

Chapter 19 REFRACTION

If light simply bounced off of everything it encountered, our study of light would now be almost finished. Fortunately, light can penetrate into different substances which it encounters. (We say fortunately because many phenomena, like our vision, depend on light's ability to penetrate.) So most generally, two things happen when light encounters a different substance. Some fraction of the light is reflected, i.e., it never enters the new substance but moves off in a direction that you already know how to determine. The remainder of the light enters the new substance, but it also changes direction. We call this penetration and accompanying change in direction the phenomenon of refraction, and it is this phenomenon which we will study in detail in this chapter.

You are probably used to the idea of light moving through the air. We will discover in this chapter that light can move through most any substance, or even through a vacuum. The substance through which light moves is called its medium. In cases where light is moving through a vacuum, we still call the vacuum the medium of the light, even though this is a rather curious usage of the word.

In the previous chapter you encountered the parabolic mirror, a device which could focus rays of light by reflection. In this chapter we will also study the lens, a most important focusing device which works the way it does because of refraction. The lens has numerous applications in everyday life: eyeglasses, magnifying devices, cameras. In fact, there is a very excellent lens within each of your eyes.

PERFORMANCE OBJECTIVES

After completing this chapter, you should be able to:

1. formulate Snell's Law and appraise its value.
2. determine the path taken by a ray of light given the direction of the incident ray, the orientation of the refracting interface, and the index of refraction of the refractive medium.
3. recognize that light may be simultaneously reflected and refracted at an interface.
4. demonstrate conditions under which total internal reflection occurs.
5. calculate the critical angle of various substances.
6. demonstrate how a prism disperses light in various substances.
7. experimentally determine the focal length of a thin lens.
8. draw ray diagrams and/or use mathematical formulas to find the relationship between object size, image size, object location, image location, and focal length of a thin lens.

1. Obtain a clear plastic block. Look through it at the printing on this page.
 - a. What do you see?
 - b. Change the angle at which you look through the block. What happens to the image?
 - c. Now obtain a calcite crystal and repeat the above. How does the image seen through the calcite crystal differ from the image seen through the plastic block?

Note...The calcite crystal behaves in a way abnormal to the behavior of the plastic block. The light undergoes DOUBLE REFRACTION. This topic is not covered in the text. If interested, you can find information in the following texts:

PHYSICS FOR SCIENCE & ENGINEERING by Resnick and Halliday
p-975

2. Obtain an evaporating dish and a beaker filled with water. Place a coin in the empty dish. Position your eyes until the coin is just out of view while looking over the edge of the dish. Now pour the water into the dish. Results?

Obtain a second beaker. Place this empty beaker in the empty evaporating dish that has a coin in it. While looking at the coin through the side of the beaker, pour water into the beaker. Where did the coin go?

3. Investigation: REFRACTION (Notes provided.)
 - a. A complete laboratory report is expected. Be sure data is in correct table form. Each group (max of 2) is to do their own investigation.
 - b. Note...Study Notes I: The Sine, Tangent and Cosine of an Angle is in this packet.
4. Read: Section 19-1 Refraction page 391
19-2 Experiments on the Angle of Refraction page 392
5. Use Figure 19-2, page 393 for the following:
 - a. What do you use as reference to measure the angles of incidence and refraction?
 - b. In which diagram is the incident ray bent the most?
 - c. How much bending would result if the angle of incidence were zero degrees?
 - d. If the angle of incidence could be viewed as changing from zero to 90 degrees, what would be happening to the angle of refraction? Be specific!
 - e. Which angle increases at a faster rate? Why?

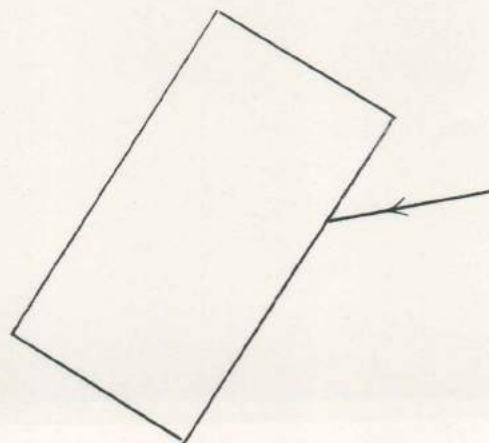
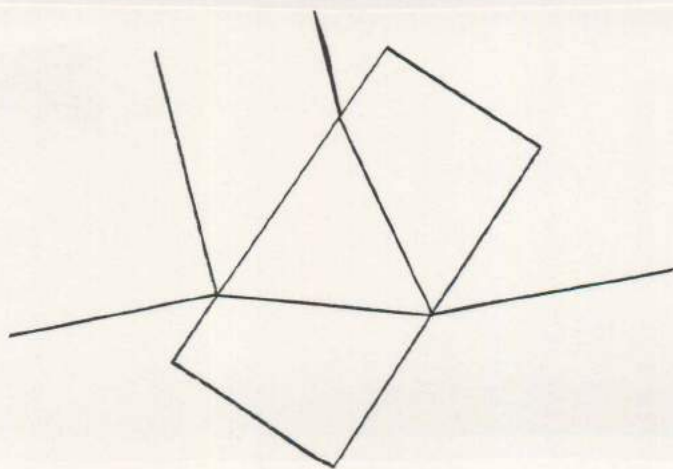
- f. If the direction of light were reversed (i.e. the light would travel the opposite direction in each case), in what direction would the refracted light rays go?

6. Problems: page 392: #1 page 395: #3 #5
7. Read: Section 19-3 The Index of Refraction: Snell's Law page 396
8. Problems: page 399: #6 #8 #10 #11 #13 page 416: #31
9. Read Section 19-4 The Passage of Light from Glass (or water) to Air: Reversibility page 400

a. Any difficulty with the discussion? If so, see your instructor.

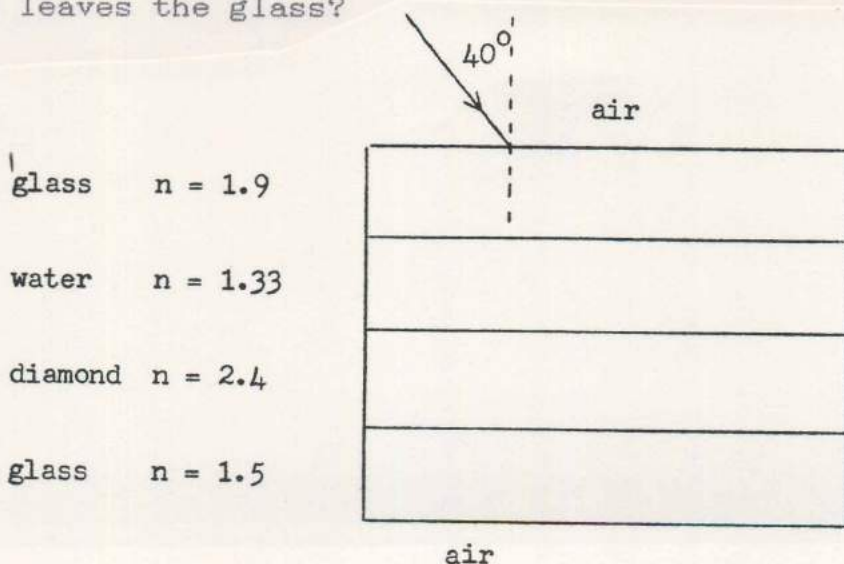
10. Examine Figure 19-7, page 400

- a. Which ray is the incoming ray? Place a plastic block over the diagram and then using the ray box, shine a single beam along each of the lines. Now using arrows, identify which way each ray is going.
b. If you take the final refracted ray and consider it to be the incident ray, will you get the same refracted and reflected rays?
c. To the right is the same object with an incident ray drawn. Draw the refracted and reflected rays. (No reference is made to the relative intensities.)

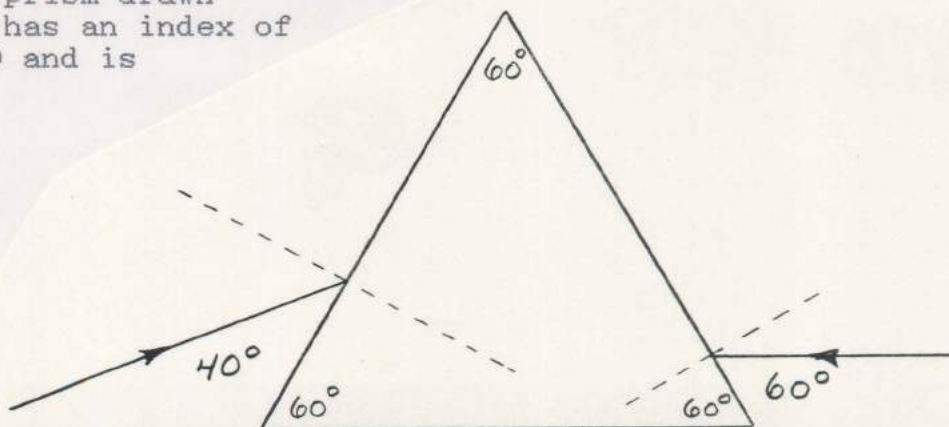


11. Problems: page 417: #32 (a must) #34 (ask to see laser photo)
12. Read: Section 19-5 The Passage of Light From Glass to Water page 403
a. Any difficulty with the discussion? If so, see your instructor.
13. What is RELATIVE INDEX OF REFRACTION? When would it be useful?
a. Note...See Study Notes: The Relative Index of Refraction
14. Problems: page 404: #18 #19 page 417: #36

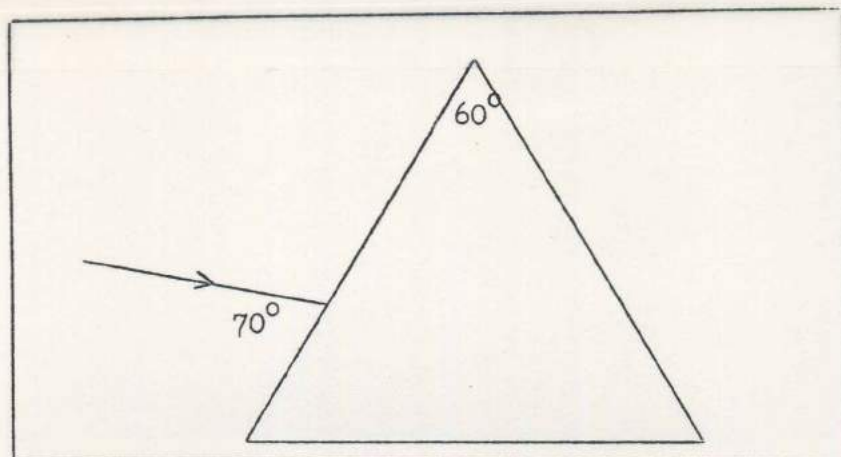
15. Light strikes the surface at an angle of incidence of 40 degrees will pass through the various layers and emerge from the glass ($n = 1.5$) into the air. What angle will it be traveling through the air after it leaves the glass?



16. Read: Section 19-6 Total Internal Reflection page 404
17. Using the semi-circular piece of plastic from the optics kit, and power source, shine the light through the optical disc from the circular side beginning with the angle of incidence at zero degrees. Rotate the disc until there is total internal reflection. This angle (the critical angle) is _____. Using $n_1 \sin \theta_1 = n_2 \sin \theta_2$, what is the index of refraction of the plastic? _____
18. Problems: page 417: #37 #38 #39 #40
-In # 38, 5 degrees is in CS_2
19. Ask to see dropper bottle which is filled with anisole.
20. How does the index of refraction of ice compare to the index of refraction of water. Place an ice cube in a glass of water (at home). Conclusion(s)?
21. Accurately trace the two rays of light through the prism drawn below. The prism has an index of refraction of 1.50 and is surrounded by air.



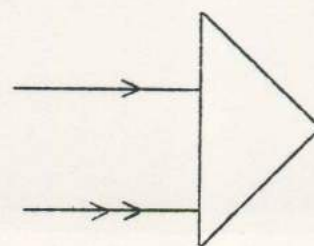
22. Accurately trace the ray of light through the air prism surrounded by glass which has an index of refraction of 1.50



23. Trace two rays of light into and out of the prism at the right.

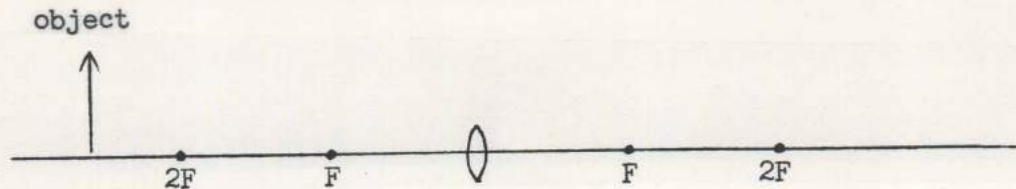
Index of refraction of prism = 1.50.

24. Optional...Obtain Appendix Information on RAINBOWS which explains how water droplets produce a rainbow.

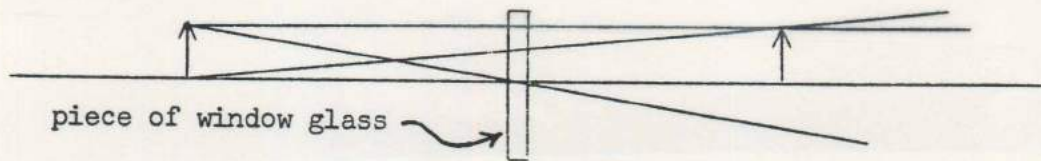


25. Using the optical bench, light source, and prism, examine what happens to the white light as it passes through the prism. What can you conclude about the index of refraction for various colors?
26. Read: Section 19-7 Refraction by Prisms: Dispersion page 406
27. Problems: page 409: #22 #23
418: #46
28. Read: Section 19-8 Lenses page 409
19-9 Real Images Formed by Lenses page 412
29. Examine Figure 19-19, page 412. Identify: H_i , H_o , S_i , S_o , F , f
- Can you draw a third ray from the object to the image for which you can predict where it will go after passing through the lens?
- Recall how light was reflected by a concave mirror. Figure 18-18, page 382 might be helpful. Can you see the similarities between reflection by concave mirrors and refraction by convex lenses?
30. Using the optical bench, parallel beam light source, and glass screen, place various glass pieces in the beam next to the glass screen. Have the piece towards the light source while observing the effect on the screen. If you wear glasses, do the same with each lens.
31. Investigation: Image Location of a Converging Lens (Notes provided.)
- a. A proper laboratory report is expected.

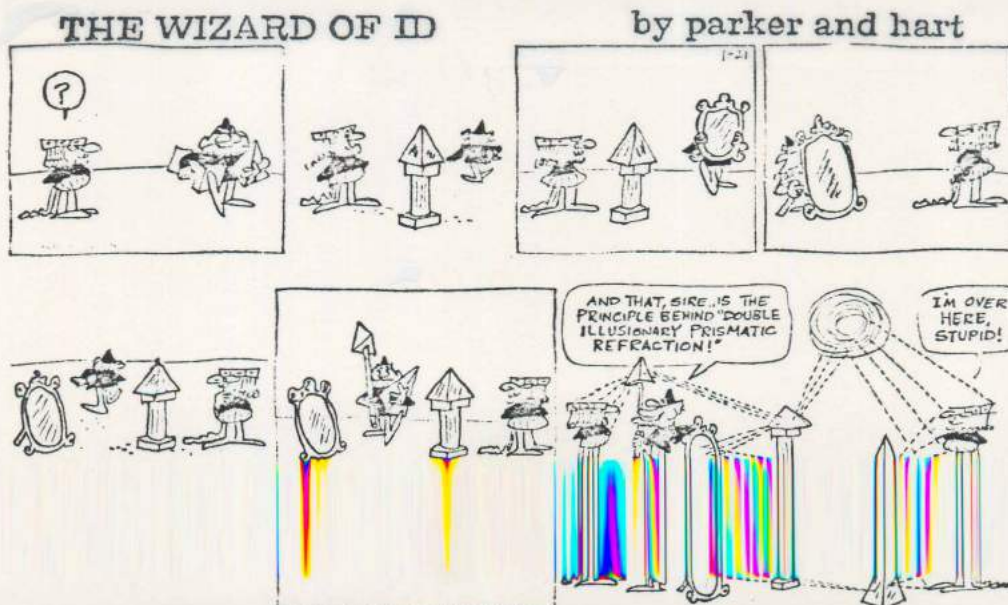
32. Using the optical bench, object source, lens, and glass screen, adjust the object, lens and/or screen until the image is in focus on the screen.
- What would happen to the image if 1/2 of the lens is covered? Try it and see.
 - Ask instructor for special broken lens demonstration.
33. Find the image of the object shown below. (The object is larger than the lens.)



34. On the enclosed sheet titled: IMAGE CONSTRUCTION OF LENSES, locate and draw the image using proper ray construction for the six situations. Indicate whether the image is real or virtual as well as whether the image is smaller or larger than the object.
35. What is wrong with the following diagram?

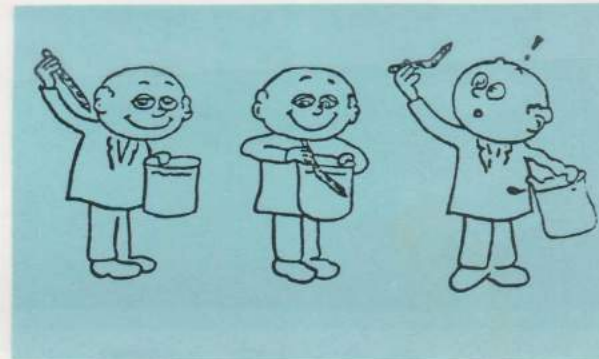


36. Problems: page 411: #26 #27
 413: #28 #29
 418: #45 #48 #49 #50 #51 #52
37. Complete written exercise and then have it evaluated.



