorescent material.

electrodes are connected to the cathode and, with no magnetic field present, they repel electrons moving toward them from the cathode and form a wedge-shaped shadow behind them (Fig. 2).

When the tube is in a uniform magnetic field parallel to the cathode, the electrons are deflected in an almost circular path as shown by the curvature of the edge of the shadow (Fig. 3).

You will put a uniform magnetic field on the tube by inserting the tube into the center of a long coil. Connect the coil and tube as shown in Fig. 4. Set the anode potential to between 90 and 250 volts and then vary the current flowing through the coil until the curvature of the edge of the shadow is estimated to be the same as some small round object whose radius can be easily measured. A dime, a piece of wooden dowel, or a pencil will do.

Make measurements for several different anode potentials. (1 volt $= 1.6 \times 10^{-19}$ joule per elementary charge.) Also, use several different magnetic fields. (How do you know the magnetic field?) Calculate the mass of the electron.

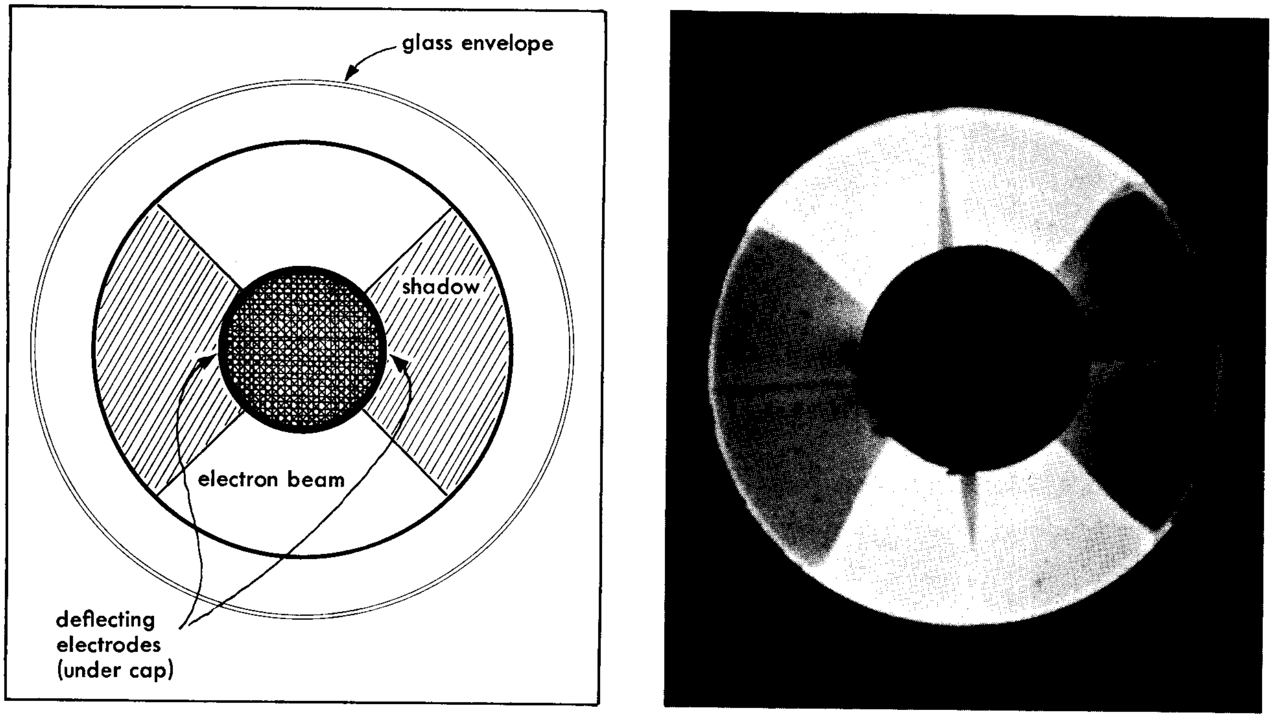


Figure 2
The drawing (left) shows the shadow of the radial beam we expect to see when there is no magnetic field. On the right is a picture of the tube in actual operation with no magnetic field applied; the two narrow shadows are caused by the wires supporting the center cap.

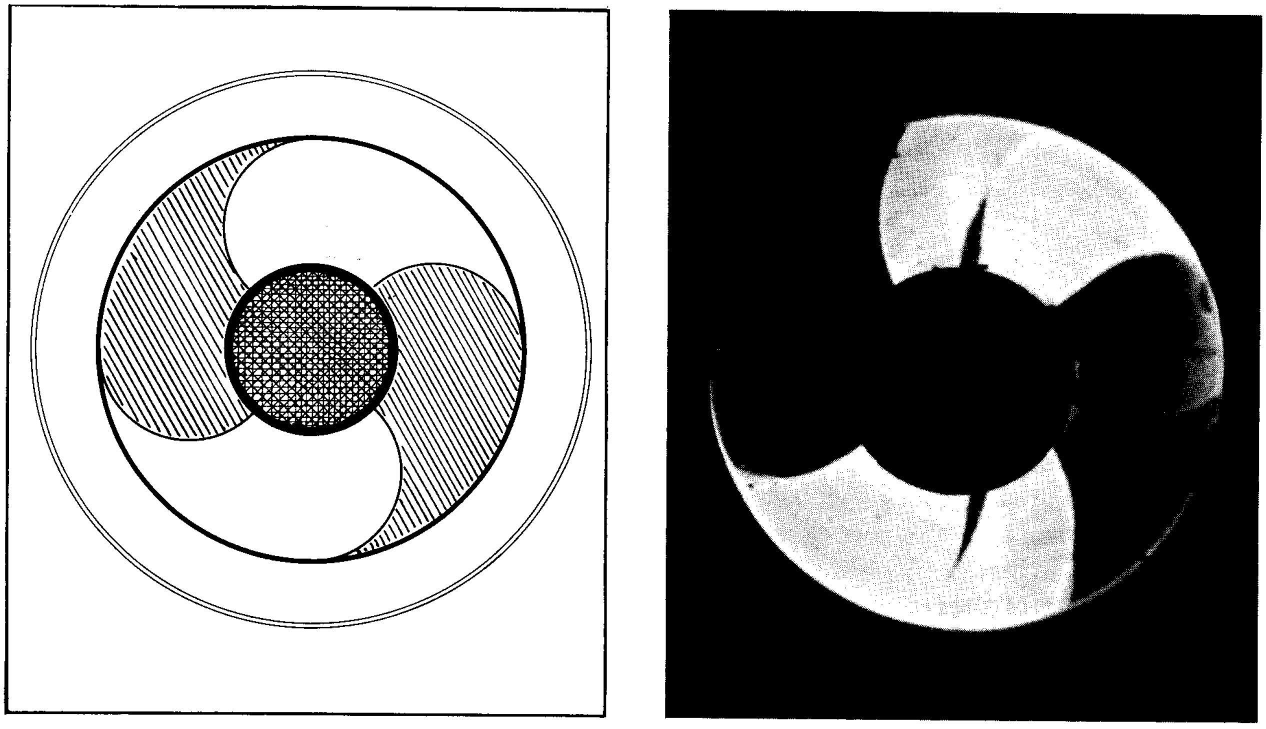


Figure 3
The shape the beam should have when the tube is in a magnetic field is shown on the left. On the right is the appearance of the beam when it is deflected by a magnetic field.

