Chapter 18 REFLECTIONS AND IMAGES

You now should be quite familiar with the general character and properties of light, but this sort of knowledge is only part of what physics is about. Physicists strive for a deeper understanding of the physical world than simple description will provide, and they seek to construct more abstract models of physical phenomena to obtain this deeper understanding. A model enables them to begin answering the more fundamental questions such as:

Why does light behave this way?
How will light behave in situations we cannot observe directly?
What is light composed of?

This sequence of chapters on optics will introduce you to almost all the properties of light, but as these properties are introduced you will see that we are also developing a model for light which can answer these more fundamental questions. This chapter introduces you in more detail to the phenomenon of reflection; in particular to the behavior of light when it strikes two common non-diffuse reflectors; plane and parabolic mirrors. In this chapter we will introduce a mathematical description of reflection which enables us to make predictions without actually running an experiment. You should also have the opportunity to verify the validity of these mathematical predictions in the laboratory.

Through an understanding of reFLEction obtained in this chapter, and an understanding of reFRAction obtained in the next chapter, we will be in a position to make a first attempt at a model for the behavior of light.

PERFORMANCE OBJECTIVES

After completing this chapter, you should:

1. be able to describe the laws of reflection.

2. be able to locate images in plane mirrors.

3. be able to demonstrate image formation by concave and convex mirrors.

4. be able to experimentally determine the focal length of a concave mirror.

5. be able to use ray diagrams to show image location.

6. be able to mathematically solve the following relationships given all but one variable:

$$\frac{H_i}{H_o} = \frac{S_i}{f} \qquad \frac{H_i}{H_o} = \frac{f}{S_o} \qquad S_o\,S_i = f^2$$

7. be able to identify an object as either real or virtual.

1. When you look into a mirror, what do you see? Where is what you see (the image) located? Where do you think the image is? Formulate an answer before proceeding further.

2. Investigation: REFLECTION FROM PLANE MIRRORS (Notes provided.)

   a. Read: THE LIGHT BOX AND OPTICAL SET (Enclosed.)

   b. A written conclusion of your findings is to be submitted along with any and all experimental evidence. Use a format that communicates in a clear, concise way. Each student is to have their own drawings.

   c. Note...The following study notes are provided for your inspection:

      I. Where is the Image in a Plane Mirror? page 1
      II. Finding the Image Using a Scale Drawing page 1-3
      Images in a Plane Mirror page 4

3. Read: Section 18-1 The Laws of Reflection page 371
   Section 18-2 Images In Plane Mirrors page 373

4. Problems: page 373: #1 #2 #4
   page 388: #22 #23
   page 375: #6 #7

5. Obtain 3 meter sticks. Then assist your instructor in examining the law of reflection in 3-D.

6. Obtain a multiple image apparatus and see what happens when the angle between the two mirrors is changed.

   See enclosed sheet titled: Multiple-Image Demonstrator

7. Optional...Two problems #12 and #13 (from 2nd Edition) have been included for your examination and solving.

8. Complete Quiz-Chapter 18 which is enclosed. Then have evaluated.

9. Investigation: REFLECTION FROM CURVED MIRRORS (Notes provided.)

   a. A written conclusion plus experimental evidence is to be presented for evaluation by <u>EACH</u> student. Use a format that communicates in a clear, concise way.

10. Read: Section 18-3 Parabolic Mirrors page 376
    Section 18-4 Astronomical Telescopes page 379

11. Problems: page 378: #9
    page 388: #24

12. Read: Section 18-5 Images and Illusions page 381 through first full paragraph on page 383.

Examine Figure 18-18, page 382

a. What do each of the following represent?

1. H_o ____.
2. H_i ____.
3. S_o ____.
4. S_i ____.
5. F ____.
6. f ____.

b. Where is the principle axis?

c. Where is the vertex of the mirror?

d. Two rays are drawn from the top of the object. Discuss with your partner(s) why both rays reflect as they do.

e. Can you think of one other ray drawn from the top of the object to the mirror for which you can easily predict where it will go?

POINT OF INFORMATION - The text considers the reflection to take place at a parabolic surface. In this situation, all incoming rays that are parallel to the principle axis will be reflected through the principle focus. Some texts such as the blue text consider the reflection surface to be circular (in 2-D) or spherical (in 3-D). When the mirror is spherical in shape, not all incoming rays parallel to the principle axis are reflected through the principle focus. Read Section 18, page 328 (of blue text) titled "Spherical Aberration" if you wish to know more.

13. Experiment FOCAL LENGTH OF A CONCAVE MIRROR (Notes provided.)

14. Experiment IMAGES FORMED BY A CONCAVE MIRROR (Notes provided.)

15. Examine your image using a convex mirror. Now, as you walk around during the next few days, look at your reflection in shiny objects such as car bumpers, toasters, etc. See if you can figure out for each situation whether the surface acts like a flat, a concave, or a convex mirror, or a combination of two or more.

16. Read: Section 18-5 Images and Illusions page 383
Section 18-6 Real and Virtual Images page 386

17. Problems:

page 386:	#11	#12	#13	#14	#15	#16	
page 387:	#17	#18	#19				
page 388:	#25	#26	#27	#28	#29	#30	#32

18. Examine Fig. 18-19, page 383 and Fig. 18-20 page 384. Verify that:

a. $H_i/H_o = f/S_o$

b. $H_i/H_o = S_i/f$

c. Having trouble? As sheet titled VERIFICATION OF LENS FORMULAE is provided for assistance. In each case, from where are S_o and S_i measured?

d. Can you use the two formula in (a) and (b) to get: So x Si = f^2?

19. Complete the worksheet titled IMAGE CONSTRUCTION USING CURVED MIRRORS.

a. Draw appropriate ray diagrams and locate the image in each case.