ANSWERS CHAPTER 19

5. (a) the normal (b) when angle i was the greatest
(c) no bending when angle i is zero
(d) increasing but at a lesser rate until some maximum angle is reached
(e) angle in air - Why? That's the way it is.
(f) light would follow a reversed path
6. (1) angle i is angle 2 angle r is angle 4
(3) on the left (5) S.A.B
8. (6)(a) AC is opposite, AP is hypotenuse (b) DF, DP (c) S.A.B. (d) 1.6
(8) appears larger (10) 1.22
(11) Diamond (13) slightly more than 1.33299
(31) (a) 20.6° (b) 27.8° (c) 1.47
11. (32) (a) 80° (b) 11 cm (34) S.A.B.
13. When neither of the two substances is air.
14. (18) less than 1 (19) 2.42 (36) 1.46
15. If it takes you longer than 60 seconds, see instructor.
17. approximately 43° , approximately 1.52
18. (37) nothing (38) (a) 8.2° (b) yes (c) It would be smaller
(39) Angle in glass is 50° , ray is reflected at glass-air surface -
thus leaves glass to water at 60°
(40) 189 cm^2
20. One needs a clear ice cube (with no air frozen in it) almost the same
21. ray from left leaves prism at 47.2° , ray from left leaves at 30°
22. ray leaving air prism is 18.9° relative to the normal
23. The double reflected rays emerge going to left. They are parallel to each other and also parallel to incoming rays.
27. (22) Is a second refraction (23) (a) 0.6° (b) 0.21 cm
(46) Ask to see transparency.
29. 3rd ray is drawn from object through center of thin lens to object.
36. (26) lenses may be made of substances with different index of refraction
(27) mirror - will not change lens - focal length will increase
(28) (a) at $2F$ (b) all would be congruent
(29) cut in half
(45) S.A.B.
(48) is greater
(49) (a) between F and the lens (b) S.A.B.
(50) $4f$
(51) Ask to see transparency
(52) S.A.B.



Chapter 19 Investigation - Refraction

When a beam of light passes from one medium to another it may or may not bend at the surface between the two media.

A beam of light passing from air into plastic, entering along the normal, is not bent from its path. However, if the path of the beam is other than along the normal, it will be bent, or refracted at the surface between the two media. In this investigation we will gather data that will show us the relationship between the angle of incidence and the angle of refraction as the light beam passes from air into plastic.

Position the semi-circular piece of plastic on top of a piece of polar graph paper as shown in Figure 1. The curved surface of the plastic should follow the curved surface of the graph paper while the straight surface of the plastic should be positioned at the center of the graph paper.

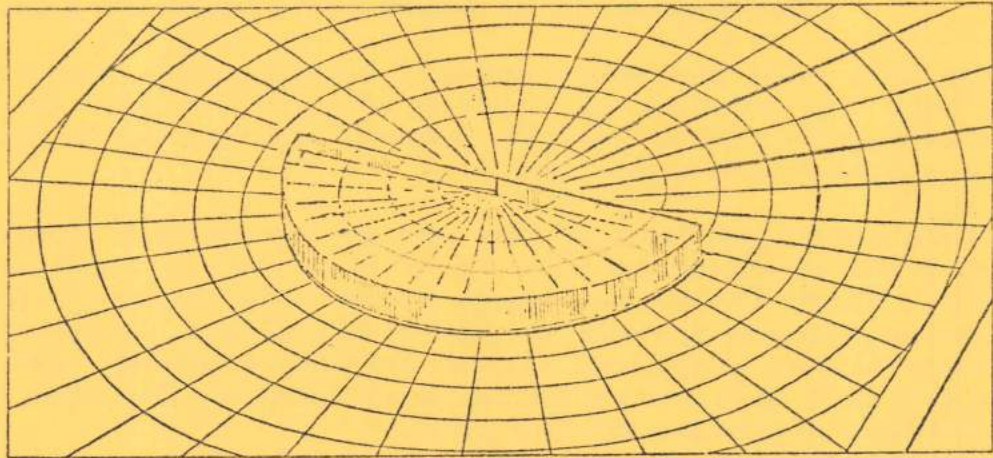


Figure 1

Aim a single beam along the normal which will strike the flat side of the plastic at an angle of incidence of 0 degrees. Note the ray path as it passes through the plastic and out into the air. Is there any bending of the ray? Does the emerging ray from the plastic continue along the same line on the graph paper as the one it followed before entering the plastic? If there is bending or if the ray does not follow the same line on the graph paper, make adjustments so that it does. Now outline the position of the plastic on the graph paper. Use care to not move the plastic as you proceed with the investigation. (Any questions or concerns, see instructor now.)

Move the light box so that the ray strikes the same central point at an angle of 10 degrees. Record this angle and the angle of refraction. Position the light box so that the ray strikes the central point at 10 degrees on the other side of the normal. How does this angle of refraction compare to the other angle of refraction. If they differ by much then the plastic block is not positioned correctly on the graph paper or the light beams were not positioned correctly. Repeat, increasing the angle of incidence in 10 degree steps (as far as possible), making measurements on both sides.

Plot the angle of incidence as a function of the angle of refraction. Also plot the sine of the angle of incidence as a function of the sine of the angle of refraction. Which relationship gives best equation which can then be used to predict the angle of refraction haven been given the angle of incidence. What is this equation? Predict the angle of refraction using the equation when the angle of incidence is 33 degrees. Then check to see how well you prediction matched the actual value.

Chapter 19 Investigation Image Location of a Converging Lens

It has been suggested that a lens can gather light from an object, bend this light according to the laws of refraction in such a manner that either a real or a virtual image is formed. The location of the image can be found using the mirror formula: $S_i S_o = f^2$.

In this investigation we will first find the focal length of a lens. Then we will make some predictions as to image location based on the position of the object. Finally we will check the predictions.

Determination of the Focal Length of a Lens

You will need the short optical bench, light bulb, mirror, and cork with a hole and cross-wires mounted in a holder that we used to determine the focal length of the concave mirror. In two other holders will be mounted a plane mirror and your lens.

Place the light bulb at one end of the optical bench. Place the cork with hole and cross-wires near it with the white cardboard side away from the bulb. Place the mirror about 25 cm from the cross-wires. The lens is then placed between the mirror and cross-wires. Align the light, hole with cross-wires, lens, and mirror in such a manner that the light passing through the lens will be reflected back through the lens and strike the white cardboard surface on the cork.

Adjust the position of the lens until a sharp image of the cross-wires is focused on the white cardboard near the cross-wire opening. The object will now be situated in the focal plane of the lens. Under this condition, rays from any point on the object will emerge from the lens as a parallel beam. They are then reflected back through the lens and brought to focus in the same plane as the object. The distance between the lens and the screen now gives the focal length of the lens which is _____ cm.

A Prediction - Image Location

With the known focal length of your lens, predict the position of the image (on the meter stick) if the object (light bulb) is placed at the 5 cm, 15 cm, 25 cm and 35 cm mark of the meter stick. (The lens is to be placed at the 50 cm mark.) Record all information in table form listing: focal length, object and image distances, and object and image location on the meter stick for the above listed object positions.

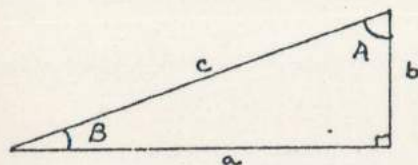
Show your predicted values to your instructor before proceeding further.

Verification - Image Location

Set up a student optical bench with your lens mounted at the 50-cm mark on the meter stick. Place the light source at the 5 cm mark on the meter stick. Next, place the screen where the predicted image will be. Finally turn on the light and see if your prediction is correct. Record the actual image position and indicate how well you did. Trouble? See your instructor. Repeat for the other object positions.

I. THE SINE, COSINE AND TANGENT OF AN ANGLE

The text briefly discusses the definition of a sine of an angle as Snell's law is developed. The sine is related to the geometry of a right triangle.



In a right triangle, the sine of an angle is the ratio of the length of the side opposite the angle to the length of the hypotenuse. The cosine of an angle is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse. The tangent of an angle is the ratio of the length of the side opposite to the length of the side adjacent to the angle.

Thus referring to the diagram, we see for angle B:

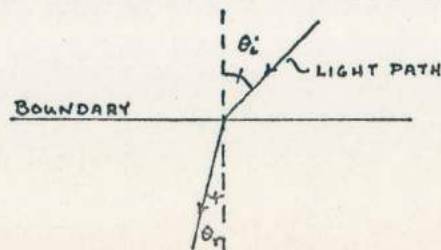
$$\sin B = b/c \quad \cos B = a/c \quad \tan B = b/a$$

and for angle A:

$$\sin A = a/c \quad \cos A = b/c \quad \tan A = a/b$$

One way to analyze the path of the refracted light at a boundary involves construction of right triangles. Most often, we use the line normal to the boundary as the adjacent side of the triangle and the path of the light ray as the hypotenuse. The triangle is completed with a line parallel to the boundary as in the figure below.

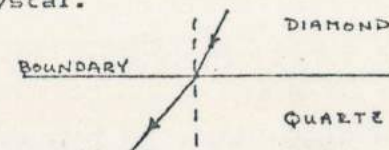
In many situations, you will need to construct such triangles and use the sine, cosine, and tangent to find the required quantities.



II. THE RELATIVE INDEX OF REFRACTION

The relative index of refraction, as discussed in the text, is useful when solving refraction problems involving two media, neither of which is a vacuum or air.

Suppose we have light moving from diamond into quartz crystal.



Snell's law says that

$$n_{\text{diamond}} \sin \theta_{\text{diamond}} = n_{\text{quartz}} \sin \theta_{\text{quartz}}$$

so

$$\frac{\sin \theta_{\text{diamond}}}{\sin \theta_{\text{quartz}}} = \frac{n_{\text{quartz}}}{n_{\text{diamond}}}$$

The value of the ratio of

$$n_{\text{quartz}} / n_{\text{diamond}}$$

is called the relative index of refraction for light going from DIAMOND into QUARTZ. Its value is

$$1.46/2.42 = 0.605$$

Relative indices of refraction are elegant because you can plug them directly into Snell's law.

They are more elegant than useful, however. So when solving refraction problems, use Snell's law in the form involving both ABSOLUTE indices

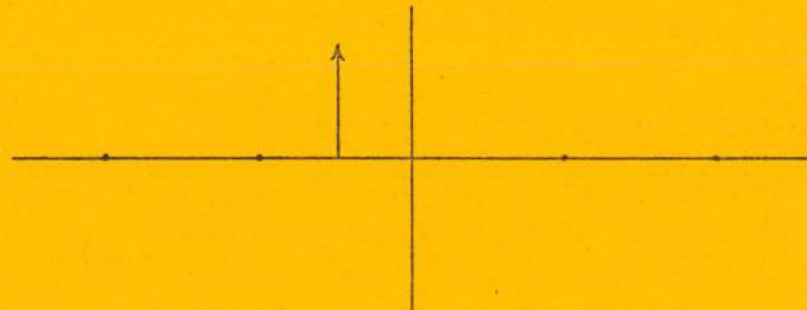
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Chapter 19 Image Construction of Lenses