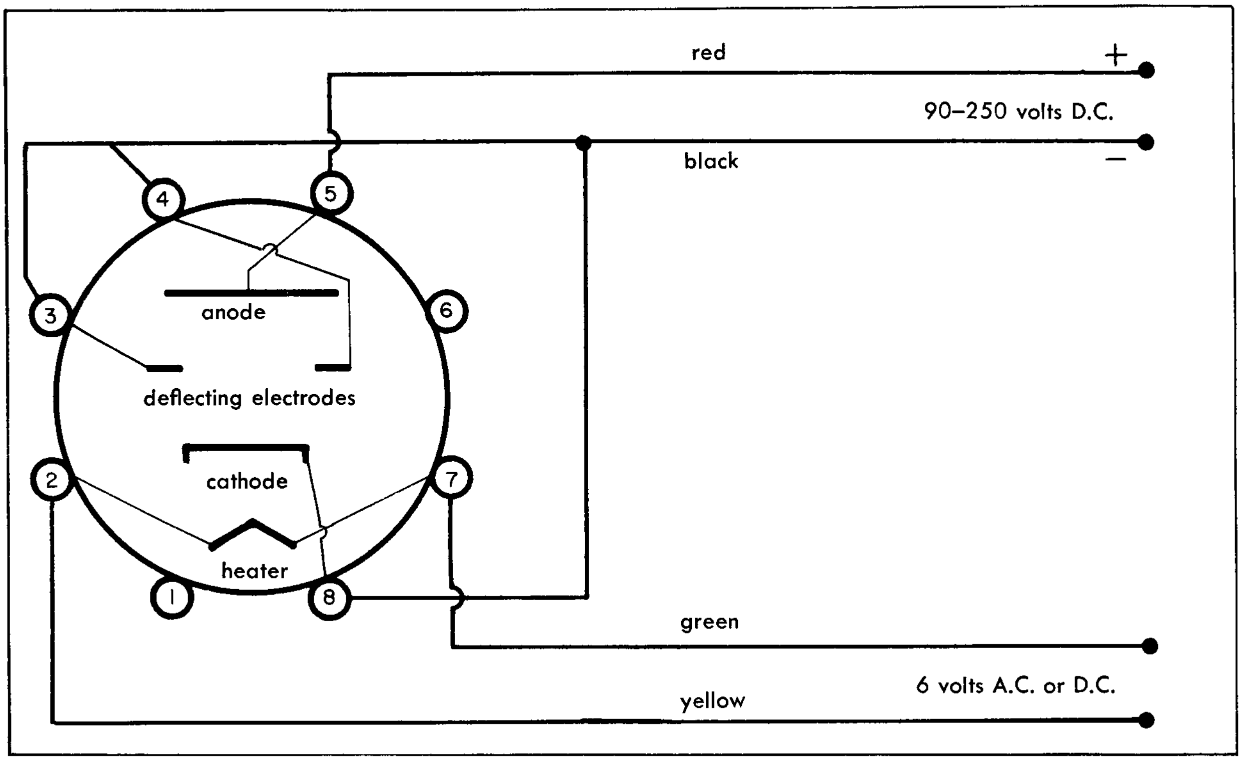


Figure 4 (a)
Circuit connections for Type 6AF6 electron ray tube.

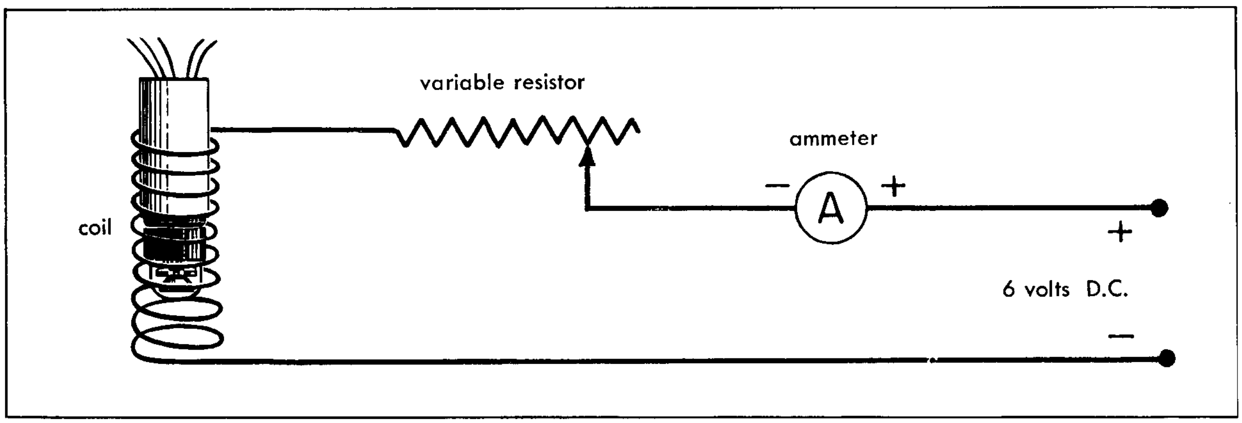


Figure 4 (b)
Circuit for coil.

Would it be possible to use the earth's magnetic field to deflect the beam? How large a tube would you need? Assuming the earth had no magnetic field, would it be practical to determine the mass of an electron by accelerating it horizontally through a known potential difference and subsequently observing its deflection in the earth's gravitational field?

44 The Magnetic Field Near a Long, Straight Wire

In Experiment 41 we used a compass to determine the magnetic field in the center of a loop of wire. Now we shall use the same method to find the field about a long, straight wire and its dependence on the distance from the wire.

Support a long, straight wire next to a sheet of graph paper which is aligned parallel to the horizontal component of the earth's field as shown in Fig. 1. The wire is held in position next to the table's edge by a piece of tape, and the graph paper is taped to the table top. Be sure there are no iron objects within 50 cm of the sheet of graph paper on which you will move the compass around. Except for the straight vertical section, all parts of the long wire should be at least 50 cm from the paper.

Allow a constant current of about 5 amperes to flow through the wire, and determine the direction of the field around it. To find the magnitude of the field, measure the deflection of the compass needle. Do this for different distances out to about 20 cm, moving the compass in steps along a line parallel to the horizontal component of the earth's field. If you try to measure the field at a distance comparable to the length of the compass needle, you will have large errors, since different parts of the needle are subject to widely varying forces. It is therefore best to start with the center of the compass about 5 cm from the wire.

How does the strength of the field due to the current vary as a function of the distance from the wire? How do you arrive at this conclusion?

Why was it necessary to keep the rest of the wire and iron objects far away from the compass?

How would the results have differed if the vertical wire had been only 20 cm long?

How would the accuracy of your results have been affected if you had used a current 100 times larger? One 100 times smaller?

Set up two parallel vertical wires about 20 cm apart in a plane parallel to the direction of the earth's field. Find how the magnetic field varies along a line between them (a) when the currents in the wires are in opposite directions; (b) when the two currents are in the same direction. How do you explain your results?

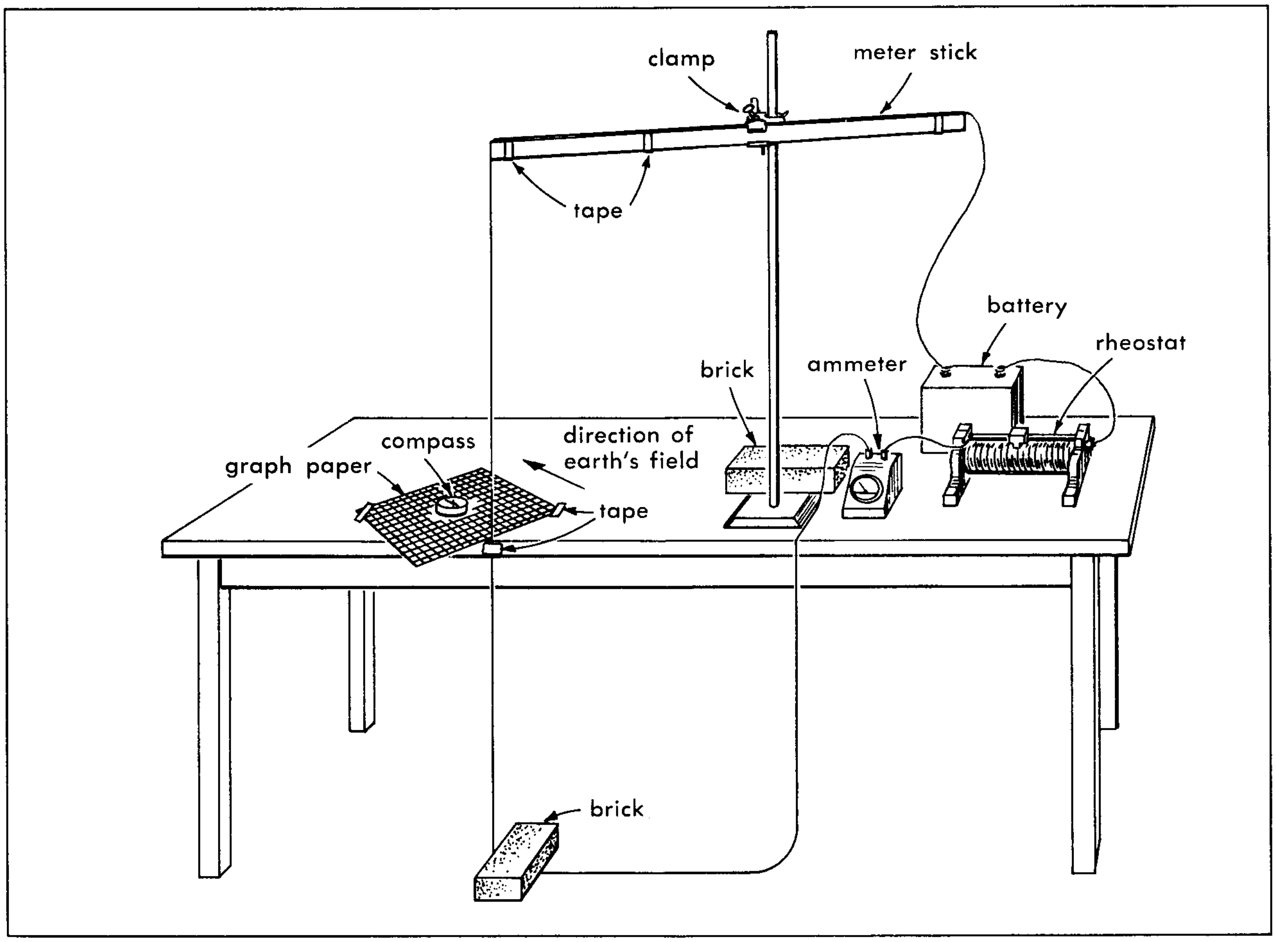

clamp
meter stick
tape
battery
rheostat
brick
ammeter
direction of
earth's field
compass
graph paper
tape
brick

Figure 1

45 Magnetic Circulation

In Experiment 44 you measured the magnetic field B near a long, straight wire carrying a current and found the relationship $B = K \dfrac{I}{r}$, with the direction of the field always perpendicular to the current. In Fig. 1 the field strengths along the two concentric circles are $B_1 = K \dfrac{I}{r_1}$ and $B_2 = K \dfrac{I}{r_2}$, with the field direction tangent to the circumferences of the circles. If in each case we multiply the field by the circumference of the circle, we obtain the circulation for each of the circular paths. The magnetic circulation is, therefore, $2\pi r_1 B_1$ in one case and $2\pi r_2 B_2$ in the other. Since $B = K \dfrac{I}{r}$, we know that $2\pi r_1 B_1 = 2\pi r_2 B_2 = 2\pi KI$, which means that the magnetic circulation is independent of the size of the circles.