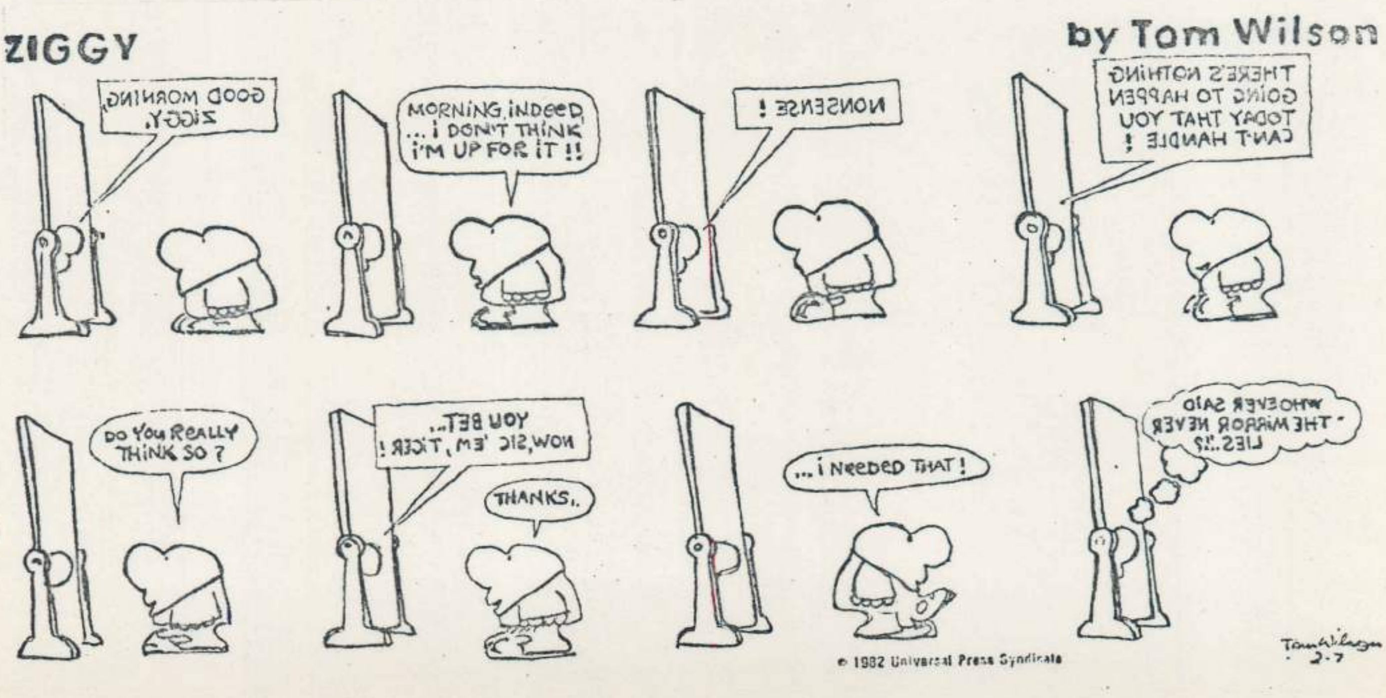
b. Identify each object as real or virtual.

c. Identify whether the image is larger or smaller than the object.

d. Can you see a pattern developing in the first 5 cases.

e. Have instructor evaluate the material when completed.

20. Complete written exercise and then have it evaluated.



4. (1) angle i is 3   angle r is 2
   (2) infinite number   (4) S.A.B
   (22) Yes, light paths are reversible
   (23) (a)(b) twice height of mirror (c) no (d) 75 cm from floor
   (6) S.A.B.   (7) 20 cm

7. (12) no   (13) 5.3 cm S.A.B.

11. (9) (a) 21.0 mm (Fig 18-11) 12.0 mm (Fig 18-12)
    (b) flatter mirror, longer f
   (24) 4 times

12. (a) (1) height of the object
   (2) height of image
   (3) distance object is from principle focus (F)
   (4) distance image is from principle focus (F)
   (5) location of principle focus
   (6) focal length
   (b) goes through principle focus (F) to vertex of mirror
   (c) is geometric center of mirror
   (d) both obey the law of reflection
   (e) one drawn from object to vertex of mirror to ________

17. (11) S.A.B.   (12) no   (13) 5.3 cm S.A.B.
   (14) 20 cm   (15) 17 cm   (16) multiplied by 4
   (17) yes, whenever $S_o < f$ (18) no   (19) yes
   (25) (26) (27) (28) (29) (30) (32) S.A.B.

Questions from THE FLYING CIRCUS OF PHYSICAL PHENOMENA by Jearl Walker

1. How does a mirror reflect light? How does the reconstructed beam know to come out at that angle?

2. If you look below the sea horizon you will see reflections of objects which are 30 degrees above the horizon. Why? Does this mean that the light is reflected off an average sea slope of 15 degrees? Try to see this reflection and then see how often you get waves with slopes of 15 degrees.

3. Do look at the moon reflected in a rippled water surface at night, for example, while standing on a pier or boat. There will be a luminous triangle in the water. If the sky is slightly hazy you will also see a dark triangle above the horizon. What causes these triangles?

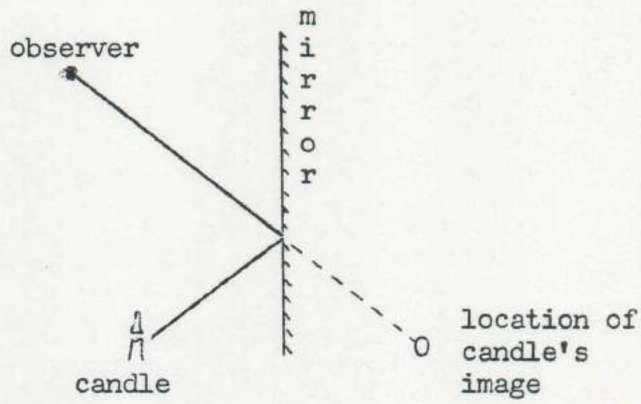
4. Often you can see the moon and the sun in the sky at the same time. If the moon is in an easily-seen crescent, mentally draw a line along the symmetry axis of the crescent. Shouldn't this line point directly to the sun? Does it?

5. When you get a suntan, what exactly happens to your skin? Can you get just as good a suntan on cloudy days as on sunny days?

Chapter 18 STUDY NOTES

I. Where is the Image in a Plane Mirror?

When you see the image in a plane mirror, it is clear that the object itself is not actually where it appears to be. The object is somewhere else, and the mirror is fooling you about the object's true location by reflecting the light which eventually reaches your eyes. Nonetheless, we refer to the location of the object's image as that place where the object appears to be as viewed in the mirror, independent of where the object really is. In a simple example of a candle, a mirror, and an observer, the location of the candle's image is as shown below. Even though the light from the candle never reaches the location of its image, the canlde appears to be at point O, and point O is therefore the location of the candle's image. Since the light from the candle never reaches the location of the candle's image, its image is what we call "virtual". We can't place a camera at point O and record a picture of the candle. On the other hand, parabolic mirrors can form "real" images. In the case of real images, reflected light actually passes through the location of the image.



II. Finding the Image Using a Scale Drawing

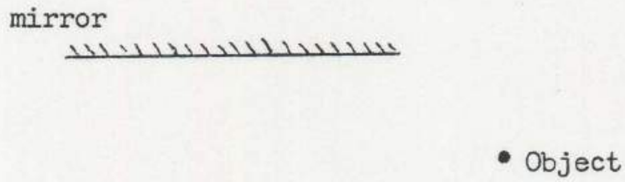
The text describes the evidence we have that light is always reflected from a smooth surface so that <u>the angle of reflection is equal to the angle of incidence</u>. This law of reflection leads us to develop a straightforward way of locating the image of an object which can be seen in a plane mirror. We need to remember three things:

a. Light travels in straight lines. (Note: Occasionally we will encounter a light path which is not straight, but that will be because the material through which it passes is not of constant thickness or of a uniform density.)

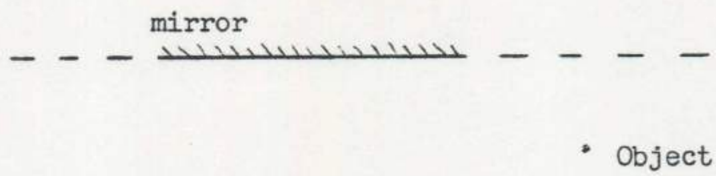
b. A straight line which is perpendicular to the plane of the mirror <u>and</u> passes through the position of the object will also pass through the position of the image.

c. The position of the image is as far behind the plane of the mirror as the object is in front of the mirror.