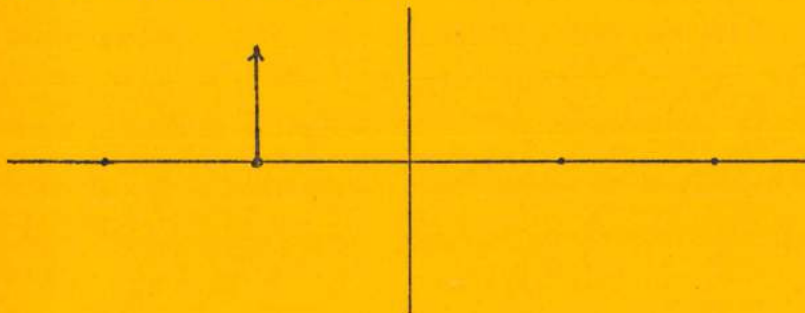
1. Converging Lens



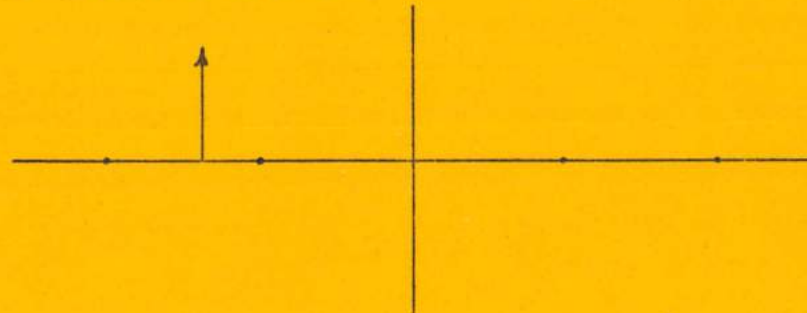
4. Double Convex Lens



2. Double Concave Lens



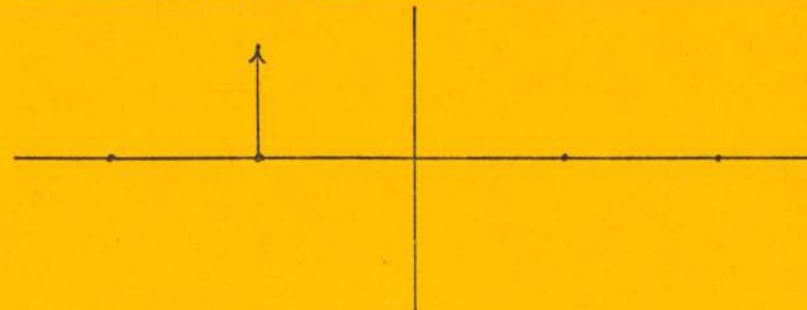
5. Double Concave Lens



3. Double Convex Lens

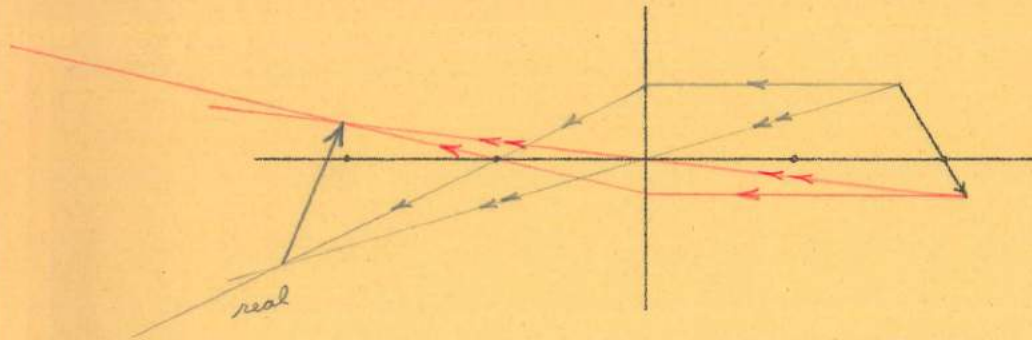


6. Double Convex Lens

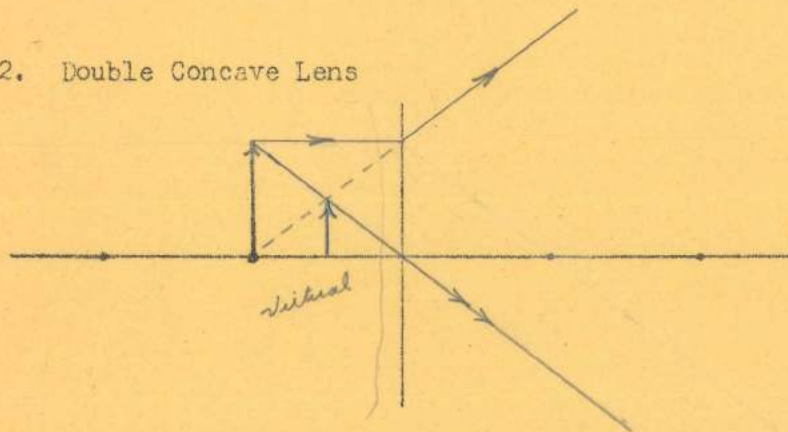


Chapter 19 Image Construction of Lenses

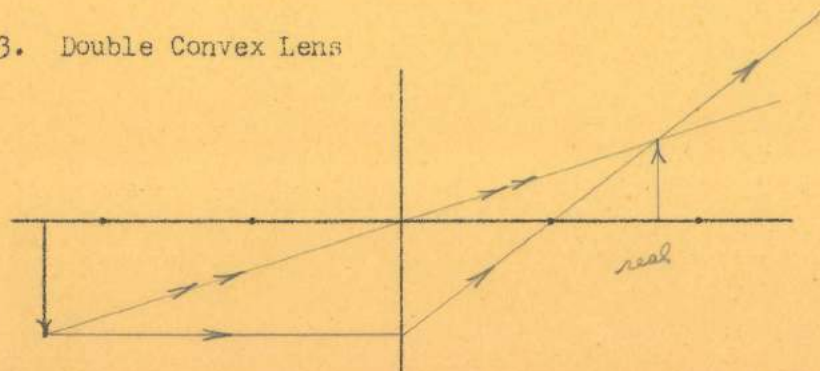
1. Converging Lens



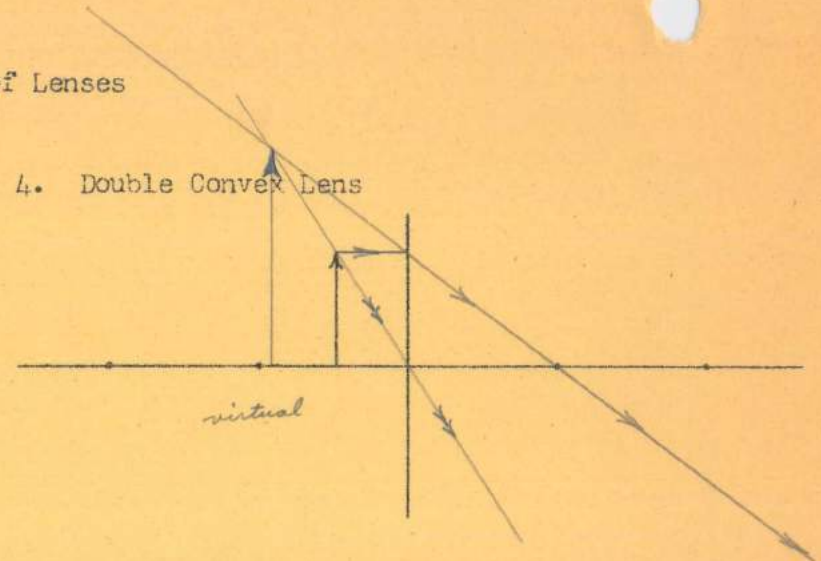
2. Double Concave Lens



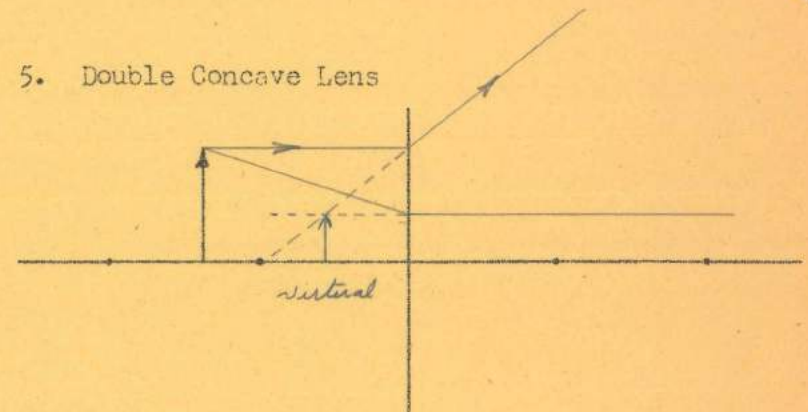
3. Double Convex Lens



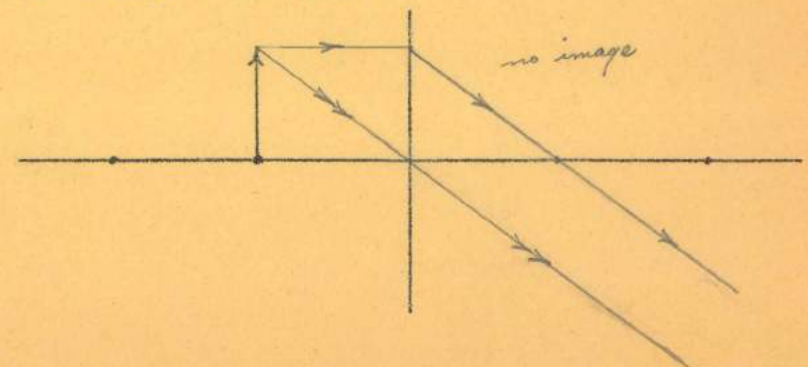
4. Double Convex Lens



5. Double Concave Lens

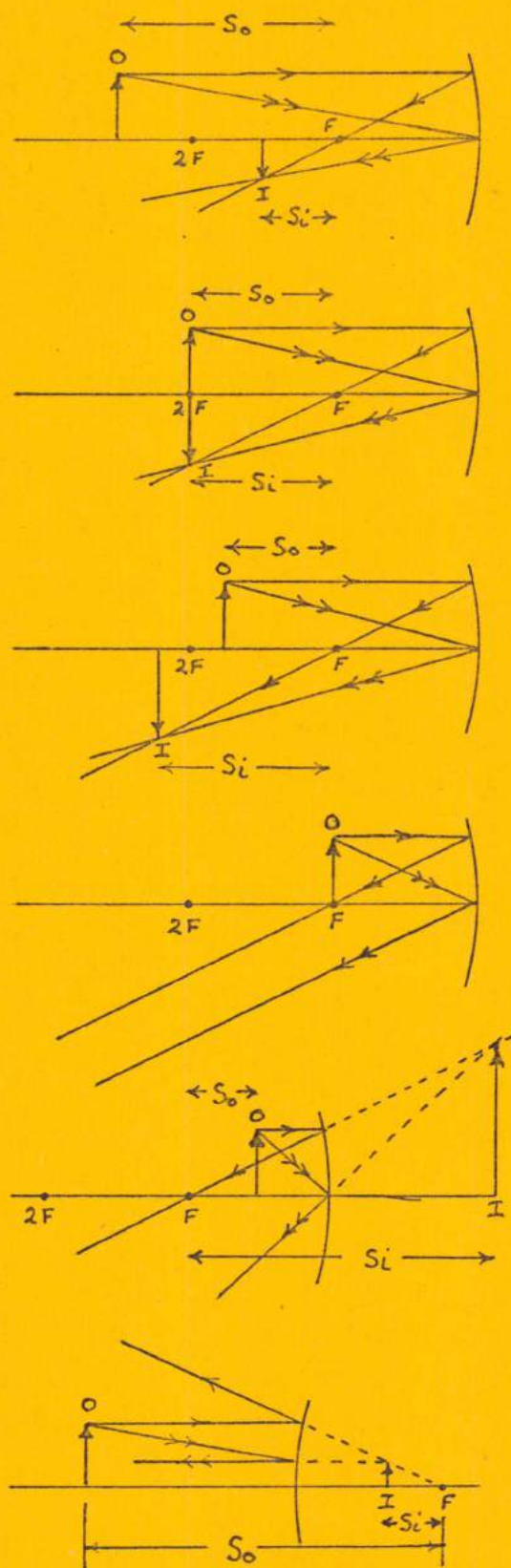


6. Double Convex Lens

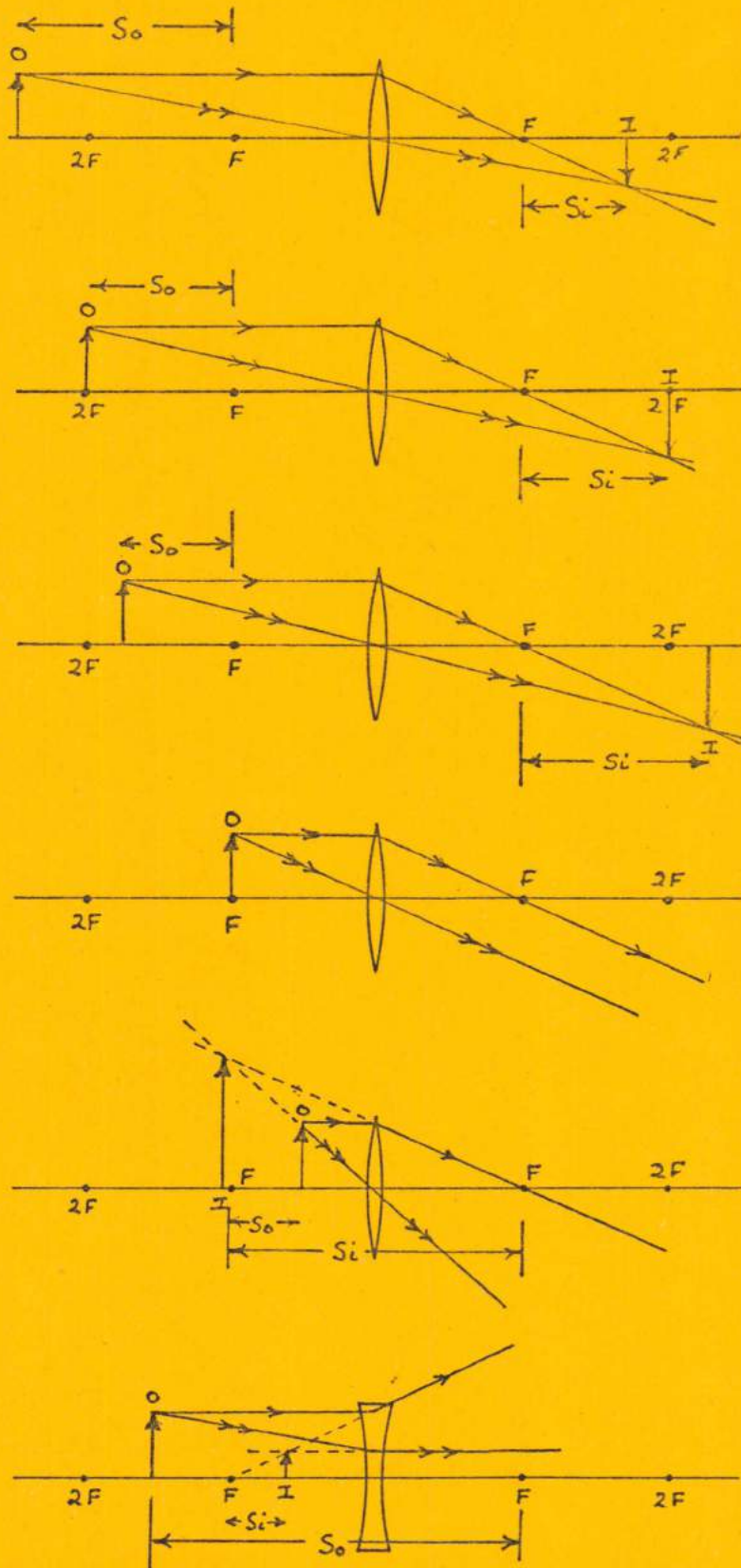


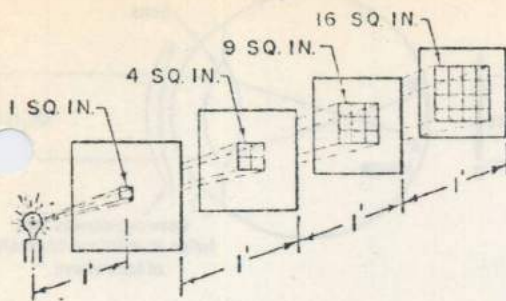
Ch 19 Comparison of Image Formation of Mirrors and Lenses

Mirrors

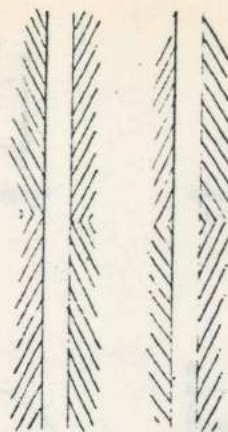


Lenses





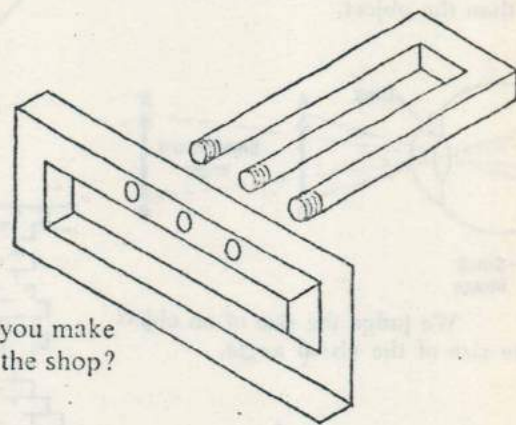
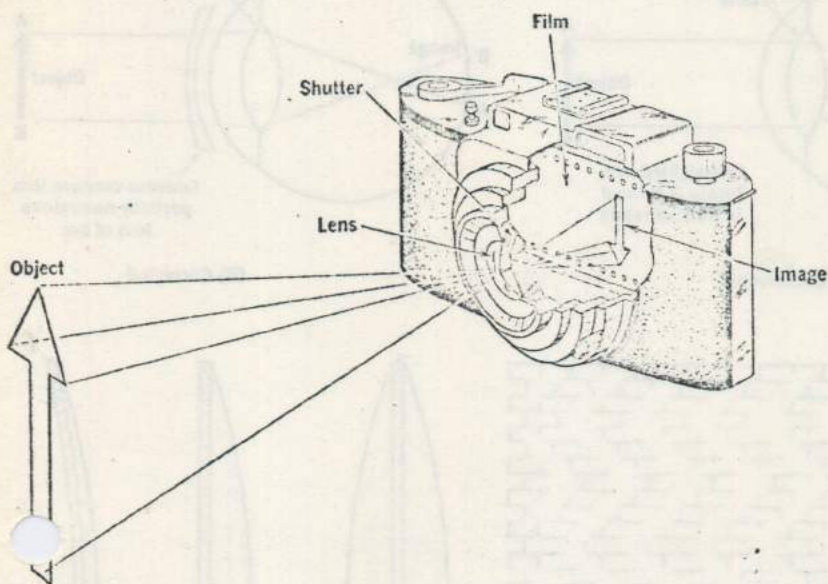
What does this sign read?



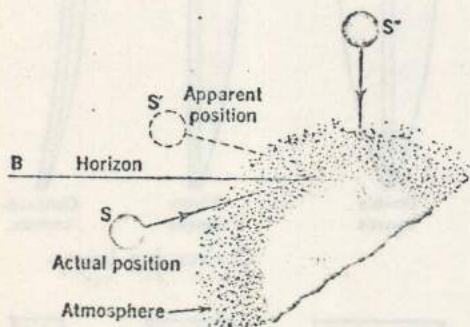
Are the vertical lines parallel?



Do these lines move?



Could you make this in the shop?



The sun is visible before actual sunrise and after sunset because of atmospheric refraction.

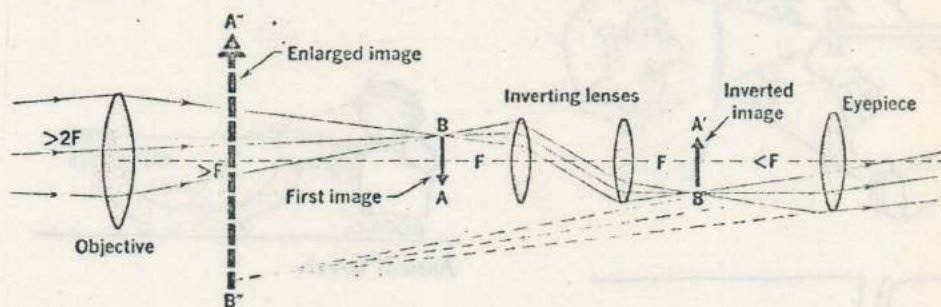
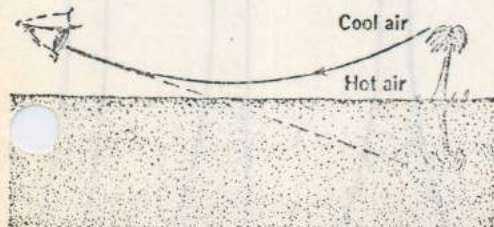
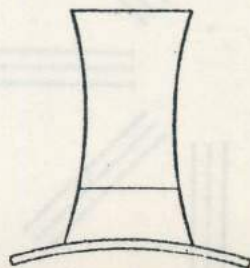


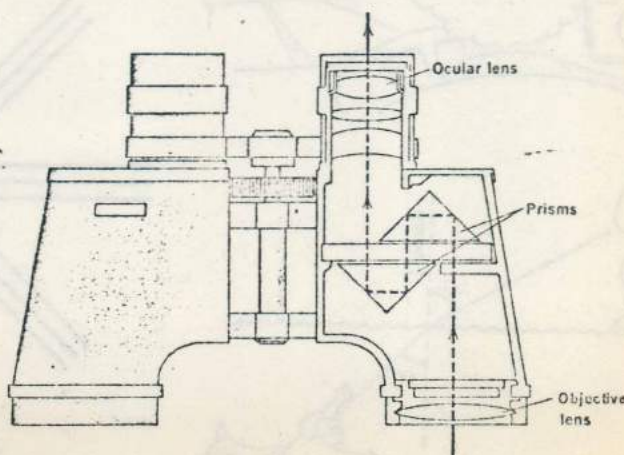
Image formation in a terrestrial telescope.

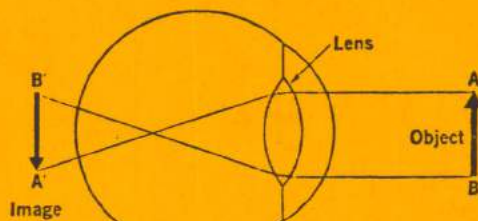
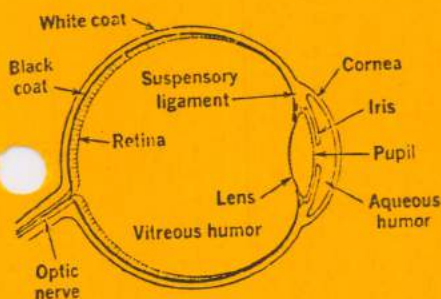


A mirage results from atmospheric refraction in the hot surface air.



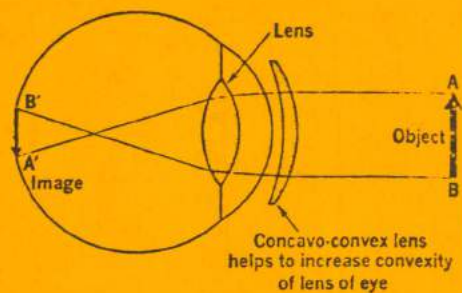
Is the hat taller than the brim is wide?





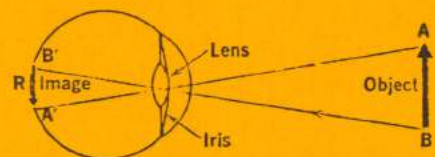
Farsighted eye.
Image is formed
behind retina

(A) Uncorrected

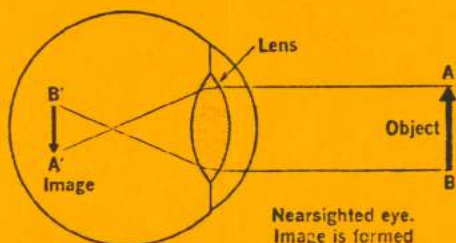


Concavo-convex lens
helps to increase convexity
of lens of eye

(B) Corrected

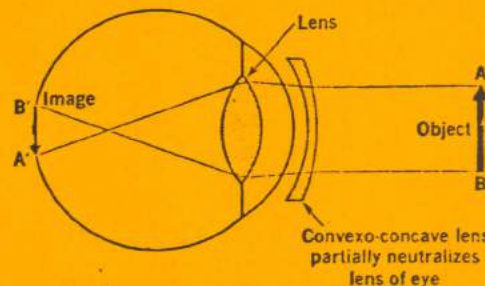


The image formed on the retina by the eye lens is real, inverted, and smaller than the object.



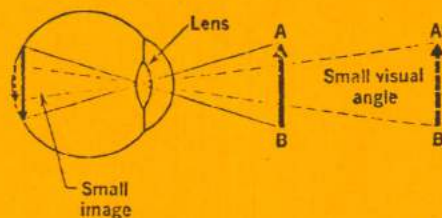
Nearsighted eye.
Image is formed
in front of retina

(A) Uncorrected



Convexo-concave lens
partially neutralizes
lens of eye

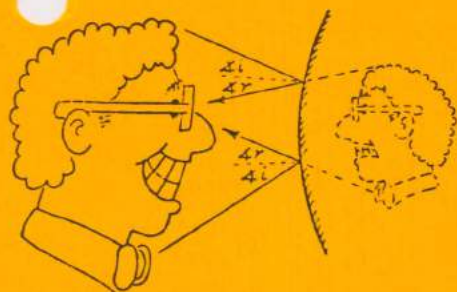
(B) Corrected



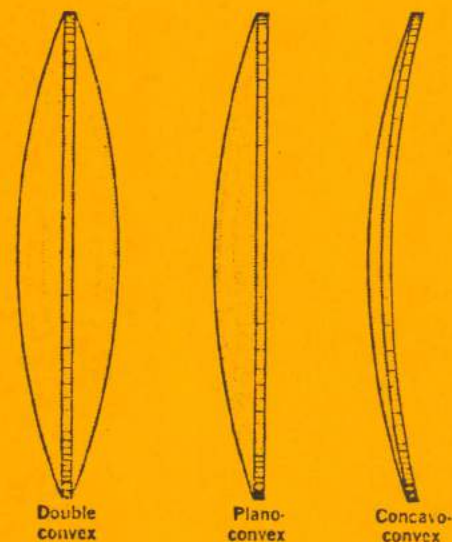
We judge the size of an object from the size of the visual angle.