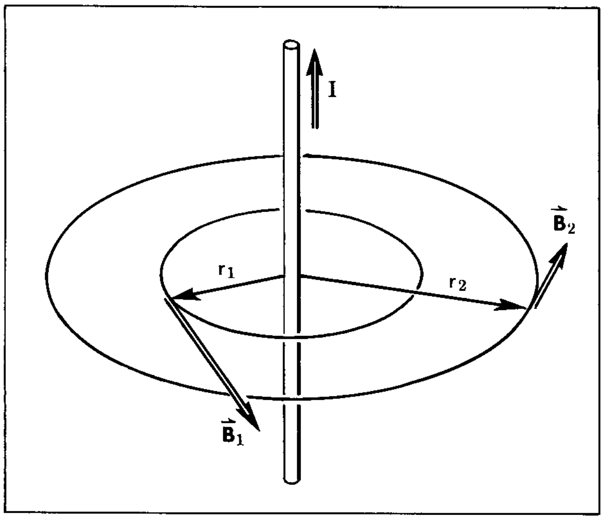


Figure 1

The question now arises as to whether or not the circulation is the same for all closed loops regardless of their shape or size. We can investigate this by measuring the circulation along an arbitrary closed loop such as that shown in Fig. 2. Since this irregular loop is not a circle concentric about the current, the

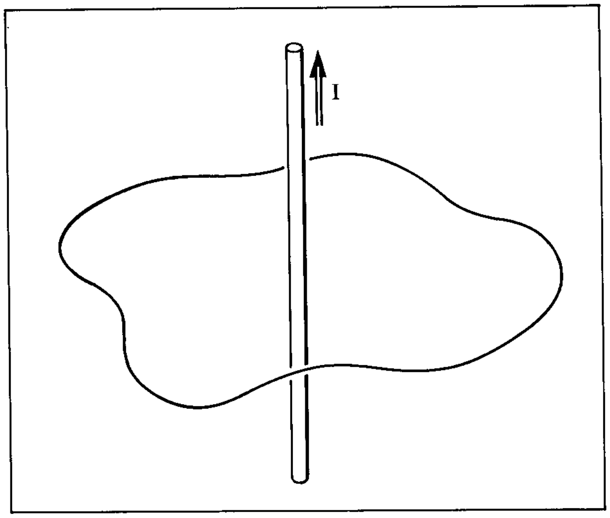


Figure 2

magnetic field is not tangent to the path except at a few points, nor does it have the same magnitude at all points along the path. We can overcome this problem by dividing the path into n small segments that are short enough to be essentially straight and along which the field does not change appreciably. The contribution of each segment to the total circulation is then $\Delta S\ B \cos\theta$ where ΔS is the length of the segment and $B \cos\theta$ is the component of B along ΔS. (See Fig. 3.) The total circulation around the loop is then equal to

$$\Delta S_1 B_1 \cos\theta_1 + \Delta S_2 B_2 \cos\theta_2 + \ldots + \Delta S_n B_n \cos\theta_n.$$

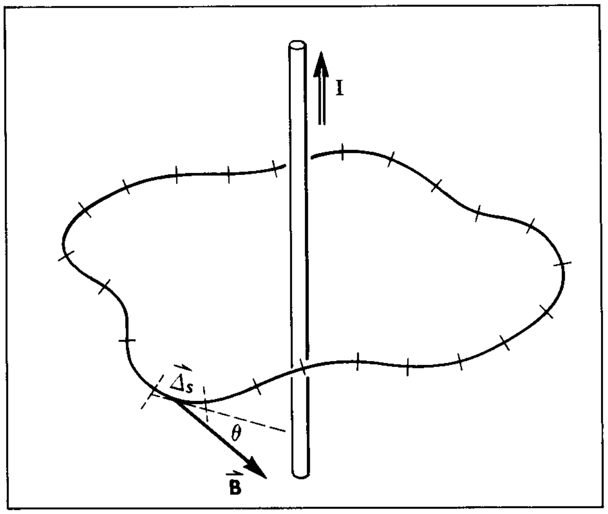


Figure 3

Set up the apparatus as shown in Fig. 4 and on the paper draw an irregular polygon of 7 to 10 sides whose lengths range from about 7 cm to 13 cm. (Don't copy the polygon shown in Fig. 4.) With the probe coil at the center of a

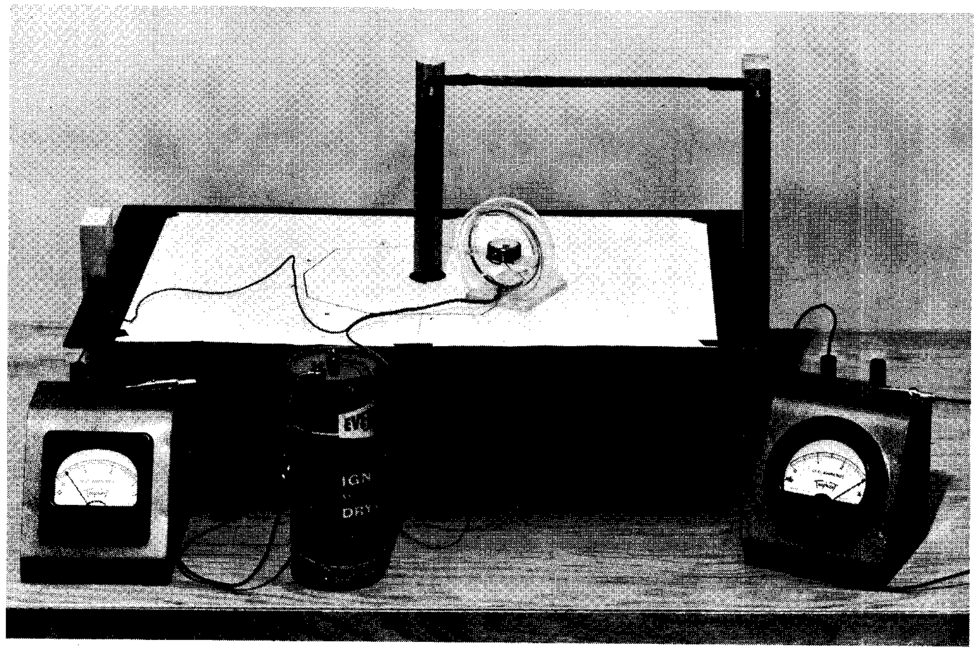
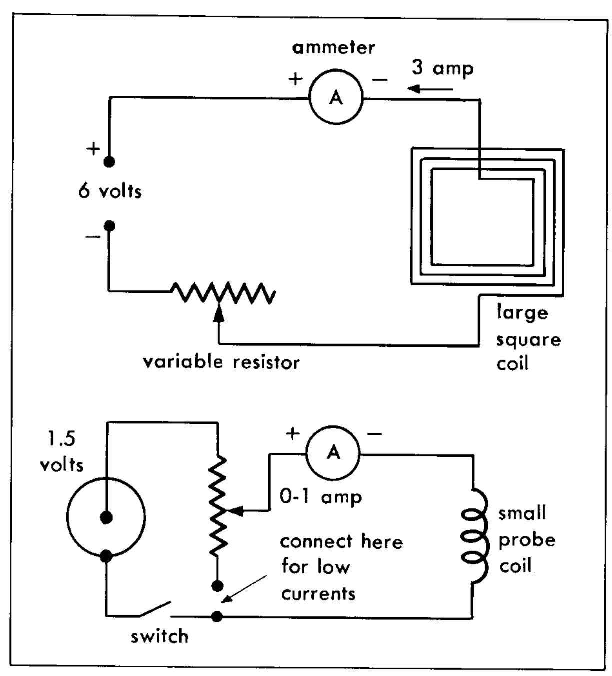


Figure 4
The photograph shows the large, 8-turn square coil which provides the current I about which the circulation is measured, and the small circular probe coil used to determine B cos θ aligned at the center of one of the segments of a polygon with its axis parallel to the segment. The two coils should be connected to separate batteries, as shown in the diagram.



straight segment and its axis parallel to the segment, a current in the probe coil that causes the needle to align itself perpendicular to the segment will produce a magnetic field at the center of the probe coil that is just equal to $-B \cos \theta$, as shown in Fig. 5.