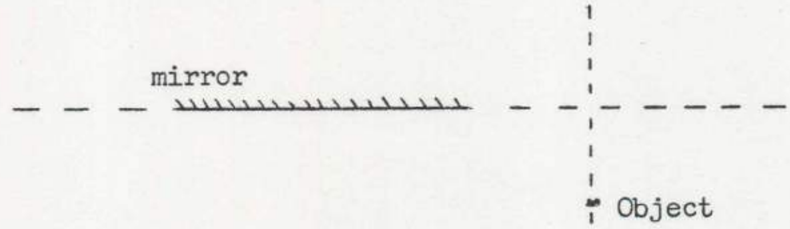
Using these three items, here is a recipe for finding the image of an object in a plane mirror using a scale drawing. We start with a scale diagram showing the positions of the object and the mirror.



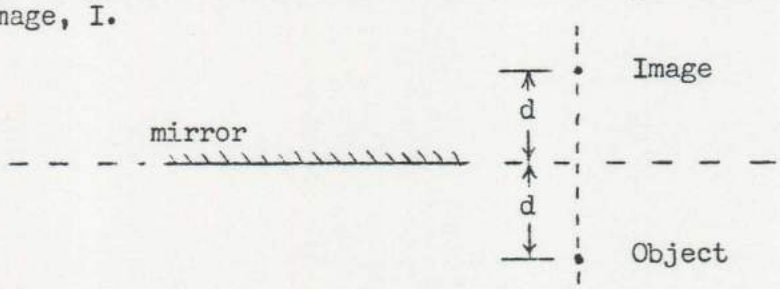
<u>Step 1.</u> If the object is not in front of the mirror (and here it is not), then draw a line along the plane of the mirror long enough so the object is in front of the line. (Shown on the next page.)



Step 2. Construct a second line which is perpendicular to the plane of the mirror and also passes through the object O (Use a protractor or drawing triangle.) Extend this line behind the plane of the mirror.

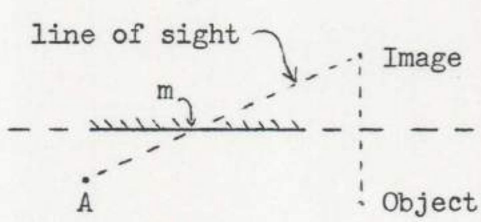


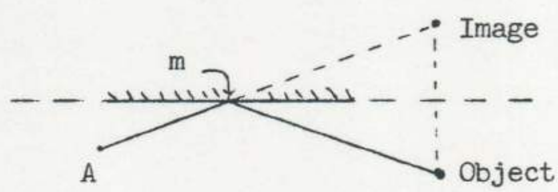
Step 3. Measure the distance d between the object O and the plane mirror. Then mark a position along the perpendicular line which is the same distance d behind the plane of the mirror. That marked position is the position of the image, I.



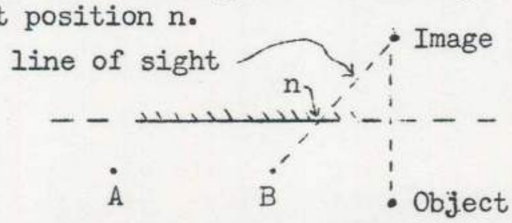
Now, as long as the object does not move, the position of the image will stay put, no matter from where we look into the mirror.

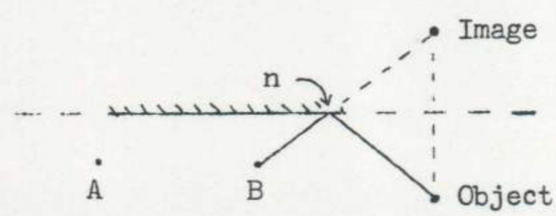
For instance, if we look into the mirror from point A, we see the image by looking at the center of the mirror. The light from the object which reactes point A must reflect from the mirror at point m and follow a straight line to point A.



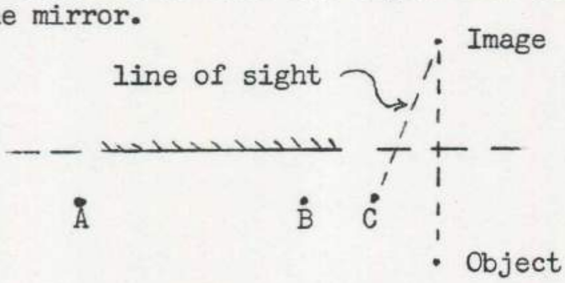


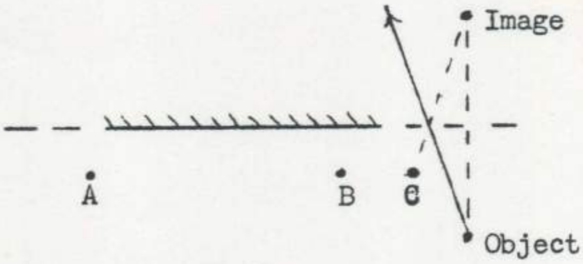
If we look into the mirror from point B, we see the image by looking at the right edge of the mirror. The light from the object which reaches point B is reflected by the mirror at position n.





Finally, if we look into the mirror from point C, we find that the image cannot be observed because the light from the object cannot reach point C after hitting the mirror.





III. Alternative Formulas for Concave Mirrors

As given in the text, the relation between object distance from the focal point, image distance from the focal point, and the focal length is:

$$S_o S_i = f^2 \qquad (1)$$

In some books on this subject you may see another formula stated where the distances of an object and its image are measured from the <u>center of the mirror</u> instead of the principal focus. This formula is:

$$1/f = 1/D_o + 1/D_i \qquad (2)$$

Comparing this with our previous notation, the object distance is given by $D_o = S_o + f$ and the image distance is given by $D_i = S_i + f$.

These two formulae (1) and (2) are equally accurate. Indeed one can be derived from the other. Let's start with:

$$S_o S_i = f^2$$

Since $D_o = S_o + f$ and $D_i = S_i + f$, we can write:

$$S_o = D_o - f \quad \text{and} \quad S_i = D_i - f$$

Substituting these expressions for S_o and S_i into the first formula, we get:

$$(D_o - f)(D_i - f) = f^2$$

Multiplying this out:

$$D_oD_i - fD_i - fD_o + f^2 = f^2$$

$$D_oD_i - fD_i - fD_o = 0$$

Now if we divide both sides of the equation by D_oD_if, we get:

$$\frac{D_oD_i}{D_oD_if} - \frac{fD_i}{D_oD_if} - \frac{fD_o}{D_oD_if} = 0$$

$$\frac{1}{f} - \frac{1}{D_o} - \frac{1}{D_i} = 0 \quad \text{or} \quad \frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$$

So if and when you come across formula (2), remember its equivalence to what we are using in this chapter, the only difference being how we write down the various distances.