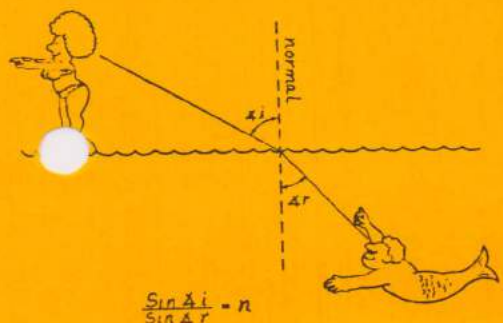
Are the tiles really crooked?



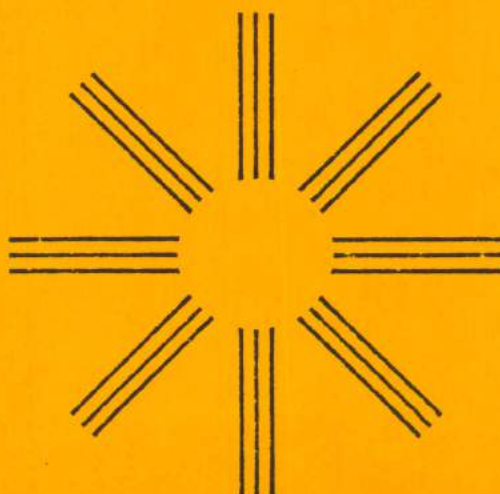
Ancient Greek



Converging lenses.



$$\frac{\sin \theta_i}{\sin \theta_r} = n$$



The lines are not all equally
distinct if the eye is astigmatic.



Diverging lenses.

all over

$$H_i = \frac{f}{S_o} H_o$$

$$\frac{13 \text{ cm} \times 2.6 \text{ cm}}{3.1 \text{ cm}}$$

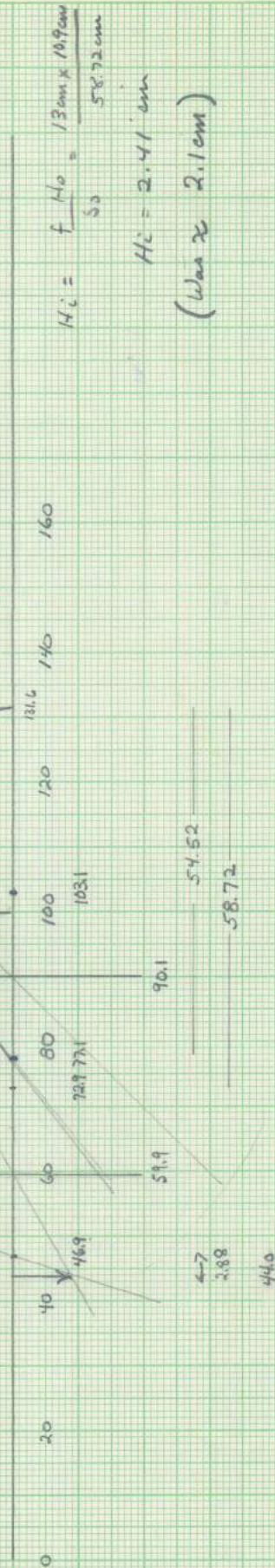
$$H_i = 10.9 \text{ cm}$$

$$H_i = \frac{f}{S_o} H_o = \frac{13 \text{ cm} \times 10.9 \text{ cm}}{58.72 \text{ cm}}$$

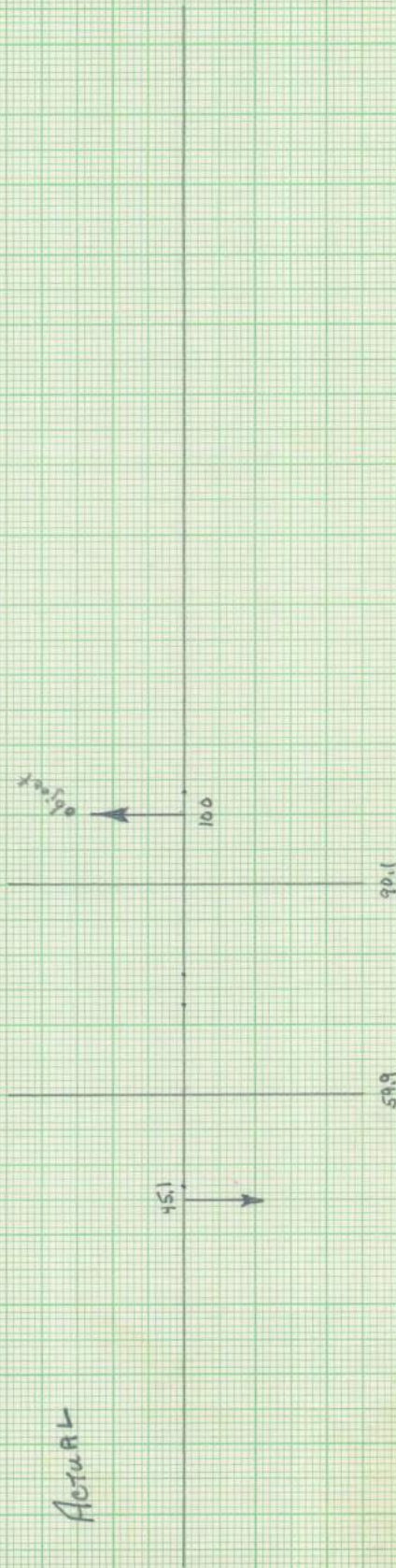
$$H_i = 2.41 \text{ cm}$$

(was x 2.1 cm)

THEORY



Actual



RAINBOWS

THE PHYSICS Teacher

p 283-286

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$$\sin 4r = \frac{3 \sin 4i}{4}$$

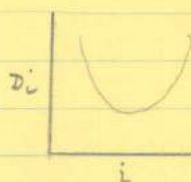
$$4r = 41^\circ$$

$$D_i = 180^\circ - 2(2r - i)$$

$$D_i = 138^\circ$$

Plot D_i vs i

D_i A MIN. AT 138°



Let $i = 62^\circ$	41.46	$D_i = 138.7$
$i = 63^\circ$	41.93	$= 138.16$
$i = 64^\circ$	42.38	$= 138.28$
$i = 65^\circ$	42.82	$= 138.43$
$i = 66^\circ$	43.25	$=$
$i = 67^\circ$	43.66	$=$
$i = 68^\circ$	44.06	$=$
$i = 69^\circ$	44.44	$=$
$i = 70^\circ$	44.81	$= 140.46$

More rays come to eye at 41° to 42° than at a smaller angle
NONE seen at larger angle

\therefore RAINDROP SERVES AS SMALL APERTURE which admits only those rays emerging at minimum deviation

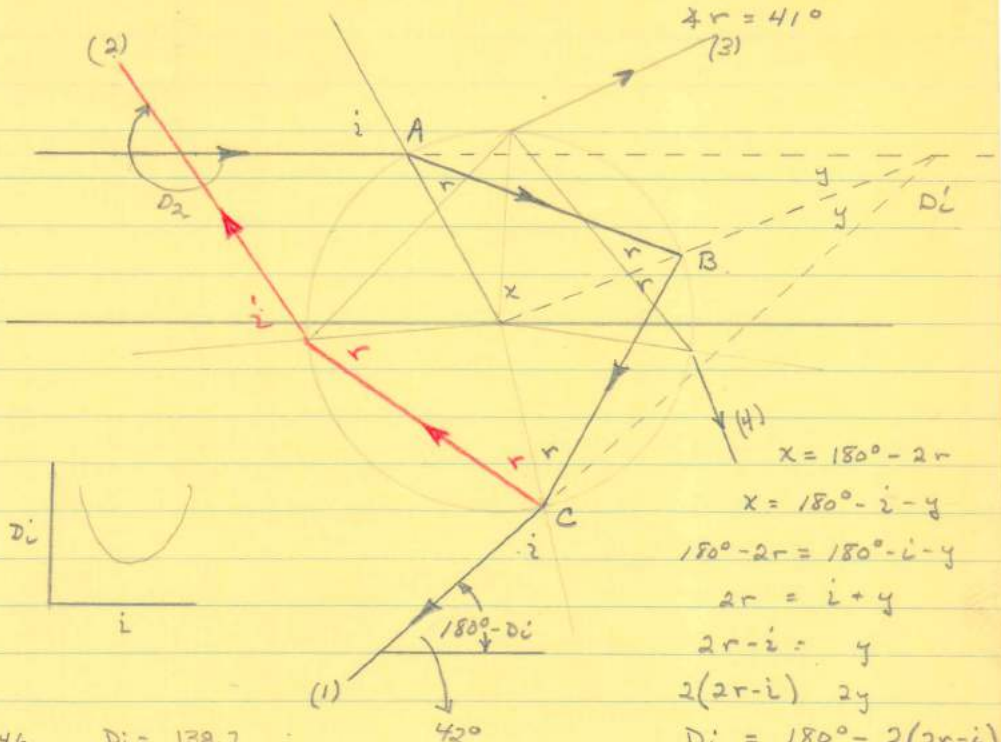
SECONDARY

$$D_2 = 2(180^\circ) - 2(3r - i) = 360^\circ - 2((3 \times 41) - 61) = 236^\circ \text{ (They say } 231^\circ)$$

$$D_3 = 3(180^\circ) - 2(4r - i) = 540^\circ - 2((4 \times 41) - 61) = 334^\circ$$

$$D_4 = 4(180^\circ) - 2(5r - i) = 720^\circ - 2((5 \times 41) - 61) = 432^\circ$$

$$D_5 = 5(180^\circ) - 2(6r - i) = 900^\circ - 2((6 \times 41) - 61) = 530^\circ$$



$$4i = 61^\circ$$

$$4r = 41^\circ$$

$$(3)$$

$$x = 180^\circ - 2r$$

$$x = 180^\circ - i - y$$

$$180^\circ - 2r = 180^\circ - i - y$$

$$2r = i + y$$

$$2r - i = y$$

$$2(2r - i) = 2y$$

$$D_i = 180^\circ - 2(2r - i)$$

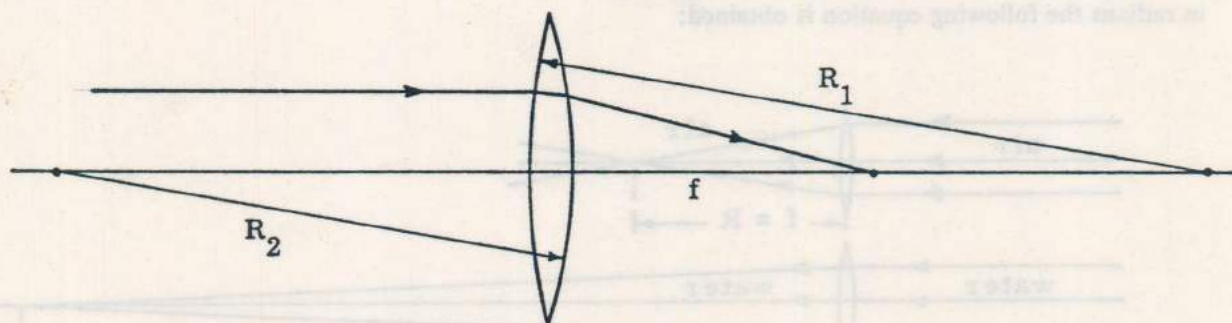
$$\frac{180}{56}$$

SUPPLEMENT TO CHAPTER 3, SECTION 10: DERIVATION OF THE LENSMAKER'S FORMULA

The following derivation involves no more than Snell's law, geometry, and radian measure.

Since all rays parallel to the principal axis of a lens

cross the axis of the lens at a common point (principal focus), it is only necessary to consider one of these rays in deriving this formula.



The above diagram is too small in the lens region through which the ray passes, so it is necessary to enlarge and exaggerate the angles in this region.

The refractions of the ray at both surfaces *A* and *B* are described respectively by Snell's law as

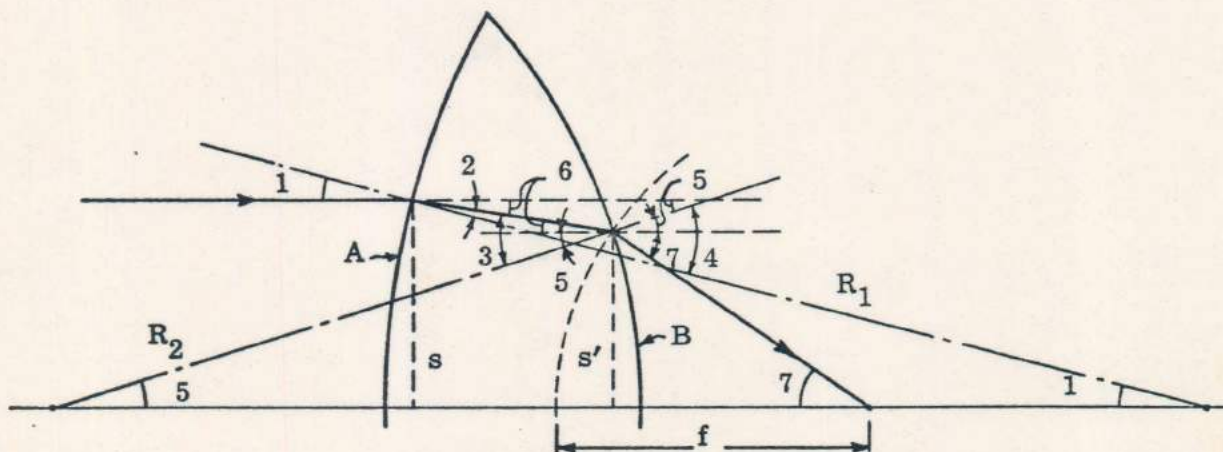
$$\sin 1 = n \sin 2$$

$$\sin 4 = n \sin 3$$

With a thin lens all of these angles are very small and thus the angles in radians may be substituted for the sines of the angles:

$$(a) \angle 1 = n \times \angle 2$$

$$(b) \angle 4 = n \times \angle 3$$



The next step is to reduce these equations to one equation with angles 1, 5, and 7 only. Additional lines have been drawn parallel to the axis of the lens and the equal angles thus formed have been labeled with the same number for simplicity. Adding (a) and (b) gives $\angle 4 + \angle 1 = n(\angle 2 + \angle 3)$, but since $\angle 4 = \angle 5 + \angle 7$ and $\angle 3 = \angle 5 + \angle 6 = \angle 5 + \angle 1 - \angle 2$, this equation (by substitution) is $\angle 5 + \angle 7 + \angle 1 = n(\angle 2 + \angle 5 + \angle 1 - \angle 2)$. By rearranging terms and factoring $\angle 7 = (n - 1)(\angle 5 + \angle 1)$. From the first drawing we can see that the lines marked s and s' in the second drawing and the length of the corresponding arcs are all so nearly equal that they can be considered equal. Also, since it doesn't really matter to what point in the lens f is measured for a thin lens, a convenient arc was drawn about the focus and f is measured to this arc. Thus, upon substituting the ratios of arcs to radii for the angles in radians the following equation is obtained:

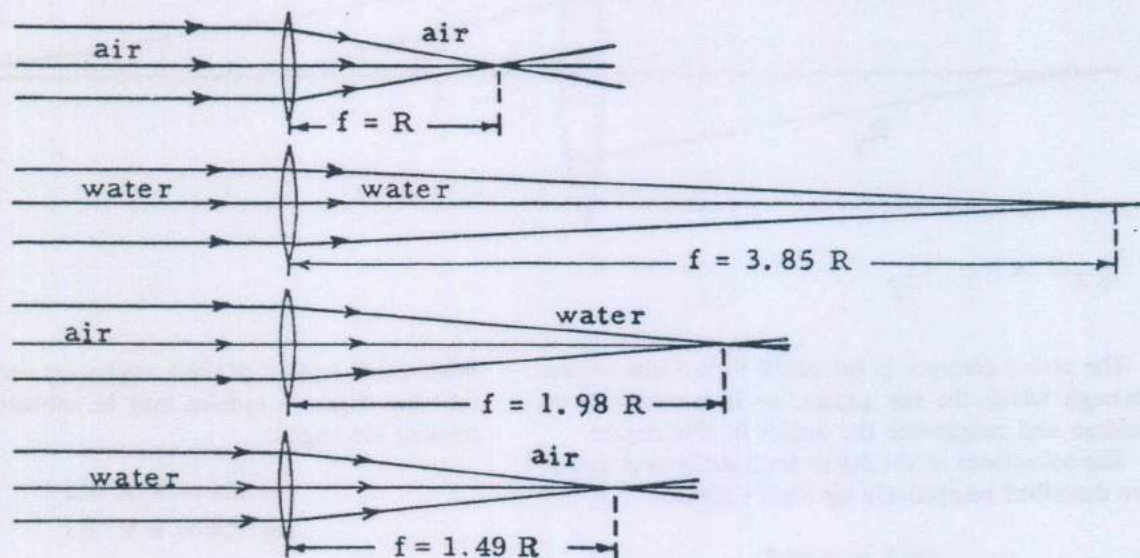
$$\frac{s'}{f} = (n - 1) \left(\frac{s'}{R_2} + \frac{s}{R_1} \right)$$

which gives the desired formula upon dividing each term by either of the equalities s or s' :

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_2} + \frac{1}{R_1} \right).$$

It should be noted that the index of refraction, n , used in the above derivation is the relative index of the lens material and the medium in which it is immersed. Thus, if there are two different media on either side of the lens the first factoring in the derivation could not be done. Diagrams which better portray these ideas are shown below:

$$(n_{\text{air}} = 1.00, n_{\text{water}} = 1.33, \text{ and } n_{\text{glass}} = 1.5)$$



TRY IT for Sect. 3.5D — Aberrations of a Magnifying Glass

You will need a focusing lens, such as a magnifying glass, preferably of large diameter. Image the grid of a window with sunlight shining through, onto a piece of paper. Can you get every point of the window grid in focus at once, or does one position of the paper provide a better focus for some parts, and other positions for others? What aberration is this? Do straight lines of the grid image as straight lines? If not, what type of distortion do you have? Does twisting the lens slightly affect the aberrations? Why should it?