Connect the probe coil and the large square coil to their respective batteries and adjust the current in the square coil to 3.0 amp. (Why must this current be kept constant during the measurement?) Now you can use the probe coil to get the total circulation around the polygon you have drawn. Since you do not know the proportionality factor relating the current I_p in the probe coil to the field that I_p produces at the center of the probe coil, you will have to express the total circulation in terms of ΔS and I_p. You can then compare the sum of all the products $\Delta S I_p$ found for each segment with the value $2\pi r I'_p$, which is the circulation around a circular path like one of those in Fig. 1, expressed in terms of ΔS and I_p.

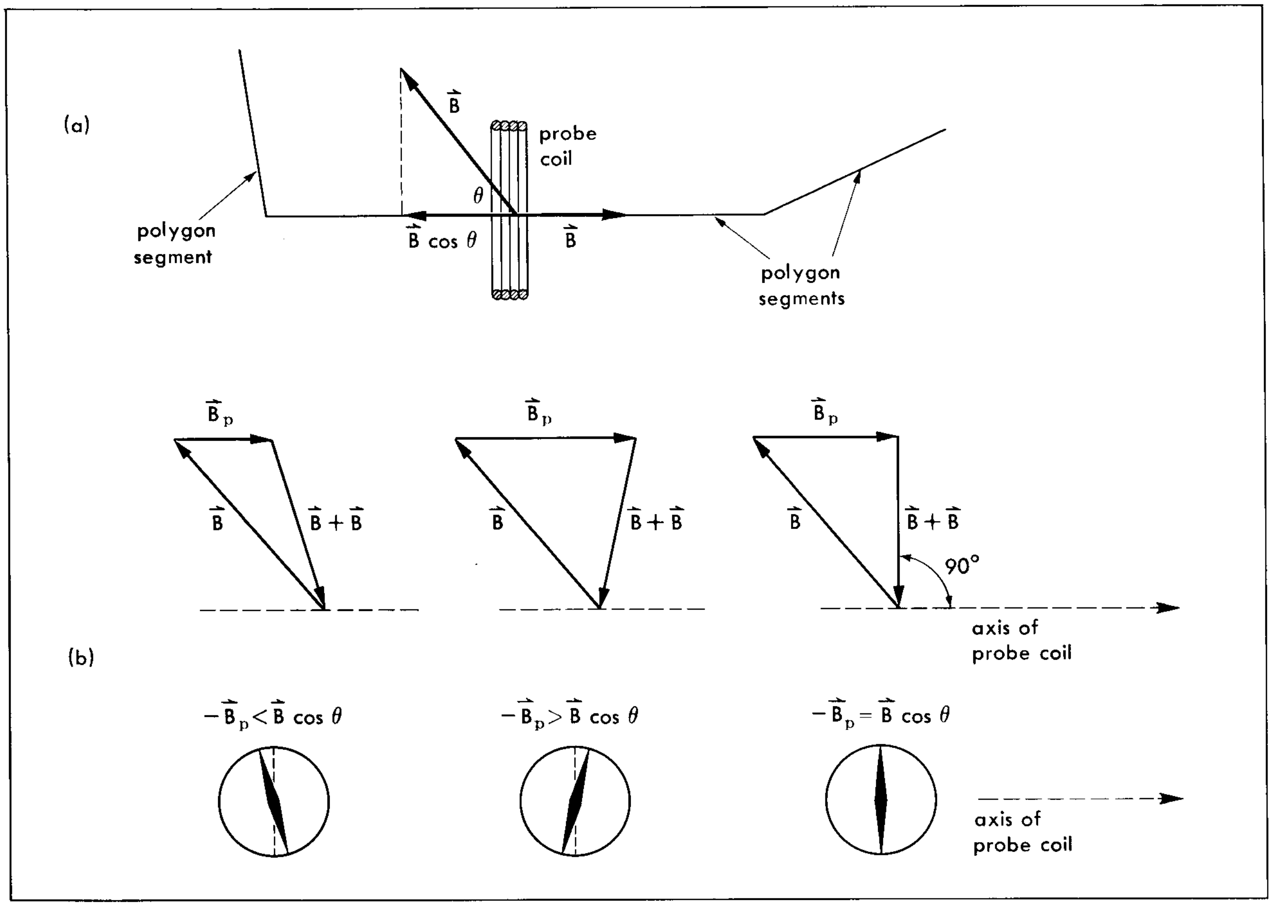


Figure 5
(a) The vector diagram showing the relationship between $\vec{B}$, $\vec{B} \cos \theta$, and $\vec{B}_p$, the field at the center of the probe coil due to the current in the probe coil. (b) Only when $-B_p$ is equal to $B \cos \theta$ does the compass needle in the probe coil point at right angles to the axis of the probe coil.

First measure $I_p \propto B \cos \theta$ and ΔS for each side of the polygon and find their sum. After making a few readings you may find a side where you must reverse the current in the probe coil in order to deflect the compass at right angles to the probe-coil axis. How does this affect the contribution to the magnetic circulation made by this side?

Now, to find the circulation around a circular path in terms of ΔS and I_p, place the probe coil so that its axis is tangent to a circle centered around a long straight wire at a point where the compass needle, acted upon only by the earth's field, points toward the long wire. When a constant current of about 5 amperes is sent through the long wire, the compass needle will be deflected. Determine the current I'_p in the probe coil which will cause the needle to point again toward the wire. This current I'_p is the current that produces a field equal and opposite to the magnetic field produced by the current in the straight wire. Now, to find the circulation around the circle all you need to do is calculate $2\pi r I'_p$ where r is the radius of the circle.

What effect does the current I and the number of turns on the square coil have on the circulation? Does the earth's magnetic field affect the circulation?

How does the circulation around the circle compare with that around the polygon? How does the circulation around your polygon compare with that of your classmates using different polygons? What do you conclude?

Now find the circulation around a polygon more or less like that in Fig. 6 that does not enclose a current. Do your results agree with the expression $2\pi KI$ for the circulation?

What would be the circulation about the path shown in Fig. 7?

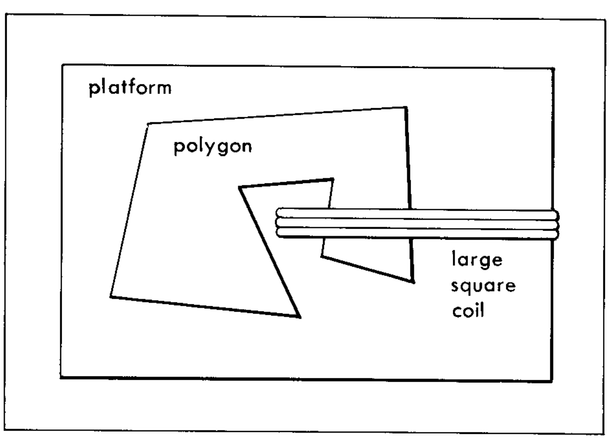


Figure 6

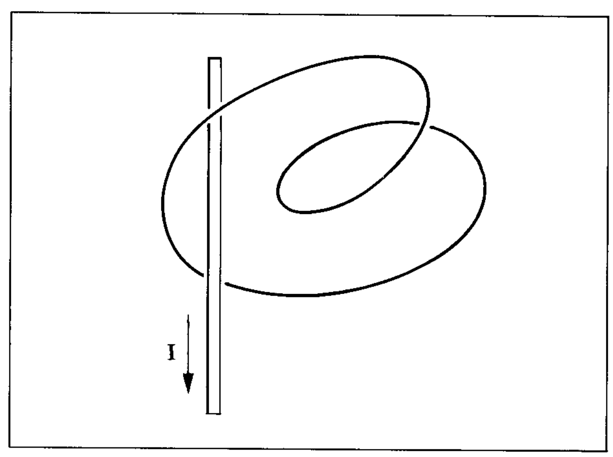


Figure 7

Randomness in Radioactive Decay

46

Radioactive elements emit particles which can be counted by a Geiger counter. Each click of the counter represents the decay of a single atomic nucleus. What can we find out about the rate at which a radioactive sample decays?

Place the probe of a Geiger counter far enough from a radioactive sample that the clicks come slowly enough to count easily. Once the probe and counter are in position, do not move them. Make a few 10-sec practice counts and then count the clicks continuously for twenty minutes, recording the number counted during each 10-sec interval. You will undoubtedly find that the number of counts per interval will vary.

Make a bar graph of your results, plotting the number of intervals N, in which k clicks are heard, as a function of k. From this graph, what do you estimate the average counting rate to be?

Now add the count obtained in the second 10-sec interval to the count obtained in the first, and divide by two to find the average counting rate over a 20-sec interval. Then add the count found in the third interval to the sum of the counts in the first two intervals and divide by three to find the average rate over a 30-sec interval. Continue this process, interval by interval, until you arrive at the average counting rate over a period of 15 to 20 minutes. How does the average rate over the whole period compare with the estimate you made from the bar graph? Plot the average counting rates obtained in this way as a function of the total count used for each successive calculation.