To study the image formed in a plane mirror

A straight line MM' is drawn across the centre of a sheet of drawing paper to represent a reflecting surface and a large letter E to serve as an object (Fig. 230). A strip of plane mirror is then stood vertically with its silvered surface over MM' and the image of the object letter E seen in the mirror is found in the following manner. An object pin is stuck into the paper at the various points O_1, O_2, etc., in turn on the object letter, and each time the images, I_1, I_2, etc., are located by the method of no-parallax, using a search pin as described in the previous section. When adjusting the search pin to coincidence with the image it is useful to keep in mind that, of the two, *the further one moves with the eye.*

Suitable measurements should now be taken on the diagram and observations recorded in the notebook to verify the following facts.

The image in a plane mirror is:

(1) *The same size as the object.*
(2) *The same distance behind the mirror as the object is in front.*
(3) *Laterally inverted.*
(4) *Virtual (it cannot be formed on a screen).*

Finally we see from this experiment that the line joining any point on the object to its corresponding point on the image cuts the mirror at right angles. It is important to remember this, as we use it when making graphical constructions of images in plane mirrors.

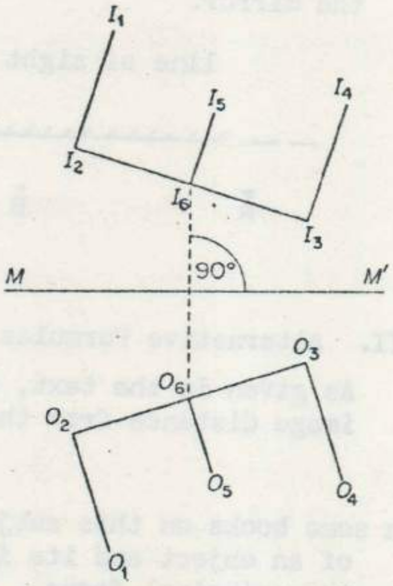


Fig. 230. Image in a plane mirror

Looking into a plane mirror

We were, of course, already familiar with some of the above facts from our everyday experience with mirrors. Looking into a mirror, we see an image of the face situated apparently behind the mirror. If we now move backwards the image will recede so that it is always the

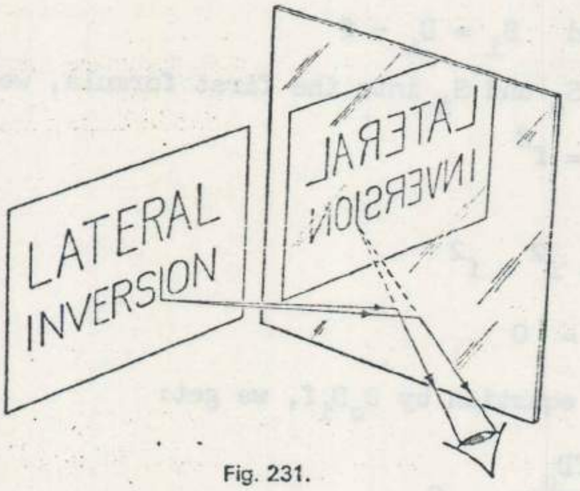


Fig. 231.

same distance behind the mirror as the object is in front. Unlike the images formed on a screen by a projector, which are said to be *real* in the sense that they are formed by the actual intersection of real rays, the image we see in the mirror cannot be formed on a screen. It is said to be *virtual* and is produced at the place *where the reflected rays appear to intersect* when their directions are produced backwards behind the mirror. This is further explained in the next paragraph. It is also to be noticed that the left ear of the image is formed from our own right ear as object. This effect is called *lateral inversion*: it is even more strikingly demonstrated when we look at the image of a printed page in a mirror (Fig. 231).

Parallax

When we look out of the window of a moving train, trees, chimneys, towers and other objects in the landscape appear to be moving relatively to each other. Thus, at one moment a tree may appear to be to the right of a church spire and a few seconds later to the left of it. Their actual positions in space have, of course, remained fixed. This apparent relative movement of two objects owing to a movement on the part of the observer is called *parallax*.

No parallax is observed between the cross surmounting a spire and the spire itself, since these two objects coincide in position.

To locate images by no-parallax

The method of no-parallax is frequently used in light experiments to locate the positions of the images of pins (Fig. 229). The eye is

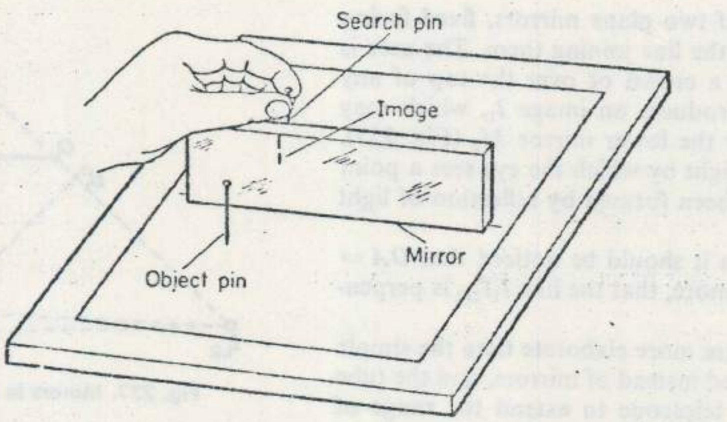


Fig. 229. Finding image position by no-parallax

moved from side to side while viewing a *search* pin placed in the neighbourhood of the image. A position is found for which both pin and image appear to coincide in the same straight line. When this condition of no-parallax holds the search pin gives the position of the image.

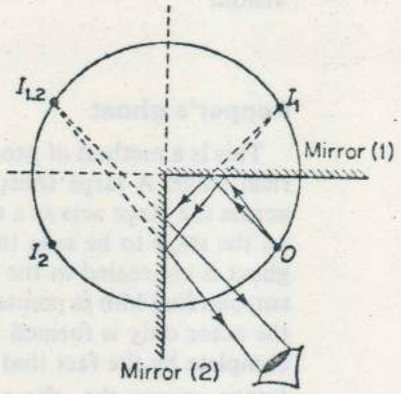


Fig. 233. Reflection from two mirrors at 90°

Images formed in two mirrors inclined at 90°

When two mirrors are inclined at right angles we have not only the images I_1 and I_2 formed by a single reflection but in addition two extra images produced by *two reflections*. The pencil of light by which the eye sees one of these, $I_{1\cdot2}$, is shown in Fig. 233. The subscript $_{1\cdot2}$ in the symbol $I_{1\cdot2}$ signifies the order in which the reflections take place from the mirrors 1 and 2.

The other image $I_{2\cdot1}$ may be seen by looking into mirror 1. Actually the images $I_{1\cdot2}$ and $I_{2\cdot1}$ are superimposed on one another.

The images I_1 and I_2 themselves act as objects for the formation of images $I_{1\cdot2}$ and $I_{2\cdot1}$, and the positions of these images are found in the usual way, i.e., a perpendicular to the mirror is drawn through the object and the object and image distances from the mirror are made equal.

Geometrically, the object and all the images lie on a circle whose centre is at the intersections of the mirrors. It is useful to remember this when drawing ray diagrams.

Fig. 234 shows, in perspective, the pencil of rays by which a point on the image $I_{2\cdot1}$ is seen.