Make a point source by covering a light source with aluminum foil with a pinhole in it. Image this source on a piece of paper. How sharp an image can you make? Is there spherical aberration? Can you see any colors around the image? What happens to the image when you twist the lens? What kind of aberrations can you see?

Look through the lens at the lines on a piece of ruled paper. The lens should be just far enough from the paper (somewhat beyond a focal length) so that the image is inverted. Your eye must be far enough from the lens so that the image is in focus for you. What kind of distortion do you see? Now move the lens closer to the paper (a little less than a focal length away) and examine the distortion again (also see Fig. 3.37).

How can there be distortion without any stop evident? The answer is that the pupil of your eye is usually the stop when you are looking through the lens. Because of this stop, a lens sometimes gives a better image when you look through it than when you use it to project an image. You can see this effect best with a large lens. Try it!

TRY IT for Sect. 4.5A — Measure Your Shutter's Exposure Time

We describe here a way to measure the exposure time that makes use of your TV. Television pictures are "painted" on the screen by scanning (see Sect. 6.3B) successive horizontal lines. A picture consisting of alternate lines of one complete frame is scanned in 1/60 second, and the remaining lines of that frame are scanned in the next 1/60 second. (In Europe this time is 1/50 second.) Thus it takes 1/30 second to scan a complete picture, but the surface of the tube is covered from top to bottom once every 1/60 second.

To test a fast exposure speed (above 1/60 second), take a picture of your TV screen so it fills most of the frame. (Include in the picture a small card on which the shutter setting is written, so you will know it after development.) From the developed picture you can judge what fraction of the TV picture is exposed. The shape of the bright region will be different, depending on the type of shutter you have.

If you have a between-the-lens shutter, you will get a horizontal band of properly exposed picture across the TV tube image. You may get a strip of picture in the middle of the tube, or part of the picture may be at the top and part at the bottom. (Count only the well-exposed part of the picture — the whole screen may actually show more faintly because it continues to glow even after the scanning beam has moved on.) For example, you may find that your exposure gave a strip of picture 1/4 as high as the full TV screen. The exposure time was therefore $(1/4) \times (1/60) \text{ sec} = 1/240 \text{ sec}$. If this is the result when your shutter was set at 1/250 sec, it is working pretty well, as

shutters go. For more accuracy, count the number of scan lines in your photo, and compute the exposure time from the fact that each line takes $(1/30) \times (1/525) \text{ sec} = 6.34 \times 10^{-5} \text{ sec}$, since in North America there are 525 lines in a frame.

If you have a focal-plane shutter, the picture will be rather different, because as the TV picture is scanned from top to bottom, the shutter scans it horizontally. The resulting photograph is a diagonal bright band that is essentially a plot of the shutter's slot position (horizontal) vs. time (vertical), so it tells you everything about the shutter motion. If the shutter moves at a constant speed, as it should, the band is straight (rather than curved). The horizontal width of the band measures the width of the slot in the shutter's curtain. The actual exposure time is the time the slot spends in front of any one point on the film. Hence, as for the between-the-lens shutter, the exposure time is measured by the vertical thickness of the band (Fig. 4.43).

To test the slower exposure speeds, try moving a thin object, such as a pencil, across the TV screen during the exposure. The result should be a sequence of shadows of the object, one for each 1/60 second the shutter was open. For example, if you see four shadows (Fig. 4.44), the exposure time was: $4 \times (1/60) \text{ sec} = 1/15 \text{ sec}$.

A somewhat better procedure is to photograph a glow lamp, which flickers with each AC current pulse through it, i.e. at 1/120 second intervals. Such lamps are used, for example, to measure the speed of revolution of hi-fi turntables. Photograph such a lamp while "panning" the camera (e.g., rotating it downward) so successive flashes of the lamp expose different points on the negative. By counting the number of images you get, you can compute the slower exposure times, as above.

Second TRY IT for Sect. 5.2A — Accommodation

With a candle, a dimly-lit room, and an accommodating friend, you can study the action of the eyelens. Hold the lit candle about 1/3 meter from your friend's eye, slightly to the side of her direction of gaze. Look carefully at the reflections of the candle in her eye. You should see something like Fig. 5.17. These are the Purkinje images. The first, which is the brightest, is due to the outer corneal surface. The image is erect, virtual and smaller than the object. The third Purkinje image, due to the front of the eyelens, is also erect, though somewhat dimmer than the first. The fourth Purkinje image is due to the rear surface of the eyelens. (The second Purkinje image, due to the inner surface of the cornea, is probably too faint to be seen.)

PONDER: Why are the first and third Purkinje images erect (and virtual) while the fourth is inverted (and real)?

Have your friend accommodate by focusing on something close, say your ear. Note carefully the positions of the Purkinje images. Now have her focus on a distant object, without changing her direction of gaze. Only the fourth Purkinje image shifts significantly, because accommodation is achieved by changing the curvature of the rear surface of the eyelens (see Fig. 5.6).

TRY IT for Sect. 9.6B -- Subtractive Mixtures of a Color With Itself

Using dyes, you can verify that a color mixed with itself subtractively may change its hue, saturation, and lightness. Rather than increasing the concentration of a dye, it is easier to decrease it -- to dilute a concentrated dye. Food coloring is a convenient transparent dye. Notice that the highly concentrated yellow dye looks red in its bottle. Compare this color to that of the dilute yellow obtained by putting a few drops from the bottle into a glass of water.

PONDER: What do you think the transmittance curve of the concentrated yellow dye looks like?

Also try diluting a spoonful of grape juice with a lot of water.

To see what happens when two different colors are mixed subtractively, you can mix two colored dyes together, or, more amusingly, use food coloring and gelatin desserts. First try to guess the resultant color, then add the dye and mix up the gelatin. Try to draw transmittance curves that explain your results.

First TRY IT for Sect. 12.5A -- Fresnel Diffraction

With a good point source of light you can see fringes in ordinary shadows. When the screen on which you display the shadows is relatively close to the object, you see Fresnel diffraction, and as the screen is moved farther away, the pattern changes into the Fraunhofer diffraction pattern.

You can easily see the Fresnel patterns using the coherent light from a laser beam as a source. To see it in ordinary light, mask the projection lens of a slide projector by aluminum foil with a pinhole (of diameter about $\frac{1}{2}$ mm). Shift the foil and change the focus until you get the brightest disk of light projected on a distant screen.

Use this light beam to cast shadows of simple objects that have sharp outlines, such as a razor or knife blade, a pin with a large round head, etc. Vary the distance of the object from the screen and observe the change in the pattern. The thin lines surrounding the main shadow of these objects are due to Fresnel diffraction. With a straight edge, such as a razor blade, observe that the intensity drops off gradually towards the dark side (there is some intensity inside the geometrical-optics shadow -- this is the light from the luminous edge). Outside the geometrical-optics shadow there are fringes, which you can think of as interference between the light forming the geometrical-optics shadow and that from the luminous edge.

At the center of the shadow of the shaft of a pin you see constructive interference of the light from the two luminous edges of the shaft. Fresnel predicted, on the basis of theory, that an opaque sphere (pin head) should have a bright spot at the center of its shadow. (At Fresnel's thesis defense, Poisson opined that the result was clearly ridiculous, hence the dissertation must be wrong. This "ridiculous" phenomenon henceforth carries the name "Poisson bright spot.") The spot is not easy to see -- but try it: Cast the pinhead's shadow on a piece of tissue paper and view the back of the paper with a magnifying glass, looking into the projector. Hold the pinhead so it is at the center of the brightest part of the beam from the projector. Have a friend move the tissue back and forth rapidly so as to blur the irregularities

of the tissue's fibers.

Another way to see Fresnel diffraction patterns is by reexamining the floaters you saw in Sect. 5.2, but this time illuminated by a pinhole. (The purpose of the pinhole is to provide a point source, which will give coherent light over a region larger than the floater's size.) Make a very small pinhole in aluminum foil, e.g., by crumpling and then flattening it, picking one of the smaller holes so produced. Hold the pinhole close to your eye, but focus your eye beyond it -- don't look at the pinhole -- and wait for floaters to drift past. You should see the Fresnel fringes around the outline of the floater, and maybe the Poisson bright spot. If you vary the pinhole distance from your eye, you can verify that the floaters must be close to the retina, because their sizes do not change appreciably.

Perhaps the simplest way to see Fresnel diffraction is to form a narrow slit between two adjacent fingers. Hold the fingers about 5 cm from your eye, and look through the slit while focusing on a distant light source. The dark lines within the slit are the Fresnel pattern.

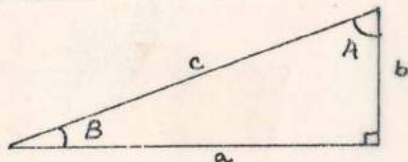
Second TRY IT for Sect. 12.5A -- Diffraction Pattern of a Hole

To see the fringe pattern of a circular hole you need a small pinhole in aluminum foil and a point light source. Place the aluminum foil on a piece of heavy paper and gently press the tip of a pin on it to make a tiny hole. For the light source, mask a light bulb with a larger pinhole, or use a distant street light, or the sun reflecting in a distant shiny object. Holding the pinhole very close to your eye, look through it at the light source, and notice the pattern of diffraction. Try pinholes of various sizes and see how the size of the rings varies. Also try pinholes of other shapes, e.g., made by cutting the foil with the tip of a sharp knife.

Chapter 19 Study Notes

I. THE SINE, COSINE AND TANGENT OF AN ANGLE

The text briefly discusses the definition of a sine of an angle as Snell's law is developed. The sine is related to the geometry of a right triangle.



In a right triangle, the sine of an angle is the ratio of the length of the side opposite the angle to the length of the hypotenuse. The cosine of an angle is the ratio of the length of the side adjacent to the angle to the length of the hypotenuse. The tangent of an angle is the ratio of the length of the side opposite to the length of the side adjacent to the angle.

Thus referring to the diagram, we see for angle B:

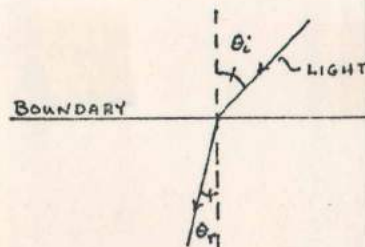
$$\sin B = b/c \quad \cos B = a/c \quad \tan B = b/a$$

and for angle A:

$$\sin A = a/c \quad \cos A = b/c \quad \tan A = a/b$$

One way to analyze the path of the refracted light at a boundary involves construction of right triangles. Most often, we use the line normal to the boundary as the adjacent side of the triangle and the path of the light ray as the hypotenuse. The triangle is completed with a line parallel to the boundary as in the figure below.

In many situations, you will need to construct such triangles and use the sine, cosine, and tangent to find the required quantities.



II. THE RELATIVE INDEX OF REFRACTION

The relative index of refraction, as discussed in the text, is useful when solving refraction problems involving two media, neither of which is a vacuum or air.

Suppose we have light moving from diamond into quartz crystal.

Snell's law says that

$$n_{\text{diamond}} \sin \theta_{\text{diamond}} = n_{\text{quartz}} \sin \theta_{\text{quartz}}$$

so

$$\frac{\sin \theta_{\text{diamond}}}{\sin \theta_{\text{quartz}}} = \frac{n_{\text{quartz}}}{n_{\text{diamond}}}$$

The value of the ratio of

$$n_{\text{quartz}} / n_{\text{diamond}}$$

is called the relative index of refraction for light going from DIAMOND into QUARTZ. Its value is

$$1.46/2.42 = 0.605$$

Relative indices of refraction are elegant because you can plug them directly into Snell's law.

They are more elegant than useful, however. So when solving refraction problems, use Snell's law in the form involving both ABSOLUTE indices

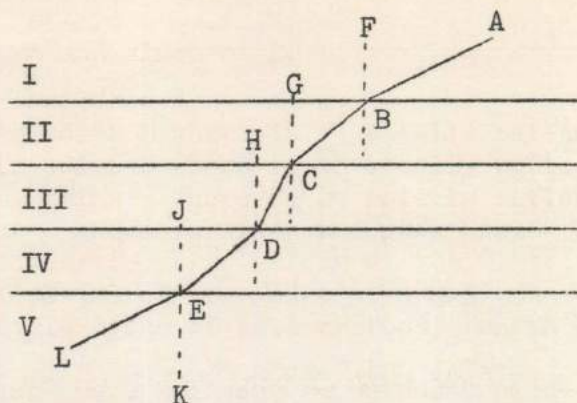
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

The following information pertains to the diagram at the right.

Angle FBA = Angle LEK
 Angle GCB = Angle JED
 Angle FBA > Angle GCB > Angle HDC

The absolute index of refractions are:

Medium I: 1.30
 Medium II: 1.40
 Medium III: 1.70



- _____ 1. What is the relative index for light going from I into II ?
- _____ 2. What is the absolute index of material IV ?
- _____ 3. What is the relative index for light going from IV into V ?
- _____ 4. What is the absolute index of material V ?
- _____ 5. If light could travel directly from material III into V, what would be the relative index?

The above drawing is a ray diagram for an incoming monochromatic yellow ray traveling from A to B.

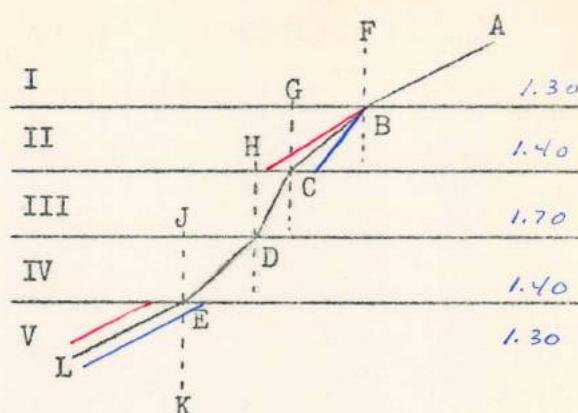
6. Replace the incoming ray with a violet ray. This will cause angle KEL to:
 - a. increase in size
 - _____ b. decrease in size
 - c. remain unchanged
7. Replacing the incoming ray at A to B with a violet ray will cause the emerging ray as a whole to:
 - a. be displaced to the right
 - _____ b. be displaced to the left
 - c. emerge from material IV at exactly the same point E as before
8. The same monochromatic source is placed at L and aimed at E.
 - a. This light will be totally reflected at E.
 - b. This light will be refracted to point D where it is totally reflected.
 - _____ c. This light will be refracted to point C where it is totally reflected.
 - d. This light will be refracted to point B where it is totally reflected.
 - e. This light will emerge at point B and move along the path BA.
9. Draw a scaled ray diagram indicating the location of the image of a 0.5 cm object that is located 9 cm from the principle focus of a 15 cm focal length lens.
10. Using the optical bench, glass screen, object box, 2 lenses in holders, place the object box at one end of the optical bench and the glass screen at the other. Then position the two lenses (that are at least 26 cm apart) between the object and screen in such a manner that a distance clear image is formed on the screen. Now make a scale drawing of the set-up. Verify that the object and image(s) in the actual situation agree with those in the scale drawing.

The following information pertains to the diagram at the right.

- Angle FBA = Angle LEK
 Angle GCB = Angle JED
 Angle FBA > Angle GCB > Angle HDC

The absolute index of refractions are:

- Medium I: 1.30
 Medium II: 1.40
 Medium III: 1.70



- 1.08 1. What is the relative index for light going from I into II ?
- 1.40 2. What is the absolute index of material IV ?
- .928 3. What is the relative index for light going from IV into V ?
- 1.30 4. What is the absolute index of material V ?
- .764 5. If light could travel directly from material III into V, what would be the relative index?

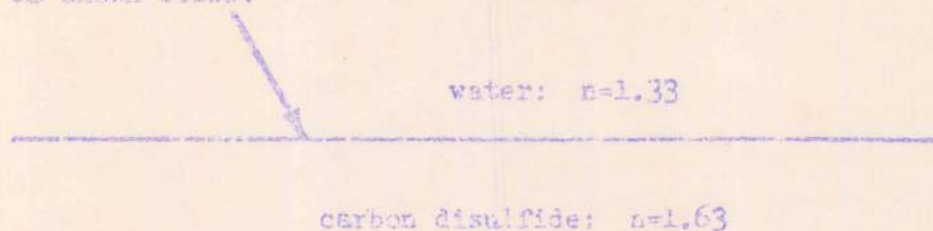
The above drawing is a ray diagram for an incoming monochromatic yellow ray traveling from A to B.

6. Replace the incoming ray with a violet ray. This will cause angle KEL to:
 - C a. increase in size
 - b. decrease in size
 - c. remain unchanged
7. Replacing the incoming ray at A to B with a violet ray will cause the emerging ray as a whole to:
 - A a. be displaced to the right
 - b. be displaced to the left
 - c. emerge from material IV at exactly the same point E as before
8. The same monochromatic source is placed at L and aimed at E.
 - E a. This light will be totally reflected at E.
 - b. This light will be refracted to point D where it is totally reflected.
 - c. This light will be refracted to point C where it is totally reflected.
 - d. This light will be refracted to point B where it is totally reflected.
 - e. This light will emerge at point B and move along the path BA.
9. Draw a scaled ray diagram indicating the location of the image of a 0.5 cm object that is located 9 cm from the principle focus of a 15 cm focal length lens.
10. Using the optical bench, glass screen, object box, 2 lenses in holders, place the object box at one end of the optical bench and the glass screen at the other. Then position the two lenses (that are at least 26 cm apart) between the object and screen in such a manner that a distance clear image is formed on the screen. Now make a scale drawing of the set-up. Verify that the object and image(s) in the actual situation agree with those in the scale drawing.

PSSC PHYSICS QUIZ # _____
Refraction (Need Protractor)

NAME _____
DATE: _____

1. A ray of light is incident on the boundary of a layer of water and glass as shown below:



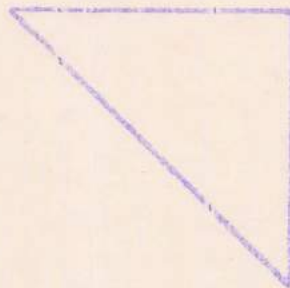
Part of the light is reflected at the boundary and part is refracted into the glass. What is the angle between the reflected and refracted rays? (4 pts.)

Show light path and angles on diagram above.

2. What is the value of the critical angle for light passing from glass to water? (2 pts.)

3. Given the glass $45^\circ - 45^\circ - 90^\circ$ prism below has an index of refraction of 1.5, show a possible path for a ray of light to travel through the prism. (2 pts.)