How does the accuracy of the measurement of the counting rate appear to depend on the total number of counts used in the calculation? What counting rate would you expect to find if you counted clicks for two hours? Would this increase your accuracy?

Since only a small fraction of the particles emitted by the sample hit the counter, the counting rate you obtained is much less than the rate of decay of the sample. How would you calculate the average number of atoms that disintegrate in each second (the rate of decay of the sample)?

Can you determine the half-life of the sample from your measurements?

You can do this experiment using a cloud chamber and a weak radioactive sample on a needle tip inside the chamber. How would the counting rate be related to the rate of decay?

47 The Spectrum of Hydrogen and Planck's Constant

The spectral lines of atomic hydrogen offer a good opportunity for comparing theory with observation. In this experiment you will measure the wavelengths of three spectral lines of hydrogen. From the numerical relationships between these wavelengths and the wave theory of atomic energy levels you can calculate Planck's constant.

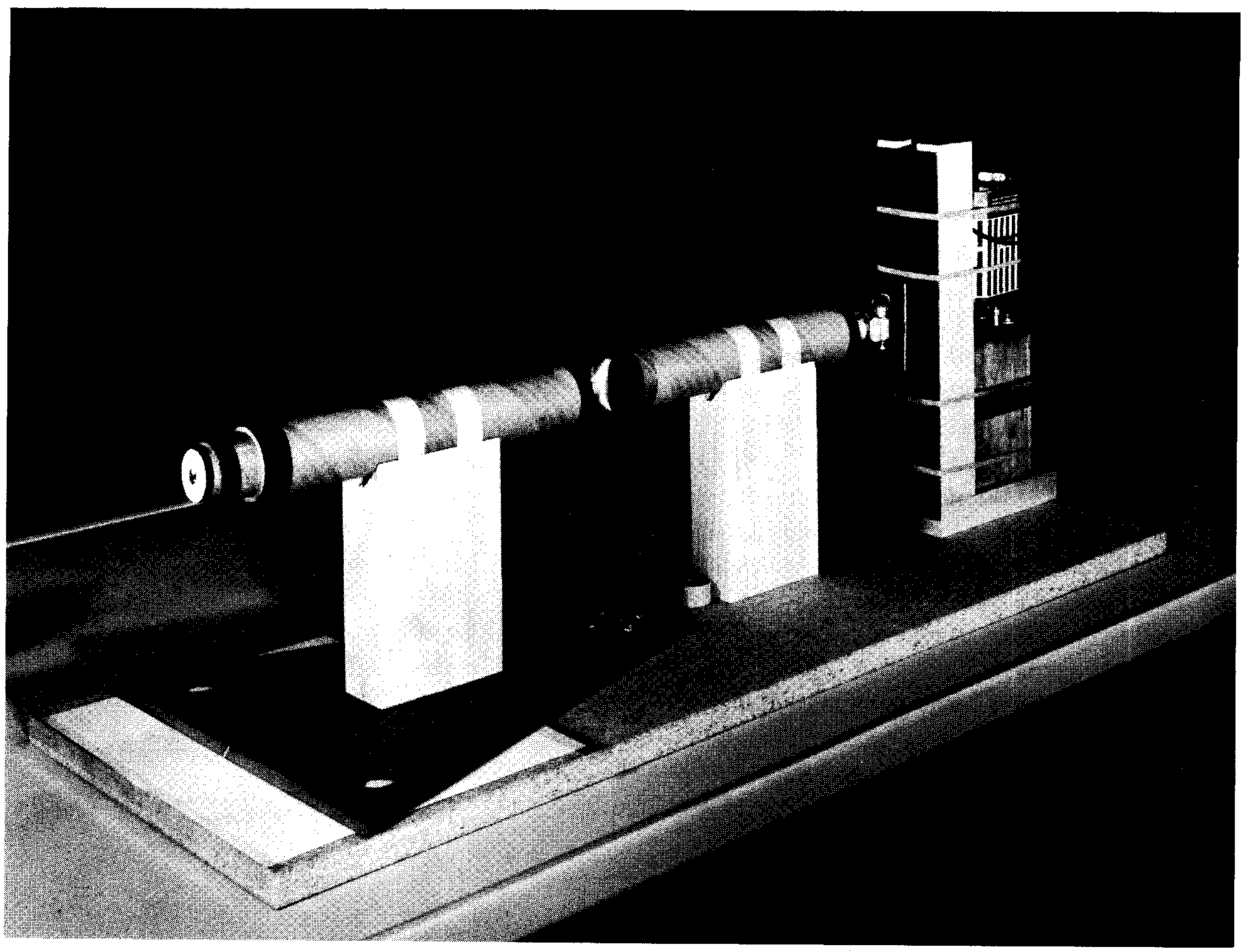


Figure 1
Make sure the sheet of paper under the telescope base is taut.

Photographs (Fig. 1 and 2) and schematic drawing (Fig. 3) show the spectrometer you will be using. Several adjustments are necessary to bring the spectrometer into operating conditions. First you want to have the telescope focused on infinity. It is best to remove the telescope from its mount for this purpose, and focus it on a distant object. With the telescope back on its mount, look at the light source and make sure that the cross hair and slit are aligned vertically. Now, with the slit cap off, move the slit back and forth until you find the position that yields the sharpest image of the slit. Where is the image of the slit? To be sure the diffraction grating is parallel to the slit, look at one of the lines (*not* the central maximum) while rotating the grating. Can you tell when the grating is parallel to the slit? Why? A narrow slit compatible with a reasonably bright image of the slit is most desirable.

You know from your study of waves that the wavelength, λ, is related to the distance between the lines of the grating, d, and the diffraction angle, θ, through the following relation:

$$\lambda = d \sin \theta.$$

You can calibrate the spectrometer by trying it on a spectral line of known wavelength and marking the positions of the notch at the center of the base of the telescope with a sharp pencil. The distance between the two marks is

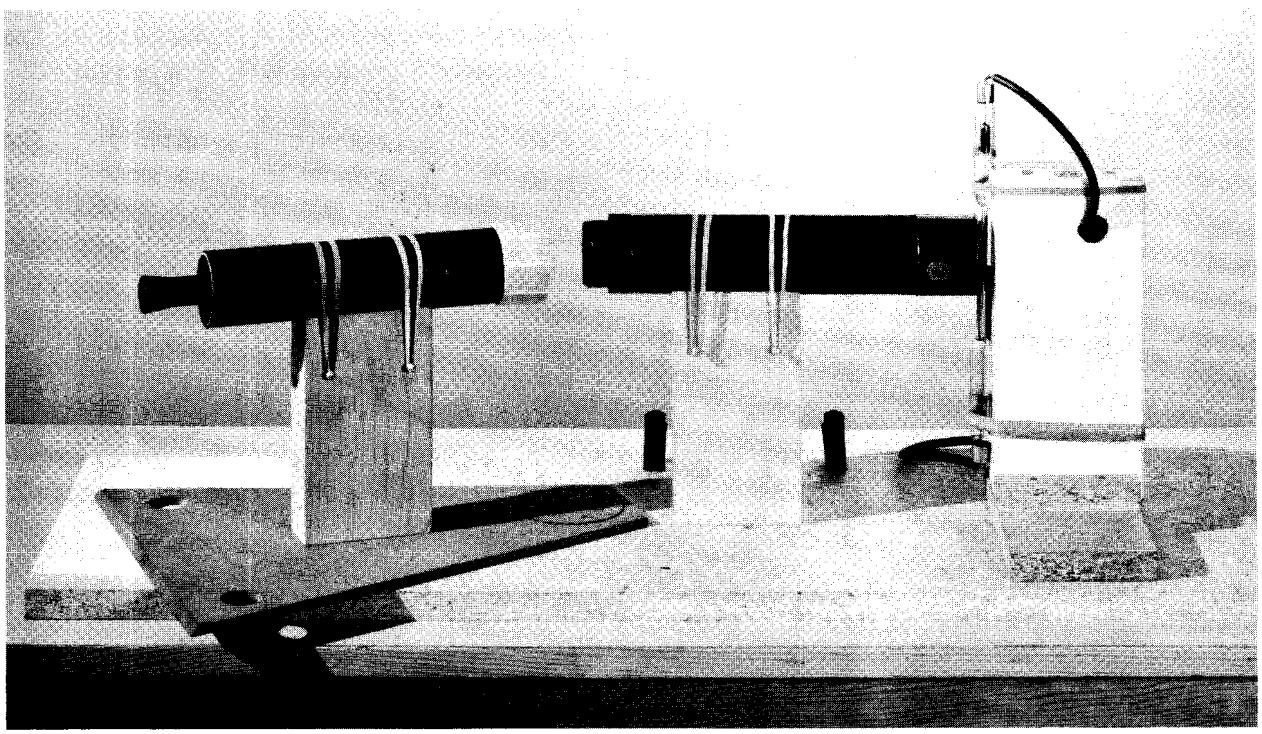


Figure 2
After completion of all the adjustments, fasten the collimator and telescope to their mounts.
Why is it important to have the grating exactly over the pivot point of the telescope?