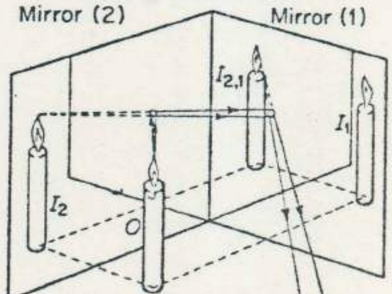


Fig. 234. Images formed by two mirrors at 90°

The kaleidoscope

The kaleidoscope consists of two strips of plane mirror M_1 and M_2 about 15 cm long, placed at an angle of 60° inside a tube (Fig. 235). At the bottom of the tube is a ground-glass plate to admit light, on which is scattered small pieces of brightly coloured glass. These pieces of coloured glass act as objects, and on looking down the tube five images are seen, which together with the object form a symmetrical pattern in six sectors.

This may be compared with Fig. 233, where the mirrors are at 90° and there are only three image spaces.

The number of different patterns obtained is unlimited, as a fresh one is produced every time the tube is shaken to rearrange the pieces.

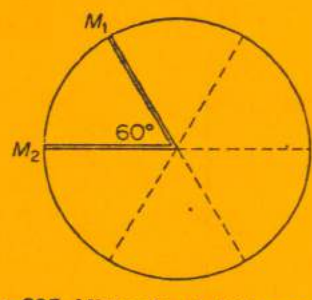


Fig. 235. Mirrors in a kaleidoscope

The periscope

The simple periscope consists of two plane mirrors, fixed facing one another at an angle of 45° to the line joining them. The user is enabled to see over the heads of a crowd or over the top of any obstacle. The upper mirror M_1 produces an image I_1, which may then be regarded as an object for the lower mirror M_2 (Fig. 237). The diagram shows the pencil of light by which the eye sees a point on the final image I_{12}, which has been formed by reflection of light from each mirror in turn.

When constructing this diagram it should be noticed that $OA = I_1A$, and $I_1B = I_{1.2}B$, and, furthermore, that the line $I_1I_{1.2}$ is perpendicular to the mirrors.

Periscopes used in submarines are more elaborate than the simple type described here. Prisms are used instead of mirrors, and the tube supporting them incorporates a telescope to extend the range of vision.

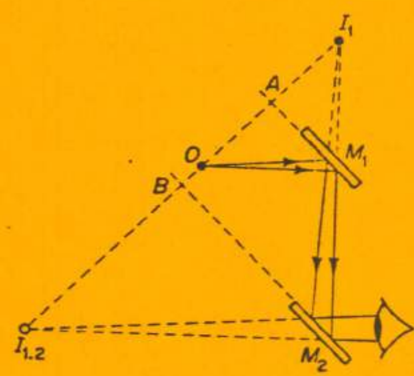
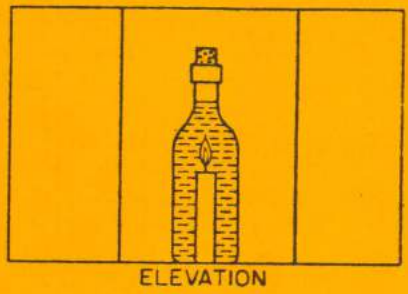


Fig. 237. Mirrors in a periscope

Pepper's ghost

This is a method of producing the illusion of a ghost on the theatrical stage. A large sheet of polished plate glass placed diagonally across the stage acts as a mirror, but at the same time permits objects on the stage to be seen through it. An actor attired to represent the ghost is concealed in the wings and is strongly illuminated. All else surrounding him is painted or draped dull black so that an image of the actor only is formed by the plate glass. The illusion is rendered complete by the fact that light from objects on the stage behind the image causes the ghost to appear transparent. Headless ghosts result simply from enclosing the actor's head in dull black cloth.

A practically identical arrangement causes a candle to appear to be burning inside a bottle full of water. How this may be done is shown in Fig. 238.



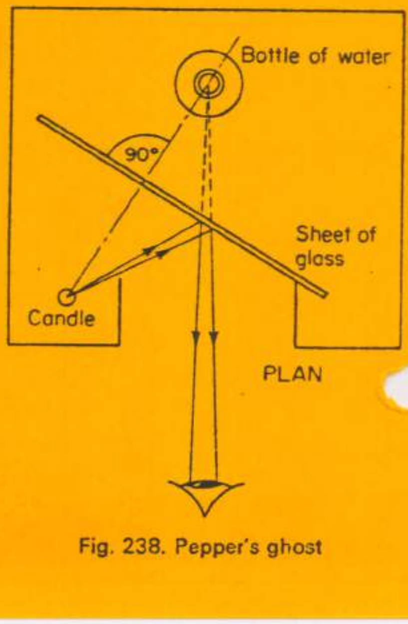


Fig. 238. Pepper's ghost

MULTIPLE-IMAGE DEMONSTRATOR

The multiple-image demonstrator was developed to show the relationship between angular separation of two hinged plane mirrors and the number of images that may be seen in the mirrors.

The multiple-image demonstrator consists of two hinged plane mirrors mounted on a common vertical rotation axis which, in turn, is mounted on a semicircular platform. Each mirror may be moved independently and the angle between the two mirrors may vary from a small angle to 180°. The actual separation may be determined from the semicircular scale at the edge of the platform.

A grey wooden dowel serves as the object whose multiple images are observed in the mirrors. It is mounted on a vertical support, which stands on the line joining the vertical axis to the midpoint of the semicircular arc.

The junction of the two plane mirrors is at the center of an imaginary circle defined by the length of each mirror, the length serving as radius (see Fig. 2). The angular separation Ω divides the imaginary circle into n sectors. Thus

$$n\,\Omega = 360^\circ \qquad (1)$$

$$n = 360^\circ/\Omega \qquad (2)$$

Now all sectors but one contain images. Thus, the number n' of images is one less than the number of sectors:

$$n' = n - 1 \qquad (3)$$

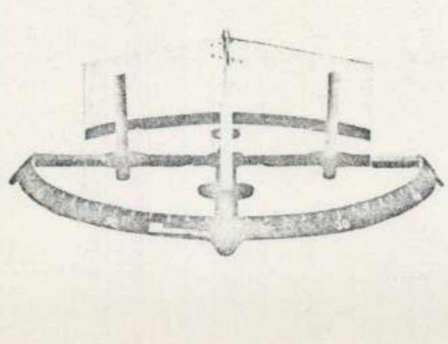
By equations (2) and (3), $$n' = \frac{360^\circ}{\Omega} - 1 \qquad (4)$$

Note that equation (4) works only for those values of Ω which are exact divisors of 360°. For other values of Ω, equation (4) gives only approximate results.

If there is an image directly behind the junction of the mirrors, that image is split. The splitting of the image is indicated in Fig. 2 by the solid line extending from the junction at the center of the imaginary circle.

If the angular separation Ω is chosen so that the object lies on the bisector of Ω, then a split image occurs only when there is an odd number of images.

To test the validity of equation (4), beginning at $\Omega = 180^\circ$, observe and record the number of images for 120°, 90°, 75°, 60°, 45°, 40°, and 36°. The number of images should be 1, 2, 3, 4, 5, 7, 8, and 9, respectively.