Please draw to proper scale and label size of angles.

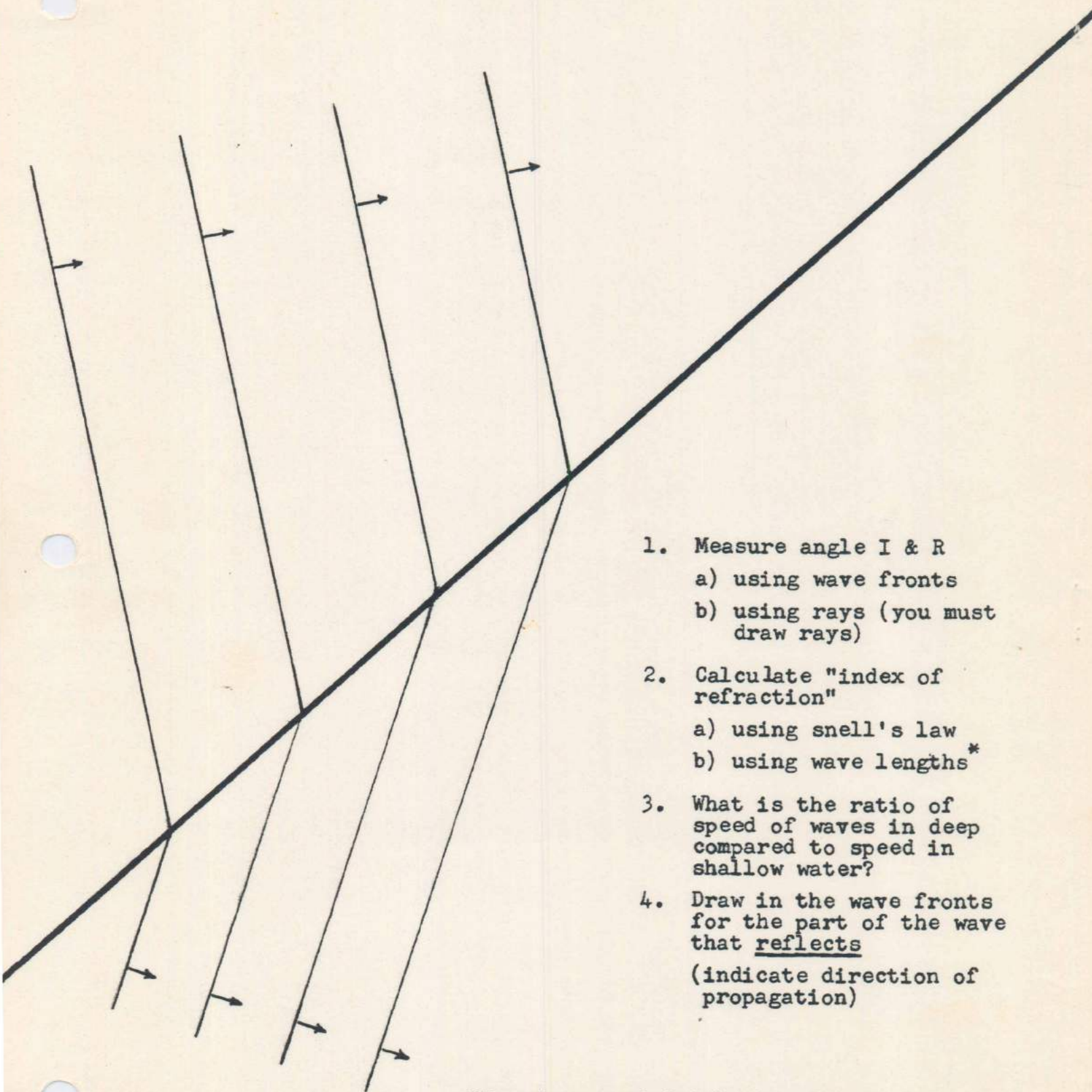


4. Explain why the refraction dish used in Expt. #3 is shaped like a semi-circle. (2 pts.)

E.P. Draw a diagram to show why an object under water appears to an observer to be closer to the surface than its true depth.

SAMPLE SET OF PROBLEMS

REFRACTION OF WAVES



1. Measure angle I & R
 - a) using wave fronts
 - b) using rays (you must draw rays)
2. Calculate "index of refraction"
 - a) using snell's law
 - b) using wave lengths*
3. What is the ratio of speed of waves in deep compared to speed in shallow water?
4. Draw in the wave fronts for the part of the wave that reflects
(indicate direction of propagation)

*How do you minimize the effect of measurement error?

Problems 1-3 refer to an object placed 15 cm in front of a converging lens of focal length 20 cm.

1. The distance of the image from the lens is _____ cm
2. Describe what/where/how the image exists.
3. If the object is 10 cm high the image is _____ cm high.

Problems 4-6 refer to an object located 30 cm in front of a spherical convex mirror of radius 20 cm.

4. The image distance is _____ cm.
5. Describe what/where/how the image exists.
6. The magnification of the image is _____ times.
7. The large telescope at Mt. Palomar has a concave objective mirror of diameter 200 inches and radius of curvature 150 ft. What is the magnifying power of the instrument when used with an eyepiece of focal length 0.5 in.?

8. An optical bench pointer 1 cm tall is placed at the 55 cm mark on the meter stick. When a diverging lens is placed at the 0 cm mark, an image is formed at the 5 cm mark. What is (a) the focal length of the lens, and (b) the size of the image?
9. The dimensions of the picture on a slide are 6.4 cm by 7.6 cm. This slide is to be projected to form an image 1.5 m by 1.8 m at a distance of 9 m from the objective lens of the projector. (a) What is the distance from the slide to the objective lens? (b) What focal length objective lens must be used?

IMAGES FORMED BY A CONVERGING LENSE

1. By focusing on distant object find focal length for both sides of your lens.
2. Have your teacher check these values before you proceed with the rest of experiment.
3. Notice "search light" effect when light source is placed at principal focus.
4. Measure several values for S_o & S_i ———→ find formula for relationship. Note that S_o and S_i are measured from focal points on their respective sides of the lens.
5. Make table to summarize properties of image for various object positions.
6. Measure S_o & S_i for one virtual image case. (See Figure #1) Check to see if the relationship found for real images (in 4- above) also holds for virtual image--- be careful that you are measuring S_o & S_i from their correct focal points. You must determine which focal point goes with image and which goes with object.
7. Arrange two lenses so that their principal axis are congruent. Distance between lenses should be $2(f_1 + f_2)$. Place object light source at $2f_1$ and locate final image. (see Figure #2) Repeat with light at $3f_1$ and again at $1f_1$. Explain how you could determine the position of these images by calculation rather than experimental measurement.
8. Extra: Something you think about.

NOTE: IF AVAILABLE YOU CAN RUN DATA ON COMPUTER PROGRAM 'JNTLEN'

Figure #1
Finding Virtual Image

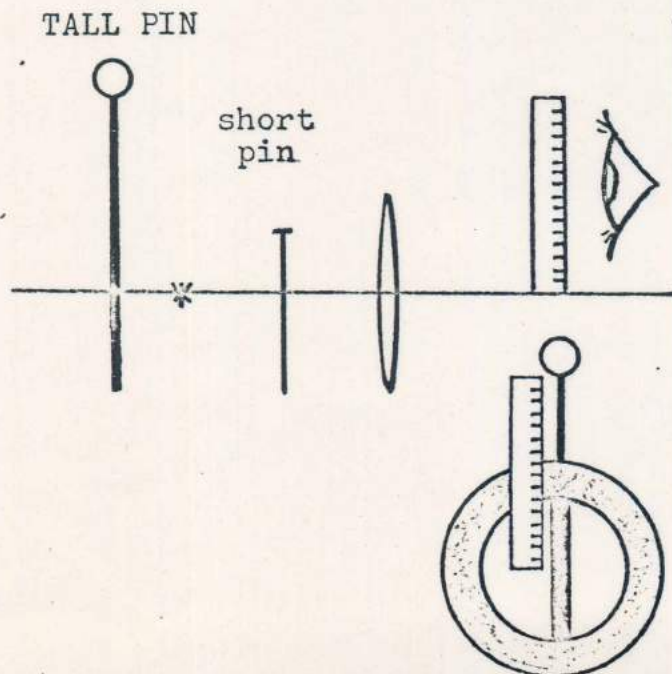
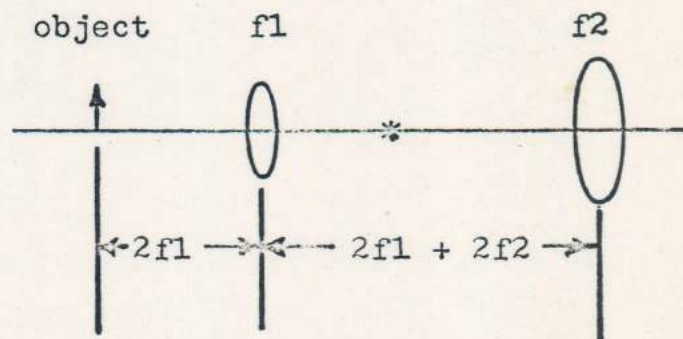


Figure #2
Location of Lenses for Step 7-



PSSC Experiment Number 3 - REFRACTION OF LIGHT -

- 1] Using the semi-circular dish and polar graph paper investigate the bending of light as it passes from air into water. You should find a relationship between the angle of incidence and the angle of refraction. NOTE: These angles are measured between the ray of light and the normal. How can you use symmetry to improve on the accuracy of your angle measurements? Since more than one conclusion for the relationship is possible, you should run your data through the computer program "JNSNEL". See sample run. We will discuss in class how you can interpret the output of the computer program.
- 2] Plot the following graphs:
 - A) angle of refraction $\angle r$ versus angle of incidence $\angle i$.
 - B) angle i /angle r versus angle i
 - C) sine of i /sine of r versus angle iIn your laboratory report explain the meaning of these graphs.
- 3] Repeat above using glycerine.

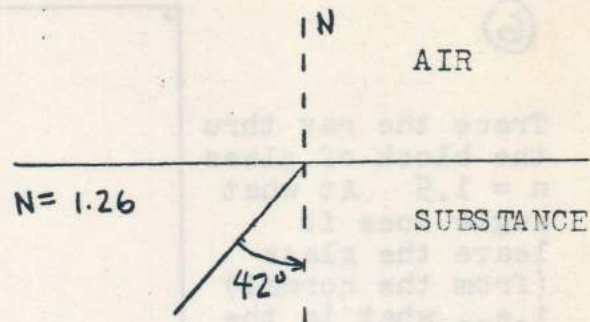
*** NOTE *** DO NOT USE POLAR GRAPH PAPER FOR 4 AND 5 BELOW:

- 4] Trace several rays of light through any transparent rectangular block of material. What conclusion can be made about the index of refraction for light going from air into glass (plastic) compared to the index of refraction for light going from glass (plastic) into air.
- 5] Trace a few rays of light through a transparent equilateral prism of glass. Make any conclusion you can about the passage of light through an equilateral prism.

REFRACTION

REFRACTION REVIEW SHEET (HONORS)

- Using the drawing at right, find the angle at which the ray leaves the material.

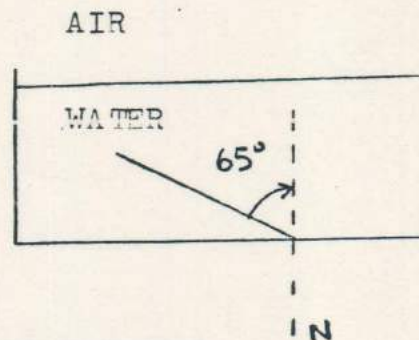


- Find the critical angle for the material in problem #1.

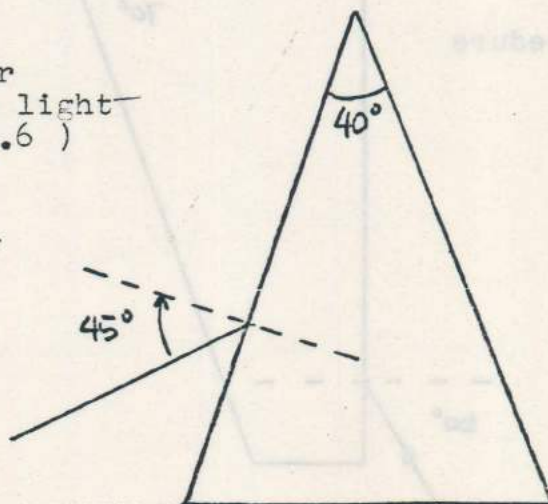
- If a light ray is incident to a material at an angle of 52° ($n = 1.83$) what is the angle of refraction?

- What is the speed of light in the material of problem #3?

- Use*
- Using the information given in the drawing at right, find the angle at which the light ray leaves the tank of water.



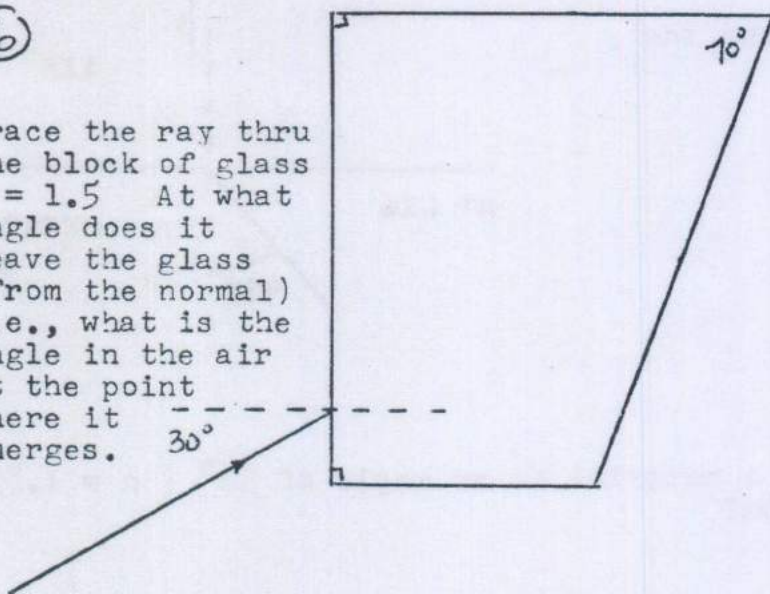
- Use*
- Complete the drawing at right after calculating the angle at which the light ray leaves the glass block ($n = 1.6$) after passing through it.



HONORS

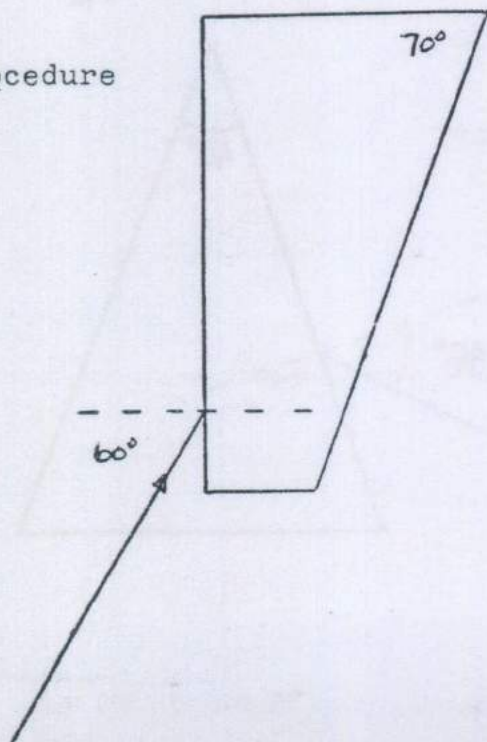
⑥

Trace the ray thru
the block of glass
 $n = 1.5$ At what
angle does it
leave the glass
(from the normal)
i.e., what is the
angle in the air
at the point
where it
emerges. 30°



⑦

Same procedure
as 6.



FERMAT'S PRINCIPLE



Fermat's principle may provide the basis for interesting discussions outside of class with students.

The following discussion is related to the time it takes light to travel from any point *A* to another point *B*.

(1) Fermat's principle, as he stated it, applies to light traveling in a homogeneous medium or to light reflected or refracted at a single plane surface. The principle states that the path light actually takes between *A* and *B* is that which requires less transit time than any neighboring path.

(2) Also related to transit time and covered by an extension of Fermat's principle, is the fact that the transit time is the same for all light rays which go through an optical system from one point on an object to the corresponding point on the image.

Fermat's principle (in its restricted sense of the light path being a minimum for plane surfaces) can be used directly to derive the following:

- (1) In a homogeneous medium, light travels in a straight line.
- (2) The laws of reflection. (The proof is given below.)
- (3) The laws of refraction. (The proof in this case is straightforward, but involves either calculus or quite complex algebra and is therefore not reproduced here.)

If you merely mention this principle and all that follows from it, the imagination of many students will be stimulated. Some will wonder from where this powerful principle comes, and whether it has a deeper, even philosophical significance. Other students may wonder how Fermat discovered it. You can start an interesting out-of-class discussion on either of these questions.

First we might consider whether Fermat's principle seems more correct or more fundamental than the rectilinear propagation of light and the laws of reflection and refraction. Many students will appreciate the simple elegance of one rule which leads to three others that had been only empirical (even though they explained many phenomena). You can continue

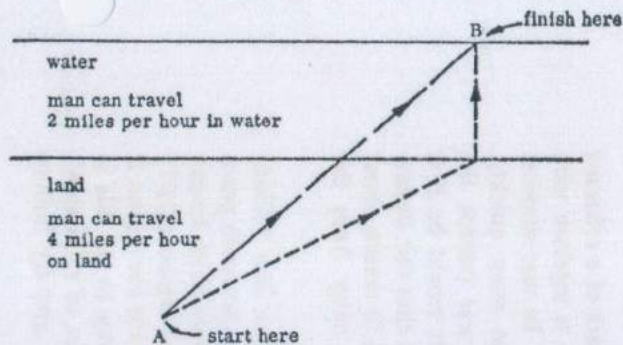
this discussion by being sure that the students realize that Fermat's principle is not like a model; it is more analogous to the laws of reflection or refraction.

Even though you cannot give a recipe indicating how Fermat discovered this principle, students will enjoy thinking about the kind of question that must have motivated Fermat. The question of what is distinctive about one particular path is typical of a fruitful class of questions in science. One way to get insight into why one thing happens is to think about why other things do not happen. Notice that in Fermat's case he probably did not get ahead by asking why light starts off in a particular direction or why the angle of reflection is equal to the angle of incidence. Instead he asked the rather indirect question: If light is going to go from *A* to *B* (perhaps touching a mirror first) why does it choose one particular path? Fermat's principle does not help you decide that light from *A* reaches any arbitrary point *B*; it merely states that *if* light does go, it chooses a certain path among the possible ones.