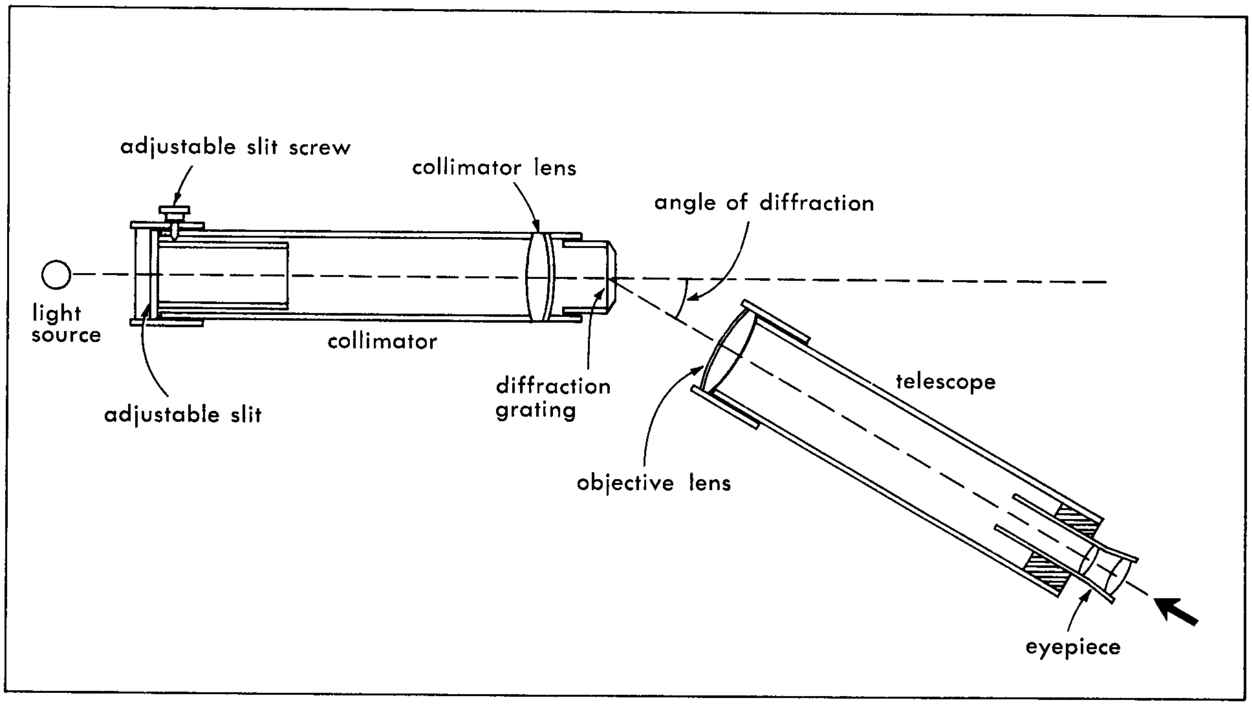


Figure 3

proportional to λ. What is the constant of proportionality? A convenient line to use is the green line of the mercury spectrum (5461 A). You can see this line and a few others by using an uncoated fluorescent bulb or a mercury discharge tube as a light source.

Now replace the fluorescent bulb with the hydrogen discharge tube. The intensity of this tube is much less than that of the fluorescent bulb, and you may find it worthwhile to work in a darkened room. Scan through the range of angles corresponding to the visible spectrum. How many spectral lines do you see? What are their wavelengths?

We know from the study of inelastic collisions between electrons and atoms (see text, Section 26–3) that the frequency of an emitted spectral line, ν, is given by $\nu = \dfrac{E_i - E_f}{h}$, where E_i and E_f are the energies of the atom before and after the emission of the photon and h is Planck's constant. For atomic hydrogen the wave theory of energy levels (text, Section 26–5) tells us that the energy levels have the following values:

$$E_n = -\frac{2\pi^2 k^2 m}{h^2} \cdot \frac{1}{n^2}.$$

Let us denote the value of n for E_i and E_f in the first equation by n_i and n_f. Then, by combining the two equations,

$$\nu = \frac{2\pi^2 k^2 m}{h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right);$$

or, since $\nu = \frac{c}{\lambda}$,

$$\frac{1}{\lambda} = \frac{2\pi^2 k^2 m}{h^3 c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right).$$

From the ratios of the inverse of the wavelengths, can you tell which energy levels were involved in emitting your spectral lines? (Hint: Assume that the lower level is the same for all your lines.)

After you are sure of the values of n_i and n_f for the three spectral lines, you know all the quantities in the last equation except Planck's constant. Calculate it. Using all the information you have, what is the accuracy of your determination of h? What is the accuracy of the best value of h obtained from a histogram of all the class results?

Table of Trigonometric Functions

sin (read down)

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
0°	.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	.0175	89°
1°	.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	.0349	88°
2°	.0349	.0366	.0384	.0401	.0419	.0436	.0454	.0471	.0488	.0506	.0523	87°
3°	.0523	.0541	.0558	.0576	.0593	.0610	.0628	.0645	.0663	.0680	.0698	86°
4°	.0698	.0715	.0732	.0750	.0767	.0785	.0802	.0819	.0837	.0854	.0872	85°
5°	.0872	.0889	.0906	.0924	.0941	.0958	.0976	.0993	.1011	.1028	.1045	84°
6°	.1045	.1063	.1080	.1097	.1115	.1132	.1149	.1167	.1184	.1201	.1219	83°
7°	.1219	.1236	.1253	.1271	.1288	.1305	.1323	.1340	.1357	.1374	.1392	82°
8°	.1392	.1409	.1426	.1444	.1461	.1478	.1495	.1513	.1530	.1547	.1564	81°
9°	.1564	.1582	.1599	.1616	.1633	.1650	.1668	.1685	.1702	.1719	.1736	80°
10°	.1736	.1754	.1771	.1788	.1805	.1822	.1840	.1857	.1874	.1891	.1908	79°
11°	.1908	.1925	.1942	.1959	.1977	.1994	.2011	.2028	.2045	.2062	.2079	78°
12°	.2079	.2096	.2113	.3130	.2147	.2164	.2181	.2198	.2115	.2233	.2250	77°
13°	.2250	.2267	.2284	.2300	.2317	.2334	.2351	.2368	.2385	.2402	.2419	76°
14°	.2419	.2436	.2453	.2470	.2487	.2504	.2521	.2538	.2554	.2571	.2588	75°
15°	.2588	.2605	.2622	.2639	.2656	.2672	.2689	.2706	.2723	.2740	.2756	74°
16°	.2756	.2773	.2790	.2807	.2823	.2840	.2857	.2874	.2890	.2907	.2924	73°
17°	.2924	.2940	.2957	.2974	.2990	.3007	.3024	.3040	.3057	.3074	.3090	72°
18°	.3090	.3107	.3123	.3140	.3156	.3173	.3190	.3206	.3223	.3239	.3256	71°
19°	.3256	.3272	.3289	.3305	.3322	.3338	.3355	.3371	.3387	.3404	.3420	70°
20°	.3420	.3437	.3453	.3469	.3486	.3502	.3518	.3535	.3551	.3567	.3584	69°
21°	.3584	.3600	.3616	.3633	.3649	.3665	.3681	.3697	.3714	.3730	.3746	68°
22°	.3746	.3762	.3778	.3795	.3811	.3827	.3843	.3859	.3875	.3891	.3907	67°
23°	.3907	.3923	.3939	.3955	.3971	.3987	.4003	.4019	.4035	.4051	.4067	66°
24°	.4067	.4083	.4099	.4115	.4131	.4147	.4163	.4179	.4195	.4210	.4226	65°
25°	.4226	.4242	.4258	.4274	.4289	.4305	.4321	.4337	.4352	.4368	.4384	64°
26°	.4384	.4399	.4415	.4431	.4446	.4462	.4478	.4493	.4509	.4524	.4540	63°
27°	.4540	.4555	.4571	.4586	.4602	.4617	.4633	.4648	.4664	.4679	.4695	62°
28°	.4695	.4710	.4726	.4741	.4756	.4772	.4787	.4802	.4818	.4833	.4848	61°
29°	.4848	.4863	.4879	.4894	.4909	.4924	.4939	.4955	.4970	.4985	.5000	60°
30°	.5000	.5015	.5030	.5045	.5060	.5075	.5090	.5105	.5120	.5135	.5150	59°
31°	.5150	.5165	.5180	.5195	.5210	.5225	.5240	.5255	.5270	.5284	.5299	58°
32°	.5299	.5314	.5329	.5344	.5358	.5373	.5388	.5402	.5417	.5432	.5446	57°
33°	.5446	.5461	.5476	.5490	.5505	.5519	.5534	.5548	.5563	.5577	.5592	56°
34°	.5592	.5606	.5621	.5635	.5650	.5664	.5678	.5693	.5707	.5721	.5736	55°
35°	.5736	.5750	.5764	.5779	.5793	.5807	.5821	.5835	.5850	.5864	.5878	54°
36°	.5878	.5892	.5906	.5920	.5934	.5948	.5962	.5976	.5990	.6004	.6018	53°
37°	.6018	.6032	.6046	.6060	.6074	.6088	.6101	.6115	.6129	.6143	.6157	52°
38°	.6157	.6170	.6184	.6198	.6211	.6225	.6239	.6252	.6266	.6280	.6293	51°
39°	.6293	.6307	.6320	.6334	.6347	.6361	.6374	.6388	.6401	.6414	.6428	50°
40°	.6428	.6441	.6455	.6468	.6481	.6494	.6508	.6521	.6534	.6547	.6561	49°
41°	.6561	.6574	.6587	.6600	.6613	.6626	.6639	.6652	.6665	.6678	.6691	48°
42°	.6691	.6704	.6717	.6730	.6743	.6756	.6769	.6782	.6794	.6807	.6820	47°
43°	.6820	.6833	.6845	.6858	.6871	.6884	.6896	.6909	.6921	.6934	.6947	46°
44°	.6947	.6959	.6972	.6984	.6997	.7009	.7022	.7034	.7046	.7059	.7071	45°
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

cos (read up)