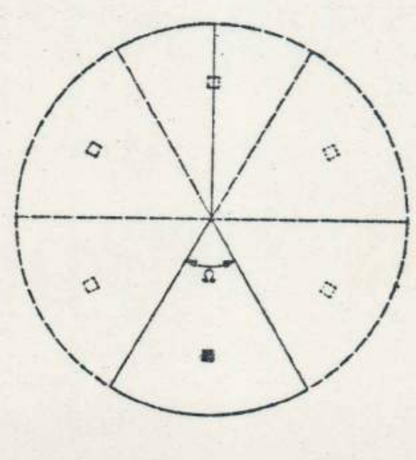
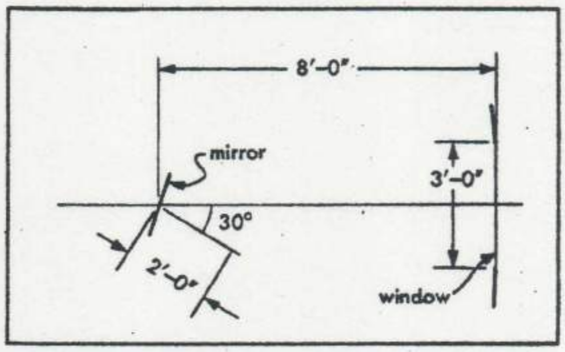


FIG. 2 - THE IMAGINARY CIRCLE DEFINED BY THE HINGED MIRRORS

Chapter 18   Supplement Problems

(for Study Guide Item 7)

12. The rearview mirror of a car is so placed that its upper and lower edges are horizontal and its center is at the same level as the center of the rear window. The driver's eye is also at this level, and the line of sight from his eye to the center of the mirror makes an angle of 30° with the line joining the centers of the mirror and the window. (See Fig. 12-31.) The distance from his eye to the mirror is 2.0 ft., and that from the mirror to the window is 8.0 ft. What is the least width of the mirror that is needed if the entire width (3.0 ft.) of the rear window is to be seen?



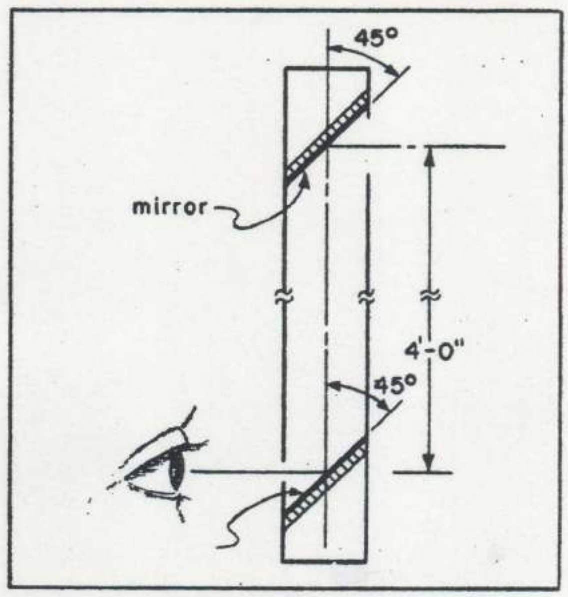
12-31 For Problem 12.

13. A *periscope* can be made by mounting two plane mirrors at the ends of a tube. (Fig. 12-32.) The mirrors face each other, are parallel, and each makes an angle of 45° with the axis of the tube. An eyehole is cut in the tube opposite the center of one mirror, and a larger hole is cut in the other end of the tube, opposite the other mirror. Suppose that the distance between the two mirrors is 4 ft. 0 in., and that you are using the periscope to look over a fence at a man 6 ft. 0 in. tall who is 50 ft. away.

(a) What is the smallest possible height of the hole in the top of the periscope?

(b) What is the smallest possible size of the top mirror?

Hint: Draw rays from the man to the eye, and remember the properties of similar triangles.



12-32 The periscope of Problem 13.

INDEX OF REFRACTION OF CROWN GLASS

∠AIR = 30°		∠GLASS
RED	1.513	19.297°
ORANGE	1.514	19.284°
YELLOW	1.517	19.244°
GREEN	1.519	19.218°
BLUE	1.528	19.100°
VIOLET	1.532	19.049°

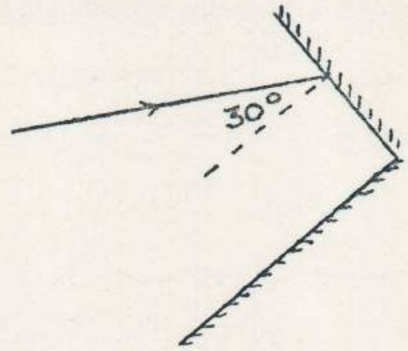
$$\frac{1.532 - 1.513}{1.513} = 1.26\%$$

$$1.0 \sin 30° = 1.513 \sin \theta$$

Chapter 18 QUIZ

Hunters A and B have their rifles pointed as shown. They are aiming at the same deer. However, a bullet reflector and light reflector happens to be in the way. Where is the deer?

2. Two mirrors meet at right angles as shown. A ray of light is incident on one at an angle of 30°. Draw the path of the reflected light ray.



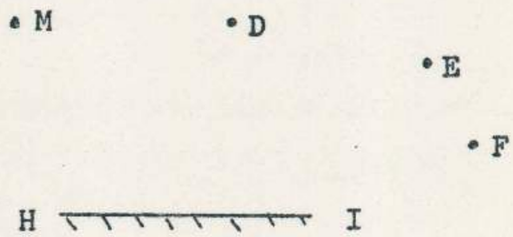
3. A man stands at M, just to one side of a plane mirror HI. (This is a front surfaced plane mirror).

   a. Can he see himself in the mirror?

   b. Can he see object D in the mirror?

   c. Can he see object E in the mirror?

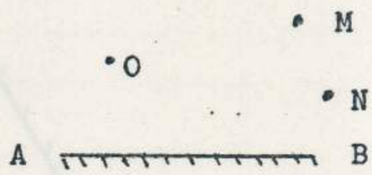
   d. Can he see object F in the morror?



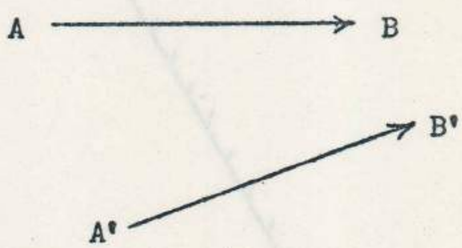
4. a. Find the position of the image of O in the front surface plane mirror AB.

   b. Can a man at M see this image?

   c. When a man at M moves to N, where does the image move?