It is not easy to verify Fermat's principle graphically or analytically unless one is extremely careful and precise. Instead of discussing numerical examples, we can try two other applications:

(1) Consider qualitatively the path of a refracted ray going from *A* in air to *B* in a medium with large index (relatively low speed). Be sure students see qualitatively that light, to go most quickly, would take a path in which it was outside the medium for a longer time than it would be if it went over the straight path. Note that this discussion makes it possible for students to reason about whether light bends toward or away from the normal.

(2) Consider a kinematic problem like "Which path should a boy take if he wants to reach point *B* from point *A*, if *A* is in a region in which he can move twice as fast as he can in the region of *B*?" (The region *A* might be ground while the region *B* might be water.) The simplest way to do this is to think about light, use the "index of refraction" equal to the ratio of the speeds, and try finding the path using Snell's law.



The fastest path is somewhere between the two indicated. Use Snell's law with refractive index $4/2 = 2$ to find the point on the river bank where $\frac{\sin(\text{angle to normal on land})}{\sin(\text{angle to normal on water})} = 2$.

There is another point which might be worth making about the time it takes light to travel in an optical system. If an optical system focuses light from one point on an object to one point on an image (i.e., if the optical system is free from aberrations or distortions), the light which goes from the object point to the corresponding image point takes exactly the same time, no matter which of the many possible ray paths it takes, even though the "ruler distance" varies considerably. Notice that you can use this interesting fact to decide which way light will bend when it goes through a lens. Since light goes more slowly through the glass, the ray which goes through the thicker part of the lens spends a long time in glass; during this extra time it spends in glass, the light which goes through the thinner part of the glass moves a greater distance in air. In order to make the two times equal, the ray which spends more time in air must bend toward the ray which goes through the thicker glass.

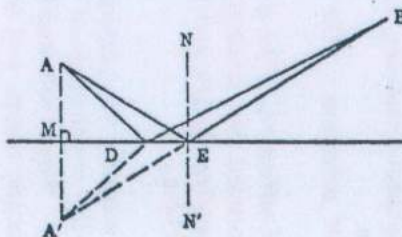
PROOF THAT FERMAT'S PRINCIPLE LEADS TO THE LAWS OF REFLECTION

The first law of reflection, that the incident ray, reflected ray, and the normal are in the same plane, is an easy result of Fermat's principle in Euclidean space.

The second law of reflection, that the angle of reflection is equal to the angle of incidence, must be proved by geometry. Since light, when reflected, stays in one particular medium, the index of refraction and therefore the speed of light may be assumed to be constant throughout the path from A to B. This means that the path over which light takes a minimum

time to travel is just the path which is a minimum distance from A to B. The shortest distance from A to B is just the straight line from A to B, but by minimum we mean the minimum path between A and B which touches (reflection) the mirror. This is what we must prove for the equal angle path.

Since the shortest distance between two points is along a straight line between them, we expect the path of light from A to the reflecting surface to be a straight line and the path from the reflecting surface to B to be a straight line. We find the point where the reflected ray hits the surface by making use of the point A', which is the point such that the line representing the surface is the perpendicular bisector of AA'. Then for any point D on the line representing the surface, AD = A'D by simple geometry. But the distance the light travels is just AD + DB = A'D + DB. The location of D such that the distance A'DB and therefore the distance ADB is a minimum, is the point E on the line between A' and B. From simple geometry we have $\angle AEM = \angle MEA'$, and therefore, if NN' is the normal to the surface at E, $\angle AEN = \angle A'EN'$. But $\angle A'EN'$ and $\angle NEB$ are vertical angles and therefore are equal; so, $\angle AEN =$



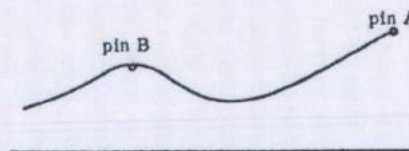
A-11

$\angle NEB$. This is just the second law of reflection, that the incident ray and the reflected ray form angles of the same size with the normal to the surface at the point of reflection.

An attempt to prove that a path near the "equal angle path" is longer, by using the direct method of taking the difference in length between it and the "equal angle path" as a function, say, of the distance between the point of reflection of the near path and the point of reflection of the "equal angle path," will end in disaster. It involves squaring an inequality three times.

Using calculus in a straightforward way to find the minimum path leads to the equal angle path and also the direct straight line path between A and B. An easy physical demonstration that $\angle i = \angle r$, when the reflected path is as short as possible, can be accomplished with two pins, and a piece of thread.

Tie the thread to one pin. Loop the free end of the thread around the other pin. Using a pencil, push the thread toward a line while letting the free end of the thread slip past pin B. Pay out as little thread as possible to allow the pencil to touch somewhere along the line. Mark the point where the pencil touches, construct the normal, and measure the angles of incidence and reflection.



The diagram above illustrates the use of Fermat's principle in the study of refraction and shows the high precision needed to recognize that the time is a minimum.

With the particular positions of A and B shown at right, path (1) corresponds to Snell's law. It makes an angle of 49.2° with the normal. Path (2) is near the actual path (1) of light as determined by Snell's law, but it makes an angle of 45° with the normal to the surface in air and an angle of 30° with the normal in the dense medium. The time it takes to go from A to B along the true path (1) is:

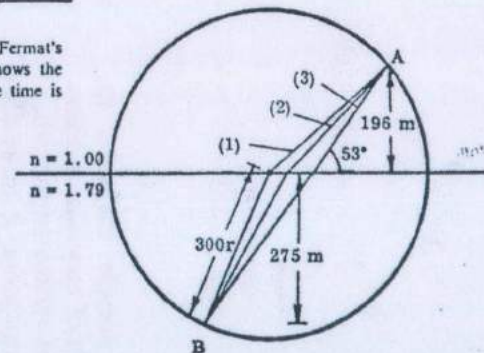
$$t = \frac{300 \text{ m}}{c} = \frac{300 \text{ m}}{3 \times 10^8 \text{ m/sec}} + \frac{300 \text{ m}}{c/n} = \frac{300 \text{ m}}{1.68 \times 10^8 \text{ m/sec}} \\ = 10^{-6} + 1.79 \times 10^{-6} \text{ sec} = 2.79 \times 10^{-6} \text{ sec.}$$

The time it would take to go from A to B along path (2) would be almost the same:

$$\frac{275/\sin 60^\circ \text{ m}}{c/n} = \frac{275/\sin 60^\circ \text{ m}}{1.68 \times 10^8 \text{ m/sec}} + \frac{196/\sin 45^\circ \text{ m}}{c} = \frac{196/\sin 45^\circ \text{ m}}{3 \times 10^8 \text{ m/sec}} \\ = 1.89 \times 10^{-6} + 0.92 \times 10^{-6} \\ = 2.81 \times 10^{-6} \text{ sec.}$$

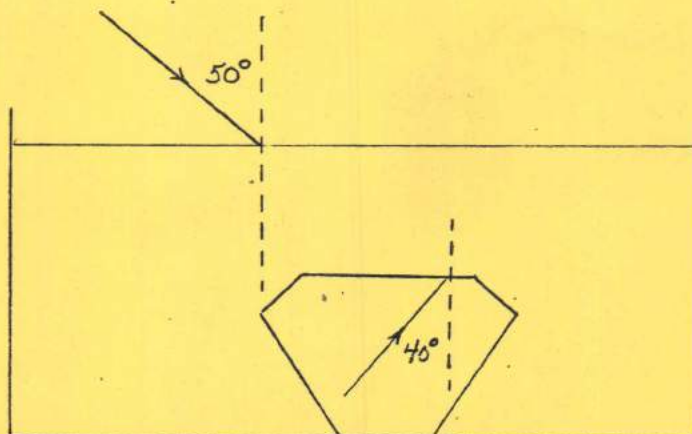
Even for path (3), the straight line between A and B, the transit time would not be very different:

$$t = \frac{275/\sin 53^\circ \text{ m}}{c/n} = \frac{275/\sin 53^\circ \text{ m}}{1.68 \times 10^8 \text{ m/sec}} + \frac{196/\sin 53^\circ \text{ m}}{c} = \frac{196/\sin 53^\circ \text{ m}}{3 \times 10^8 \text{ m/sec}} \\ = 2.05 \times 10^{-6} + 0.82 \times 10^{-6} \text{ sec} \\ = 2.87 \times 10^{-6} \text{ sec.}$$



A-12

1. A diamond is emersed in a container of water as shown in the drawing below. Light is incident on the water surface at an angle of 50° . The index of refraction is 1.33 for water, and 2.42 for a diamond.
- What is the refracted angle of the light ray in the water?
 - The light ray in the water eventually hits the water-diamond surface. What is the angle between the light ray which penetrates the diamond and the ~~normal to the~~ normal to the water-diamond surface?
 - Part of the light beam which hits the water-diamond surface is reflected up through the water again. Trace this ray up through the water and out into the air labeling each angle of incidence and refraction.



- A light ray is projected upward through the diamond at an angle of 40° relative to the normal. What would be the angle between the light ray which penetrates the water and the normal to the water-diamond surface?
clarify - towards at 1st surface interface
- An object 6 mm high is 24 cm to the left of a converging lens of focal length of 8 cm.
 - Where is the image?
 - Is it real or virtual?
 - Erect or inverted?
 - How large is it?
 - A double convex lens of focal length 20 cm forms a real image 5 times as large as the object. How far apart are the object and the image?
 - A movie theater projectionist must know how to select the proper projection lens to fill the screen with an image of the movie film. The picture on the movie film is 3.4 cm wide. In a particular theater the screen is 10 m wide and is located 30 m from the projector.
 - What magnification is needed to fill the projection screen?
 - What object distance is needed to produce this magnification?
 - What should be the focal length of the lens?

1. AIR (1) $n_1 = 1.00$ $\theta_1 = 50^\circ$
 H₂O (2) $n_2 = 1.33$
 DIAMOND (3) $n_3 = 2.42$

(a) AIR \rightarrow H₂O

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.0 \sin 50^\circ = 1.33 \sin \theta_2$$

$$\theta_2 = 35.2^\circ$$

(b) H₂O \rightarrow DIAMOND

$$n_2 \sin \theta_1 = n_3 \sin \theta_3$$

$$1.33 \sin 50^\circ = 2.42 \sin \theta_3$$

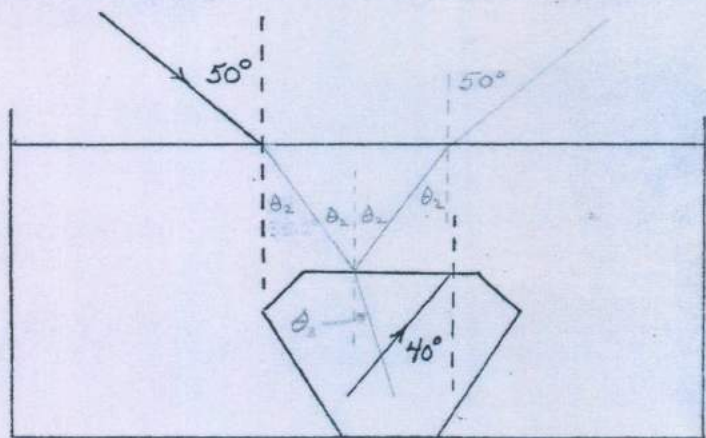
$$\theta_3 = 18.5^\circ$$

(c) SEE DRAWING

$$n_3 \sin \theta_3 = n_2 \sin \theta_2$$

$$2.42 \sin 40^\circ = 1.33 \sin \theta_2$$

$\theta_2 = 40^\circ$ \therefore TOTAL INTERNAL REFLECTION



2. $H_o = 6 \text{ mm}$

$$f + S_o = 24 \text{ cm}$$

$$f = 8 \text{ cm}$$

$$S_o = 16 \text{ cm}$$

$$(a) S_i = \frac{f^2}{S_o} = \frac{8 \text{ cm} \times 8 \text{ cm}}{16 \text{ cm}}$$

$$S_i = 4 \text{ cm}$$

OR
 12 CM BEYOND LENS

(b) REAL (c) INVERTED

$$(d) \frac{H_i}{H_o} = \frac{f}{S_o} \Rightarrow H_i = \frac{f H_o}{S_o} = \frac{8 \text{ cm} \times 6 \text{ mm}}{16 \text{ cm}} \therefore H_i = 3 \text{ mm}$$

$$3. \frac{H_i}{H_o} = 5 \quad f = 20 \text{ cm}$$

$$\frac{H_i}{H_o} = \frac{f}{S_o} \therefore S_o = \frac{f}{5} = 4 \text{ cm}$$

$$\frac{H_i}{H_o} = \frac{S_i}{f} \therefore S_i = 5f = 100 \text{ cm}$$

$$D_T = 144 \text{ cm}$$

4. $H_o = 3.4 \text{ cm}$

$$H_i = 10 \text{ M}$$

$$S_i + f = 30 \text{ M} \quad S_i = 30 \text{ M} - f$$

$$(a) M_{ay} = \frac{H_i}{H_o} = \frac{10 \text{ M}}{3.4 \text{ cm}} \times \frac{10^2 \text{ cm}}{1 \text{ M}} = 294.1$$

$$(b) \frac{H_i}{H_o} = \frac{S_i}{f} = \frac{30 \text{ M} - f}{f}$$

$$294.1 f = 30 \text{ M} - f$$

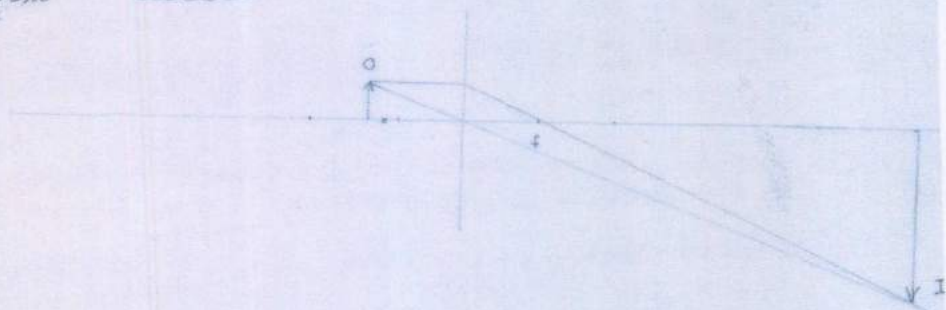
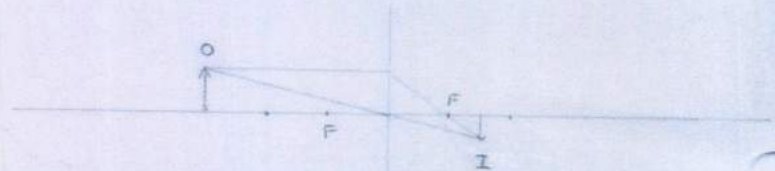
$$295.1 f = 30 \text{ M}$$

$$f = 30 \text{ M} = 100 \text{ cm}$$

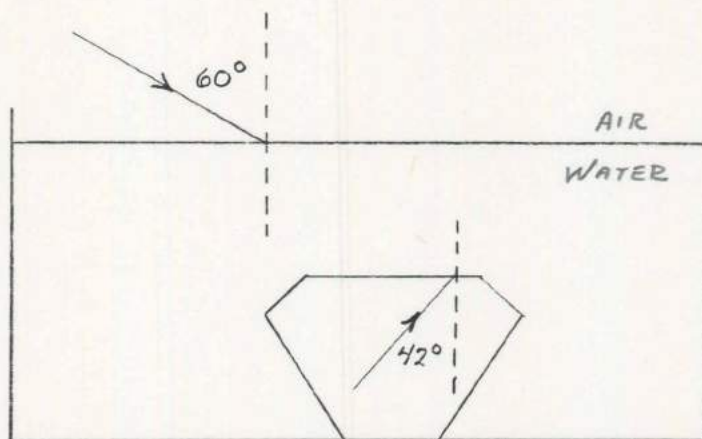
$$(b) \frac{H_i}{H_o} = \frac{f}{S_o} = 294.1$$

$$S_o = \frac{f}{294.1}$$

$$S_o = 0.035 \text{ cm}$$



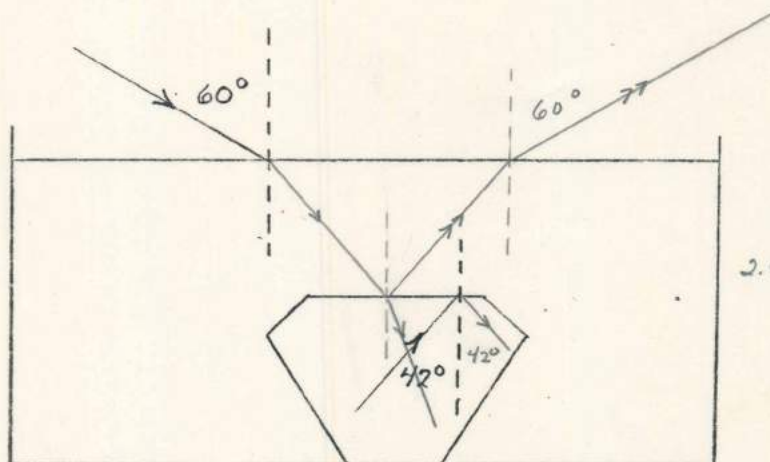
1. A diamond is emersed in a container of water as shown in the drawing below. Light is incident on the water surface at an angle of 60° . The index of refraction is 1.33 for water, and 2.42 for a diamond.
- What is the refracted angle of the light ray in the water?
 - The light ray in the water eventually hits the water-diamond surface. What is the angle between the light ray which penetrates the diamond and the normal to the water-diamond surface?
 - Part of the light beam which hits the water-diamond surface is reflected up through the water again. Trace this ray up through the water and out into the air labeling each angle of incidence and reiraction.



- A light ray is projected upward through the diamond at an angle of 42° relative to the normal. What would be the angle between the light ray which penetrates the water and the normal to the water-diamond surface?
2. An object 6 mm high is 6 cm to the left of a converging lens of focal length of 8 cm.
- Where is the image?
 - Is it real or virtual?
 - Erect or inverted?
 - How large is it?
3. A double convex lens of focal length 20 cm forms a real image 2 times as large as the object. How far apart are the object and the image?
4. A movie theater projectionist must know how to select the proper projection lens to fill the screen with an image of the movie film. The picture on the movie film is 3.4 cm wide. In a particular theater the screen is 10 m wide and is located 30 m from the projector.
- What magnification is needed to fill the projection screen?
 - What object distance is needed to produce this magnification?
 - What should be the focal length of the lens?