Table of Trigonometric Functions

sin (read down)

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
45°	.7071	.7083	.7096	.7108	.7120	.7133	.7145	.7157	.7169	.7181	.7193	**44°**
46°	.7193	.7206	.7218	.7230	.7242	.7254	.7266	.7278	.7290	.7302	.7314	**43°**
47°	.7314	.7325	.7337	.7349	.7361	.7373	.7385	.7396	.7408	.7420	.7431	**42°**
48°	.7431	.7443	.7455	.7466	.7478	.7490	.7501	.7513	.7524	.7536	.7547	**41°**
49°	.7547	.7559	.7570	.7581	.7593	.7604	.7615	.7627	.7638	.7649	.7660	**40°**
50°	.7660	.7672	.7683	.7694	.7705	.7716	.7727	.7738	.7749	.7760	.7771	**39°**
51°	.7771	.7782	.7793	.7804	.7815	.7826	.7837	.7848	.7859	.7869	.7880	**38°**
52°	.7880	.7891	.7902	.7912	.7923	.7934	.7944	.7955	.7965	.7976	.7986	**37°**
53°	.7986	.7997	.8007	.8018	.8028	.8039	.8049	.8059	.8070	.8080	.8090	**36°**
54°	.8090	.8100	.8111	.8121	.8131	.8141	.8151	.8161	.8171	.8181	.8192	**35°**
55°	.8192	.8202	.8211	.8221	.8231	.8241	.8251	.8261	.8271	.8281	.8290	**34°**
56°	.8290	.8300	.8310	.8320	.8329	.8339	.8348	.8358	.8368	.8377	.8387	**33°**
57°	.8387	.8396	.8406	.8415	.8425	.8434	.8443	.8453	.8462	.8471	.8480	**32°**
58°	.8480	.8490	.8499	.8508	.8517	.8526	.8536	.8545	.8554	.8563	.8572	**31°**
59°	.8572	.8581	.8590	.8599	.8607	.8616	.8625	.8634	.8643	.8652	.8660	**30°**
60°	.8660	.8669	.8678	.8686	.8695	.8704	.8712	.8721	.8729	.8738	.8746	**29°**
61°	.8746	.8755	.8763	.8771	.8780	.8788	.8796	.8805	.8813	.8821	.8829	**28°**
62°	.8829	.8838	.8846	.8854	.8862	.8870	.8878	.8886	.8894	.8902	.8910	**27°**
63°	.8910	.8918	.8926	.8934	.8942	.8949	.8957	.8965	.8973	.8980	.8988	**26°**
64°	.8988	.8996	.9003	.9011	.9018	.9026	.9033	.9041	.9048	.9056	.9063	**25°**
65°	.9063	.9070	.9078	.9085	.9092	.9100	.9107	.9114	.9121	.9128	.9135	**24°**
66°	.9135	.9143	.9150	.9157	.9164	.9171	.9178	.9184	.9191	.9198	.9205	**23°**
67°	.9205	.9212	.9219	.9225	.9232	.9239	.9245	.9252	.9259	.9265	.9272	**22°**
68°	.9272	.9278	.9285	.9291	.9298	.9304	.9311	.9317	.9323	.9330	.9336	**21°**
69°	.9336	.9342	.9348	.9354	.9361	.9367	.9373	.9379	.9385	.9391	.9397	**20°**
70°	.9397	.9403	.9409	.9415	.9421	.9426	.9432	.9438	.9444	.9449	.9455	**19°**
71°	.9455	.9461	.9466	.9472	.9478	.9483	.9489	.9494	.9500	.9505	.9511	**18°**
72°	.9511	.9516	.9521	.9527	.9532	.9537	.9542	.9548	.9553	.9558	.9563	**17°**
73°	.9563	.9568	.9573	.9578	.9583	.9588	.9593	.9598	.9603	.9608	.9613	**16°**
74°	.9613	.9617	.9622	.9627	.9632	.9636	.9641	.9646	.9650	.9655	.9659	**15°**
75°	.9659	.9664	.9668	.9673	.9677	.9681	.9686	.9690	.9694	.9699	.9703	**14°**
76°	.9703	.9707	.9711	.9715	.9720	.9724	.9728	.9732	.9736	.9740	.9744	**13°**
77°	.9744	.9748	.9751	.9755	.9759	.9763	.9767	.9770	.9774	.9778	.9781	**12°**
78°	.9781	.9785	.9789	.9792	.9796	.9799	.9803	.9806	.9810	.9813	.9816	**11°**
79°	.9816	.9820	.9823	.9826	.9829	.9833	.9836	.9839	.9842	.9845	.9848	**10°**
80°	.9848	.9851	.9854	.9857	.9860	.9863	.9866	.9869	.9871	.9874	.9877	**9°**
81°	.9877	.9880	.9882	.9885	.9888	.9890	.9893	.9895	.9898	.9900	.9903	**8°**
82°	.9903	.9905	.9907	.9910	.9912	.9914	.9917	.9919	.9921	.9923	.9925	**7°**
83°	.9925	.9928	.9930	.9932	.9934	.9936	.9938	.9940	.9942	.9943	.9945	**6°**
84°	.9945	.9947	.9949	.9951	.9952	.9954	.9956	.9957	.9959	.9960	.9962	**5°**
85°	.9962	.9963	.9965	.9966	.9968	.9969	.9971	.9972	.9973	.9974	.9976	**4°**
86°	.9976	.9977	.9978	.9979	.9980	.9981	.9982	.9983	.9984	.9985	.9986	**3°**
87°	.9986	.9987	.9988	.9989	.9990	.9990	.9991	.9992	.9993	.9993	.9994	**2°**
88°	.9994	.9995	.9995	.9996	.9996	.9997	.9997	.9997	.9998	.9998	.9998	**1°**
89°	.9998	.9999	.9999	.9999	.9999	1.000	1.000	1.000	1.000	1.000	1.000	**0°**
		.9	**.8**	**.7**	**.6**	**.5**	**.4**	**.3**	**.2**	**.1**	**.0**	

cos (read up)

Table of Trigonometric Functions

tan (read down)