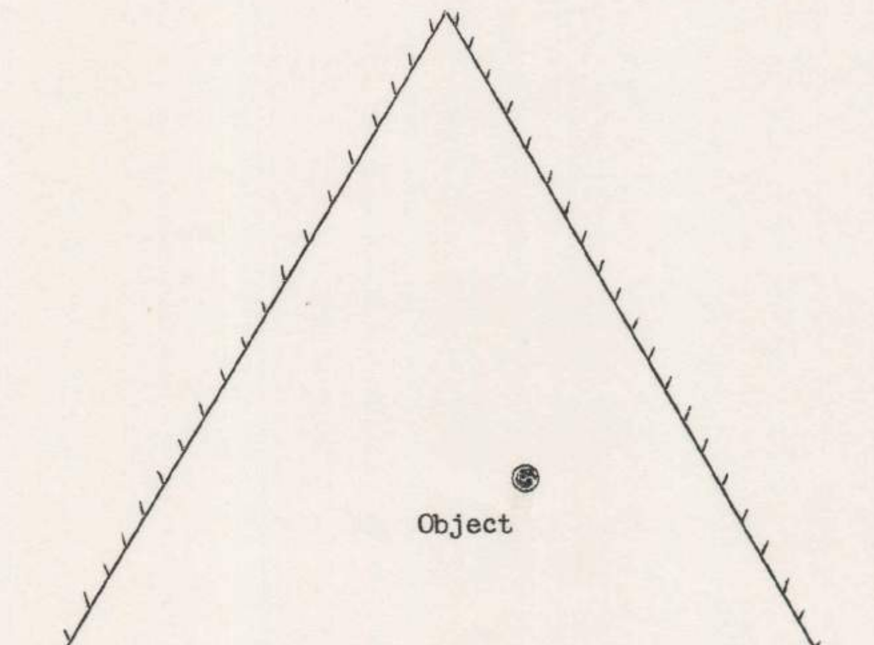
5. Can A'B' be the image of AB in any plane mirror? Prove it.



6. On the worksheet provided, locate the images of the object that you would see when the two mirrors are at an angle of 60°.

7. What is a one way mirror? Be sure to have your instructor show you one when he evaluates this work.

8. Can one use image location to improve ones pool playing?


Object

Investigation: Reflection From Plane Mirrors - Summary

1. Reflection - Single Ray

   a. What is the incident ray? the reflected ray? the normal?

   b. How do we measure the angle of incidence? the angle of reflection?

2. Reflection - Divergent Rays

   a. What is the law of reflection?

   b. What happens to converging, diverging, and parallel reflected rays?

3. Images in a Plane Mirror

   a. What is a focal point?

   b. What was noted about real and reflected rays?

   c. What was noted about real and reflected focal points?

   d. What was location of real and reflected focal points relative to the mirror?

   e. What did the process of parallax verify?

4. Images in a Plane Mirror -2-

   a. Where is the image of a pin?

   b. Where do reflected rays of the pin appear to come from?

   c. When you look into a mirror, where is your image?

5. Multiple Reflections

   a. Why were there multiple reflections?

6. More Multiple Reflections

   a. When a ray of light is reflected from 2 mirrors at 90° where does it go?

   b. What if reflected from 3 mirrors each at 90° to each other?

7. Rotation of a Plane Mirror

   a. If a mirror rotates through 15°, the reflected ray rotates through how many degrees? Explain.

8. Additional

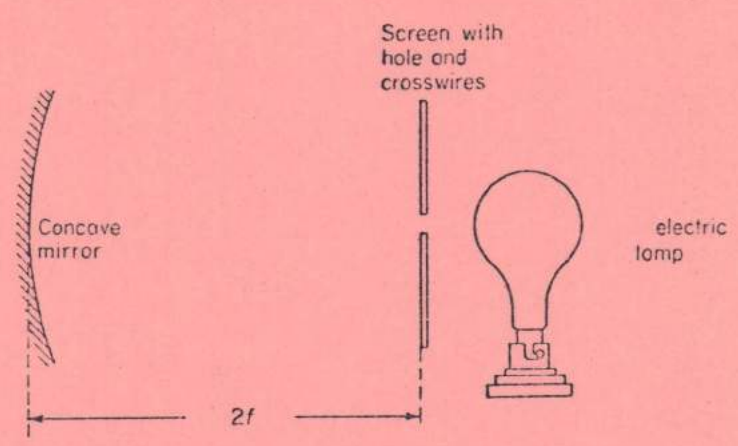
   a. How much of your face can you see with a 4-cm wide mirror?

   b. How is your image related to you, the object?

   c. How large a mirror (length) is needed for all in the class to see from head to toe?

Focal Length of A Concave Mirror

The focal length of a concave mirror can be determined using the set-up as shown. The object used in this experiment consists of a hole cut in a white screen made from cardboard and illuminated behind by an electric lamp. Sharpness of focus is greatly assisted by the use of a thin cross-wire placed across the hole.



The concave mirror, mounted in a holder is moved to and fro in front of the screen until a sharp image of the object is formed on the screen adjacent to the object. [The image should be the same size as the object.] When this has been done, both the object and the image are the same distance from the mirror.

Measure the distance between the screen and the mirror. The distance is ______. [This distance represents twice the focal length of the mirror.] Why this is so will not be discussed at this time.

Thus the focal length of the mirror is __________.

Images Formed by a Concave Mirror

Look at your image in a concave mirror. Is it right side up or upside down? Do the size and position of the image change when you move the mirror towards you or away from you?

Using the student optical bench, (your) concave mirror, mirror holder, and light source, place the mirror at one end of the meter stick and the light source at the other as shown in Figure 1.

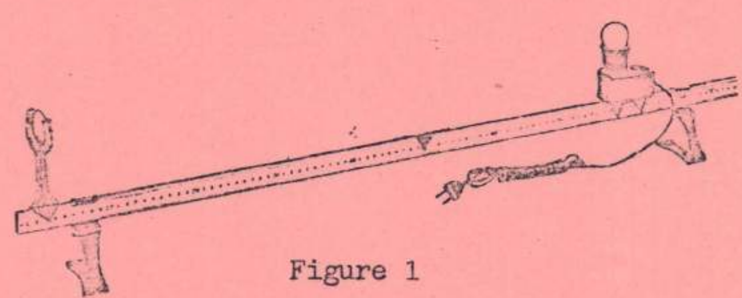


Figure 1

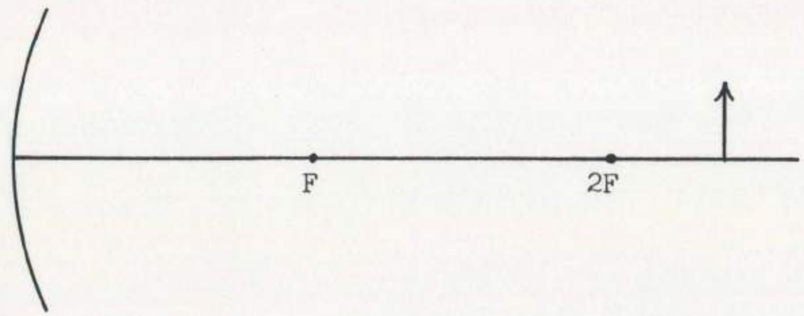
Locate the image using a small cardboard screen. Then record both the position of the object and the image. Next determine the value of both the object distance (S_o) and the image distance (S_i) and record these distances. [Be sure you determine these distances correctly.] Now move the object towards the mirror in small steps, recording and determining the information as previously suggested. Continue until the image moves off the end of the meter stick. How does the change in position of the image compare with that of the object? Where do you expect the image to be when the object is very near the principle focus? Where would you expect the image to be if the object is several meters away from the mirror?