1. A diamond is emersed in a container of water as shown in the drawing below. Light is incident on the water surface at an angle of 60° . The index of refraction is 1.33 for water, and 2.42 for a diamond.

- (3) a. What is the refracted angle of the light ray in the water?
(3) b. The light ray in the water eventually hits the water-diamond surface. What is the angle between the light ray which penetrates the diamond and the normal to the water-diamond surface?
(7) c. Part of the light beam which hits the water-diamond surface is reflected up through the water again. Trace this ray up through the water and out into the air labeling each angle of incidence and refraction.



$$1.0 \sin 60^\circ = 1.33 \sin \theta_1$$

$$\theta_1 = 40.62^\circ$$

$$1.0 \sin 60^\circ = 2.42 \sin \theta_2$$

$$\theta_2 = 20.97^\circ$$

$$2.42 \sin 42^\circ = 1.33 \sin \theta_3$$

- (3) d. A light ray is projected upward through the diamond at an angle of 42° relative to the normal. What would be the angle between the light ray which penetrates the water and the normal to the water-diamond surface?

2. An object 6 mm high is 6 cm to the left of a converging lens of focal length of 8 cm.

$$H_o = 6 \text{ mm}$$

$$S_o = 2 \text{ cm}$$

$$f = 8 \text{ cm}$$

$$S_i =$$

- (3) a. Where is the image?

$$S_i = \frac{f^2}{S_o} = \frac{8 \text{ cm} \times 8 \text{ cm}}{2 \text{ cm}} = 32 \text{ cm}$$

24 cm TO LEFT OF LENS

- (1) b. Is it real or virtual?

- (1) c. Erect or inverted?

- (3) d. How large is it?

$$H_i = \frac{f}{S_o} H_o = \frac{8 \text{ cm}}{2 \text{ cm}} \times 6 \text{ mm} = 24 \text{ mm}$$

$$f = 20 \text{ cm}$$

$$\frac{H_i}{H_o} = 2$$

3. A double convex lens of focal length 20 cm forms a real image 2 times as large as the object. How far apart are the object and the image?

$$2 = \frac{S_i}{S_o} \quad S_i = 40 \text{ cm} \quad S_o = \frac{f}{2} = 10 \text{ cm} \quad D = 40 \text{ cm} + 10 \text{ cm} = 50 \text{ cm}$$

4. A movie theater projectionist must know how to select the proper projection lens to fill the screen with an image of the movie film. The picture on the movie film is 3.4 cm wide. In a particular theater the screen is 10 m wide and is located 30 m from the projector.

$$H_o = 3.4 \text{ cm}$$

$$H_i = 10 \text{ m}$$

$$S_i = 30 \text{ m}$$

- (3) a. What magnification is needed to fill the projection screen?

- (3) b. What object distance is needed to produce this magnification?

- (3) c. What should be the focal length of the lens?

$$(a) M = \frac{H_i}{H_o} = \frac{10 \text{ m}}{3.4 \text{ cm}} \times \frac{10^2 \text{ cm}}{1 \text{ m}} = 294.1$$

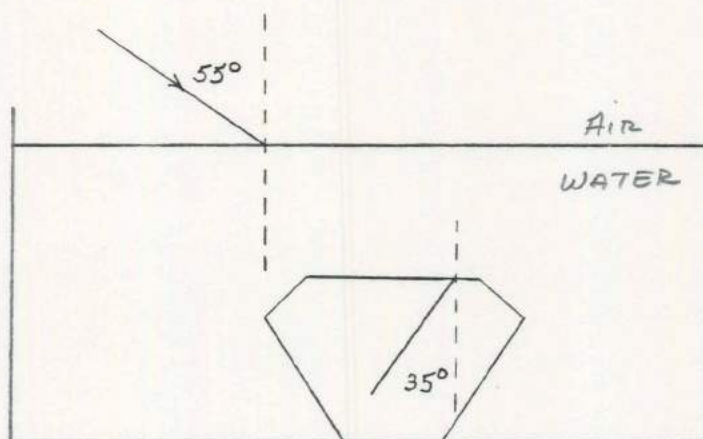
$$(c) \frac{f}{S_i} = \frac{H_o}{H_i} \quad f = \frac{H_o}{H_i} \cdot S_i = \frac{H_o}{H_i} (30 \text{ m} - f)$$

$$f = .0034 (30 \text{ m} - f)$$

$$(b) \frac{H_o}{H_i} = \frac{S_o}{f} \quad S_o = \frac{H_o}{H_i} \times f = \frac{3.4 \text{ cm}}{10 \text{ m}} \times \frac{10^2 \text{ cm}}{1 \text{ m}} = 0.035 \text{ cm}$$

$$.9966 f = .102 \text{ m} \\ = .102 \text{ m} = 10.2 \text{ cm}$$

1. A diamond is emersed in a container of water as shown in the drawing below. Light is incident on the water surface at an angle of 55° . The index of refraction is 1.33 for water, and 2.42 for a diamond.
- What is the refracted angle of the light ray in the water?
 - The light ray in the water eventually hits the water-diamond surface. What is the angle between the light ray which penetrates the diamond and the normal to the water-diamond surface?
 - Part of the light beam which hits the water-diamond surface is reflected up through the water again. Trace this ray up through the water and out into the air labeling each angle of incidence and refraction.



- A light ray is projected upward through the diamond at an angle of 35° relative to the normal. What would be the angle between the light ray which penetrates the water and the normal to the water-diamond surface?
2. An object 6 mm high is 20 cm to the left of a converging lens of focal length of 8 cm.
- Where is the image?
 - Is it real or virtual?
 - Erect or inverted?
 - How large is it?
3. A double convex lens of focal length 20 cm forms a real image 4 times as large as the object. How far apart are the object and the image?
4. A movie theater projectionist must know how to select the proper projection lens to fill the screen with an image of the movie film. The picture on the movie film is 3.4 cm wide. In a particular theater the screen is 10 m wide and is located 30 m from the projector.
- What magnification is needed to fill the projection screen?
 - What object distance is needed to produce this magnification?
 - What should be the focal length of the lens?

1. A diamond is emersed in a container of water as shown in the drawing below. Light is incident on the water surface at an angle of 55° . The index of refraction is 1.33 for water, and 2.42 for a diamond.
- 3 a. What is the refracted angle of the light ray in the water? 38.02°
- 3 b. The light ray in the water eventually hits the water-diamond surface. What is the angle between the light ray which penetrates the diamond and the normal to the water-diamond surface? 19.78°
- 7 c. Part of the light beam which hits the water-diamond surface is reflected up through the water again. Trace this ray up through the water and out into the air labeling each angle of incidence and refraction.

(16)

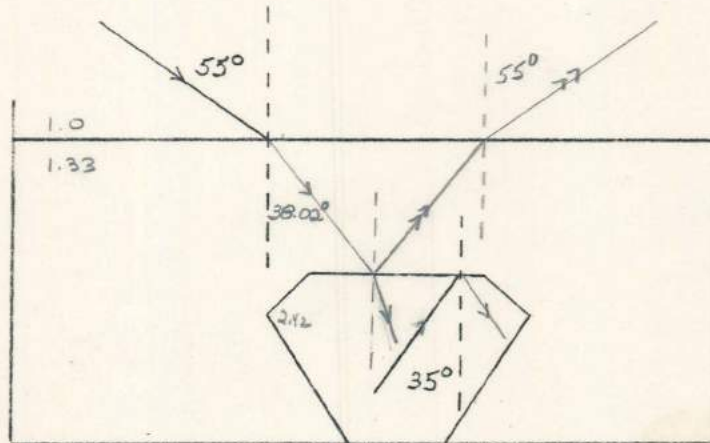
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.0 \sin 55^\circ = 1.33 \sin \theta_2$$

$$\theta_2 = 38.02^\circ$$

$$1.0 \sin 55^\circ = 2.42 \sin \theta_3$$

$$\theta_3 = 19.78^\circ$$



$$2.42 \sin 35^\circ = 1.33 \sin \theta_4$$

- 3 d. A light ray is projected upward through the diamond at an angle of 35° relative to the normal. What would be the angle between the light ray which penetrates the water and the normal to the water-diamond surface?

2. An object 6 mm high is 20 cm to the left of a converging lens of focal length of 8 cm.

$H_o = 6 \text{ mm}$
 $S_o = 12 \text{ cm}$
 $f = 8 \text{ cm}$
 $S_i =$

$$S_i = \frac{f^2}{S_o} = \frac{8 \text{ cm} \times 8 \text{ cm}}{12 \text{ cm}} = 5.33 \text{ cm}$$

13.33 cm To Right of Lens

- 3 a. Where is the image?
- 1 b. Is it real or virtual?
- 1 c. Erect or inverted?

- 3 d. How large is it? $M_i = \frac{f H_o}{S_o} = \frac{8 \text{ cm} \times 6 \text{ mm}}{12 \text{ cm}} = 4 \text{ mm}$

3. A double convex lens of focal length 20 cm forms a real image 4 times as large as the object. How far apart are the object and the image?

$f = 20 \text{ cm}$
 $\frac{H_i}{H_o} = 4$
 $4 = \frac{S_i}{f}$ $S_i = 80 \text{ cm}$ $S_o = \frac{f}{4} = 5 \text{ cm}$ $D_T = 80 \text{ cm} + 5 \text{ cm} + 40 \text{ cm} = 125 \text{ cm}$

4. A movie theater projectionist must know how to select the proper projection lens to fill the screen with an image of the movie film. The picture on the movie film is 3.4 cm wide. In a particular theater the screen is 10 m wide and is located 30 m from the projector.

- 3 a. What magnification is needed to fill the projection screen?
- 3 b. What object distance is needed to produce this magnification?
- 3 c. What should be the focal length of the lens?

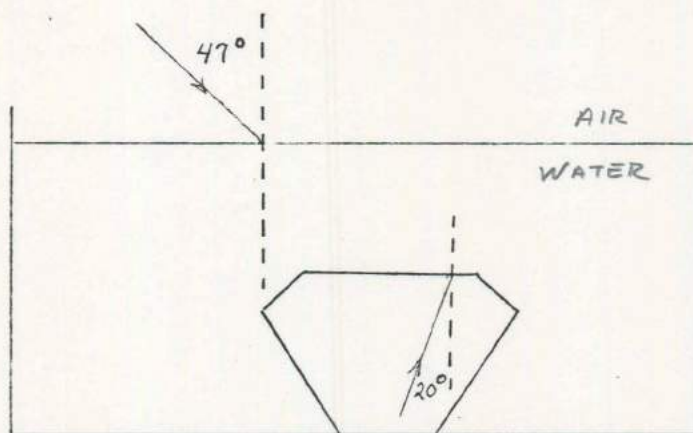
(a) $M = \frac{H_i}{H_o} = \frac{10 \text{ m}}{3.4 \text{ cm}} \times \frac{10^2 \text{ cm}}{1 \text{ m}} = 294.1$

$$\frac{f}{S_i} = \frac{H_o}{H_i} \quad f = \frac{H_o}{H_i} \cdot S_i = \frac{3.4 \text{ cm}}{10 \text{ m}} \times 30 \text{ m} = 10.2 \text{ cm}$$

(b) $\frac{H_o}{H_i} = \frac{S_o}{f}$ $S_o = \frac{H_o}{H_i} \cdot f = \frac{3.4 \text{ cm}}{10 \text{ m}} \times 10.2 \text{ cm} \times \frac{1 \text{ m}}{10^2 \text{ cm}} = 0.035 \text{ cm}$

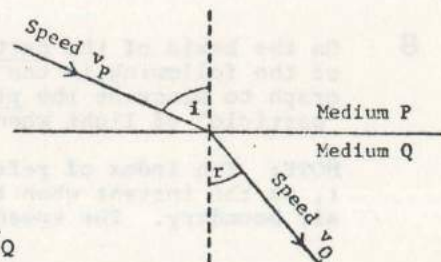
$S_o = 0.035 \text{ cm}$

1. A diamond is emersed in a container of water as shown in the drawing below. Light is incident on the water surface at an angle of 47° . The index of refraction is 1.33 for water, and 2.42 for a diamond.
 - a. What is the refracted angle of the light ray in the water?
 - b. The light ray in the water eventually hits the water-diamond surface. What is the angle between the light ray which penetrates the diamond and the normal to the water-diamond surface?
 - c. Part of the light beam which hits the water-diamond surface is reflected up through the water again. Trace this ray up through the water and out into the air labeling each angle of incidence and refraction.



- d. A light ray is projected upward through the diamond at an angle of 20° relative to the normal. What would be the angle between the light ray which penetrates the water and the normal to the water-diamond surface?
2. An object 6 mm high is 22 cm to the left of a converging lens of focal length of 8 cm.
 - a. Where is the image?
 - b. Is it real or virtual?
 - c. Erect or inverted?
 - d. How large is it?
3. A double convex lens of focal length 20 cm forms a real image $\frac{1}{2}$ times as large as the object. How far apart are the object and the image?
4. A movie theater projectionist must know how to select the proper projection lens to fill the screen with an image of the movie film. The picture on the movie film is 3.4 cm wide. In a particular theater the screen is 10 m wide and is located 30 m from the projector.
 - a. What magnification is needed to fill the projection screen?
 - b. What object distance is needed to produce this magnification?
 - c. What should be the focal length of the lens?

- 1 A ray of light travels from medium P into medium Q and is refracted toward the normal.



Consider the following statements:

- I. $\sin i / \sin r = v_P / v_Q$
- II. $\sin i / \sin r = v_Q / v_P$
- III. $v_P > v_Q$
- IV. $v_Q > v_P$

Which combination of the above statements is true according to the particle model?

- (A) I and III only
- (B) II and IV only
- (C) I and IV only
- (D) II and III only
- (E) None of these.

- 2 A small radioactive source emits beta particles uniformly in all directions such that the average number passing through a small opening at a distance of 10 cm from the source is 180/s. If the source were moved a further 20 cm from the opening, the number of beta particles passing through the opening each second would be

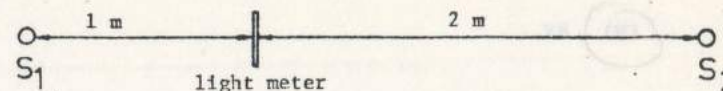
- (A) 20
- (B) 45
- (C) 60
- (D) 90
- (E) 180

1. B 2. A 3. E 4. A 5. E 6. B 7. B
8. E 9. B

Key
1

- 3 The intensity of illumination from a point source varies directly as the power of the source and inversely as the square of the distance from the source.

Two small light sources, S_1 of 20 W and S_2 of 10 W are placed 3 m apart. A light meter is placed one metre from S_1 on the line joining $S_1 S_2$. The light meter is turned so that it first faces S_1 and then S_2 . The intensities I_1 and I_2 of light falling on the meter are measured.



The ratio of the intensities $I_1 : I_2$ will be

- (A) 1:4
- (B) 1:2
- (C) 1:1
- (D) 4:1
- (E) 8:1

- 4 The particle model of light requires some or all of the following assumptions:

- I. The particles are small in size compared to the spacing between them.
- II. The particles have very small mass.
- III. The particles are travelling very quickly.

A beam of red light is observed to pass through a beam of blue light. In order to explain this observation, which of the above assumptions is/are required?

- (A) I only
- (B) III only
- (C) I and II only
- (D) I and III only
- (E) I, II and III

The intensity of illumination is observed to be Y units at a distance d from a source. What is the intensity at a distance $0.5d$ if the power of the source is doubled?

- (A) $0.5Y$
- (B) Y
- (C) $2Y$
- (D) $4Y$
- (E) $8Y$

- 6 A light beam is observed to travel in a straight line from a source A to a screen B 2.0 m away. A small sphere projected from A toward B at a speed of 20 m/s follows a parabolic path and strikes the screen below B. If we are to adopt a particle model for light, then on the basis of these observations only it must be concluded that

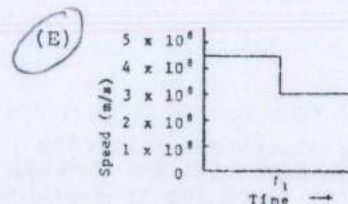
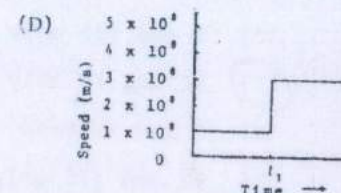
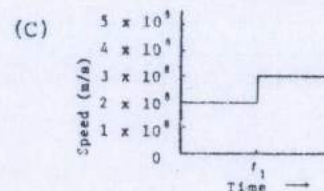
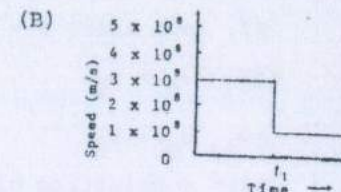
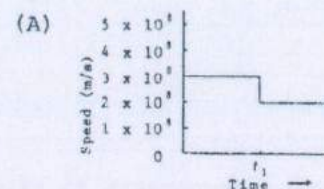
- (A) the particles have very small mass
- (B) the particles are travelling very much faster than 20 m/s
- (C) the particles have very small momentum
- (D) the particles have a very large energy compared to that of the sphere
- (E) a particle model is unsatisfactory

- 7 A simple particle model for light can account for all of the following phenomena but one. Which one is not accounted for?

- (A) The intensity of light from a point source varying inversely as the square of the distance from the source.
- (B) Color effects in thin soap films.
- (C) Radiation pressure.
- (D) Snell's law of refraction.
- (E) The formation of multiple images by two plane mirrors.

- 8 On the basis of the particle model for light, which of the following is the most suitable speed-time graph to describe the predicted behavior of a "particle" of light when passing from GLASS to AIR?

NOTE: The index of refraction of glass is 1.5 and t_1 is the instant when the particle crosses the glass-air boundary. The speed of light in air is 3×10^8 m/s.



- 9 Two identical lamps, X and Y, are located at the centres of two hollow spheres. The sphere around X has a radius of 300 cm and the sphere around Y has a radius of 200 cm. For a fixed time interval, what is the ratio of the total amount of light striking sphere X to the total amount of light striking sphere Y?

- (A) 2:3
- (B) 1:1
- (C) 3:2
- (D) 9:4
- (E) 4:9