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
0°	.0000	.0017	.0035	.0052	.0070	.0087	.0105	.0122	.0140	.0157	.0175	**89°**
1°	.0175	.0192	.0209	.0227	.0244	.0262	.0279	.0297	.0314	.0332	.0349	**88°**
2°	.0349	.0367	.0384	.0402	.0419	.0437	.0454	.0472	.0489	.0507	.0524	**87°**
3°	.0524	.0542	.0559	.0577	.0594	.0612	.0629	.0647	.0664	.0682	.0699	**86°**
4°	.0699	.0717	.0734	.0752	.0769	.0787	.0805	.0822	.0840	.0857	.0875	**85°**
5°	.0875	.0892	.0910	.0928	.0945	.0963	.0981	.0998	.1016	.1033	.1051	**84°**
6°	.1051	.1069	.1086	.1104	.1122	.1139	.1157	.1175	.1192	.1210	.1228	**83°**
7°	.1228	.1246	.1263	.1281	.1299	.1317	.1334	.1352	.1370	.1388	.1405	**82°**
8°	.1405	.1423	.1441	.1459	.1477	.1495	.1512	.1530	.1548	.1566	.1584	**81°**
9°	.1584	.1602	.1620	.1638	.1655	.1673	.1691	.1709	.1727	.1745	.1763	**80°**
10°	.1763	.1781	.1799	.1817	.1835	.1853	.1871	.1890	.1908	.1926	.1944	**79°**
11°	.1944	.1962	.1980	.1998	.2016	.2035	.2053	.2071	.2089	.2107	.2126	**78°**
12°	.2126	.2144	.2162	.2180	.2199	.2217	.2235	.2254	.2272	.2290	.2309	**77°**
13°	.2309	.2327	.2345	.2364	.2382	.2401	.2419	.2438	.2456	.2475	.2493	**76°**
14°	.2493	.2512	.2530	.2549	.2568	.2586	.2605	.2623	.2642	.2661	.2679	**75°**
15°	.2679	.2698	.2717	.2736	.2754	.2773	.2792	.2811	.2830	.2849	.2867	**74°**
16°	.2867	.2886	.2905	.2924	.2943	.2962	.2981	.3000	.3019	.3038	.3057	**73°**
17°	.3057	.3076	.3096	.3115	.3134	.3153	.3172	.3119	.3211	.3230	.3249	**72°**
18°	.3249	.3269	.3288	.3307	.3327	.3346	.3365	.3385	.3404	.3424	.3443	**71°**
19°	.3443	.3463	.3482	.3502	.3522	.3541	.3561	.3581	.3600	.3620	.3640	**70°**
20°	.3640	.3659	.3679	.3699	.3719	.3739	.3759	.3779	.3799	.3819	.3839	**69°**
21°	.3839	.3859	.3879	.3899	.3919	.3939	.3959	.3979	.4000	.4020	.4040	**68°**
22°	.4040	.4061	.4081	.4101	.4122	.4142	.4163	.4183	.4204	.4224	.4245	**67°**
23°	.4245	.4265	.4286	.4307	.4327	.4348	.4369	.4390	.4411	.4431	.4452	**66°**
24°	.4452	.4473	.4494	.4515	.4536	.4557	.4578	.4599	.4621	.4642	.4663	**65°**
25°	.4663	.4684	.4706	.4727	.4748	.4770	.4791	.4813	.4834	.4856	.4877	**64°**
26°	.4877	.4899	.4921	.4942	.4964	.4986	.5008	.5029	.5051	.5073	.5095	**63°**
27°	.5095	.5117	.5139	.5161	.5184	.5206	.5228	.5250	.5272	.5295	.5317	**62°**
28°	.5317	.5340	.5362	.5384	.5407	.5430	.5452	.5475	.5498	.5520	.5543	**61°**
29°	.5543	.5566	.5589	.5612	.5635	.5658	.5681	.5704	.5727	.5750	.5774	**60°**
30°	.5774	.5797	.5820	.5844	.5867	.5890	.5914	.5938	.5961	.5985	.6009	**59°**
31°	.6009	.6032	.6056	.6080	.6104	.6128	.6152	.6176	.6200	.6224	.6249	**58°**
32°	.6249	.6273	.6297	.6322	.6346	.6371	.6395	.6420	.6445	.6469	.6494	**57°**
33°	.6494	.6519	.6544	.6569	.6594	.6619	.6644	.6669	.6694	.6720	.6745	**56°**
34°	.6745	.6771	.6796	.6822	.6847	.6873	.6899	.6924	.6950	.6976	.7002	**55°**
35°	.7002	.7028	.7054	.7080	.7107	.7133	.7159	.7186	.7212	.7239	.7265	**54°**
36°	.7265	.7292	.7319	.7346	.7373	.7400	.7427	.7454	.7481	.7508	.7536	**53°**
37°	.7536	.7563	.7590	.7618	.7646	.7673	.7701	.7729	.7757	.7785	.7813	**52°**
38°	.7813	.7841	.7869	.7898	.7926	.7954	.7983	.8012	.8040	.8069	.8098	**51°**
39°	.8098	.8127	.8156	.8185	.8214	.8243	.8273	.8302	.8332	.8361	.8391	**50°**
40°	.8391	.8421	.8451	.8481	.8511	.8541	.8571	.8601	.8632	.8662	.8693	**49°**
41°	.8693	.8724	.8754	.8785	.8816	.8847	.8878	.9810	.8941	.8972	.9004	**48°**
42°	.9004	.9036	.9067	.9099	.9131	.9163	.9195	.9228	.9260	.9293	.9325	**47°**
43°	.9325	.9358	.9391	.9424	.9457	.9490	.9523	.9556	.9590	.9623	.9657	**46°**
44°	.9657	.9691	.9725	.9759	.9793	.9827	.9861	.9896	.9930	.9965	1.000	**45°**
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

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Table of Trigonometric Functions

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	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		
45°	1.000	1.003	1.007	1.011	1.014	1.018	1.021	1.025	1.028	1.032	1.036	**44°**
46°	1.036	1.039	1.043	1.046	1.050	1.054	1.057	1.061	1.065	1.069	1.072	**43°**
47°	1.072	1.076	1.080	1.084	1.087	1.091	1.095	1.099	1.103	1.107	1.111	**42°**
48°	1.111	1.115	1.118	1.122	1.126	1.130	1.134	1.138	1.142	1.146	1.150	**41°**
49°	1.150	1.154	1.159	1.163	1.167	1.171	1.175	1.179	1.183	1.188	1.192	**40°**
50°	1.192	1.196	1.200	1.205	1.209	1.213	1.217	1.222	1.226	1.230	1.235	**39°**
51°	1.235	1.239	1.244	1.248	1.253	1.257	1.262	1.266	1.271	1.275	1.280	**38°**
52°	1.280	1.285	1.289	1.294	1.299	1.303	1.308	1.313	1.317	1.322	1.327	**37°**
53°	1.327	1.332	1.337	1.342	1.347	1.351	1.356	1.361	1.366	1.371	1.376	**36°**
54°	1.376	1.381	1.387	1.392	1.397	1.402	1.407	1.412	1.418	1.423	1.428	**35°**
55°	1.428	1.433	1.439	1.444	1.450	1.455	1.460	1.466	1.471	1.477	1.483	**34°**
56°	1.483	1.488	1.494	1.499	1.505	1.511	1.517	1.522	1.528	1.534	1.540	**33°**
57°	1.540	1.546	1.552	1.558	1.564	1.570	1.576	1.582	1.588	1.594	1.600	**32°**
58°	1.600	1.607	1.613	1.619	1.625	1.632	1.638	1.645	1.651	1.658	1.664	**31°**
59°	1.664	1.671	1.678	1.684	1.691	1.698	1.704	1.711	1.718	1.725	1.732	**30°**
60°	1.732	1.739	1.746	1.753	1.760	1.767	1.775	1.782	1.789	1.797	1.804	**29°**
61°	1.804	1.811	1.819	1.827	1.834	1.842	1.849	1.857	1.865	1.873	1.881	**28°**
62°	1.881	1.889	1.897	1.905	1.913	1.921	1.929	1.937	1.946	1.954	1.963	**27°**
63°	1.963	1.971	1.980	1.988	1.997	2.006	2.014	2.023	2.032	2.041	2.050	**26°**
64°	2.050	2.059	2.069	2.078	2.087	2.097	2.106	2.116	2.125	2.135	2.145	**25°**
65°	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	2.246	**24°**
66°	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	2.356	**23°**
67°	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	2.475	**22°**
68°	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	2.605	**21°**
69°	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	2.747	**20°**
70°	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	2.904	**19°**
71°	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	3.078	**18°**
72°	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.251	3.271	**17°**
73°	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	3.487	**16°**
74°	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	3.732	**15°**
75°	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	4.011	**14°**
76°	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	4.331	**13°**
77°	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	4.705	**12°**
78°	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	5.145	**11°**
79°	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	5.671	**10°**
80°	5.671	5.730	5.789	5.850	5.912	5.976	6.041	6.107	6.174	6.243	6.314	**9°**
81°	6.314	6.386	6.460	6.535	6.612	6.691	6.772	6.855	6.940	7.026	7.115	**8°**
82°	7.115	7.207	7.300	7.396	7.495	7.596	7.700	7.806	7.916	8.028	8.144	**7°**
83°	8.144	8.264	8.386	8.513	8.643	8.777	8.915	9.058	9.205	9.357	9.514	**6°**
84°	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	11.43	**5°**
85°	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	14.30	**4°**
86°	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	19.08	**3°**
87°	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	28.64	**2°**
88°	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	57.29	**1°**
89°	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	**0°**
		.9	.8	.7	.6	.5	.4	.3	.2	.1	.0	

cot (read up)

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Physical Constants

Avogadro's number: $N_0 = 6.02 \times 10^{23}$

Speed of light: $c = 2.99793 \times 10^8$ m/sec

Gravitational constant: $G = 6.670 \times 10^{-11} \dfrac{\text{m}^3}{\text{kg-sec}^2}$

Boltzmann's constant: $k = 1.3805 \times 10^{-23} \dfrac{\text{joule}}{°\text{K}}$

Constant in Coulomb's law: $k = 2.306 \times 10^{-28} \dfrac{\text{newton-m}^2}{(\text{elem. ch.})^2} = 8.988 \times 10^9 \dfrac{\text{newton-m}^2}{\text{coulomb}^2}$

Mass of electron: $m_e = 9.109 \times 10^{-31}$ kg

Mass of proton: $m_p = 1.672 \times 10^{-27}$ kg

Constant in Ampère's circuital law: $K = 2 \times 10^{-7} \dfrac{\text{newton}}{\text{amp}^2}$ (exact, by definition)

Planck's constant: $h = 6.626 \times 10^{-34}$ joule-sec $= 4.136 \times 10^{-15}$ ev-sec

Conversion Factors

1 atomic mass unit $= 1.66 \times 10^{-27}$ kg

1 electron volt $= 1.602 \times 10^{-19}$ joule

1 coulomb $= 6.242 \times 10^{18}$ elem. ch.

1 2 3 4 5 6 7 8 9