Examine the object distance S_o and the image distance S_i for the various observations. How does S_i relate to S_o? Try plotting S_i as some function of S_o. [Your choice.] Is the graph a straight line passing through the origin? If not, plot S_i as a different function of S_o. Continue until the graph is a straight line through the origin. Once you have this graph, determine the mathematical relation between S_i and S_o.

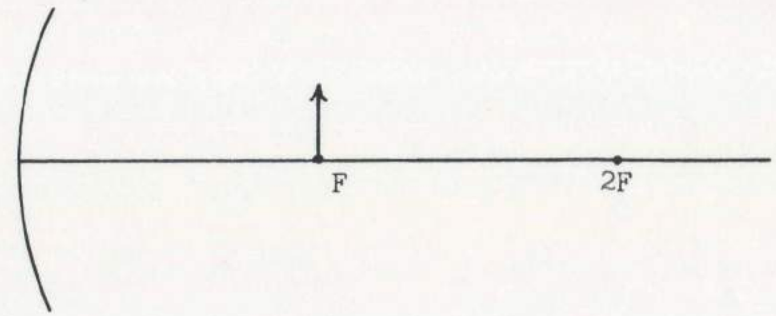
A formal laboratory write-up is expected.

Chapter 18 Image Construction using Curved Mirrors

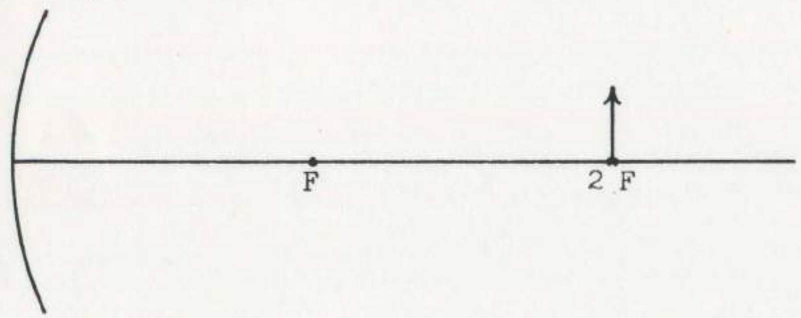
1. Object beyond 2F



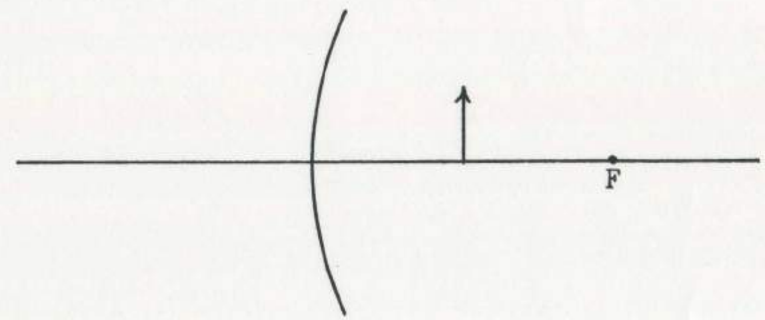
4. Object at F



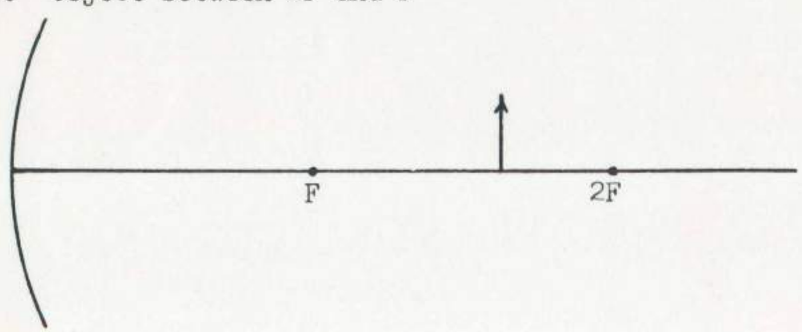
2. Object at 2F



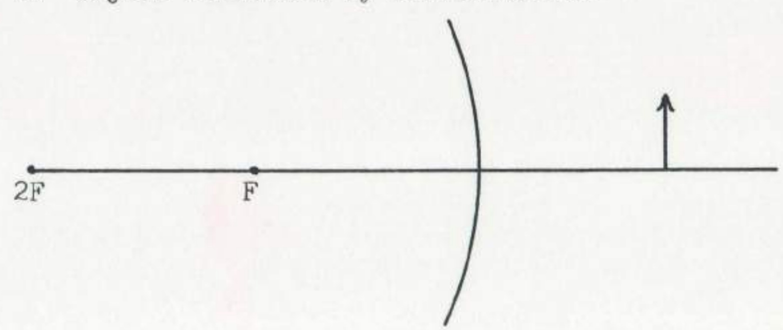
5. Object between F and mirror



3. Object between 2F and F

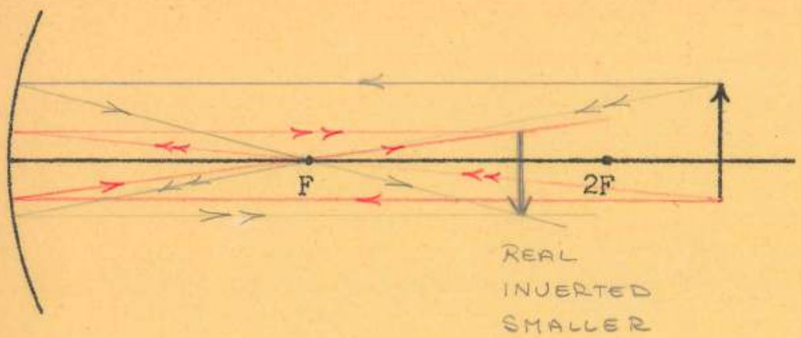


6. Object reflected by Convex mirror

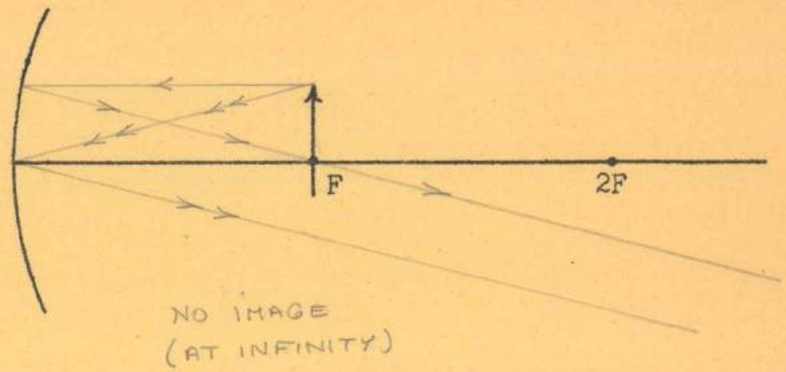


Chapter 2 Image Construction using Curved Mirrors

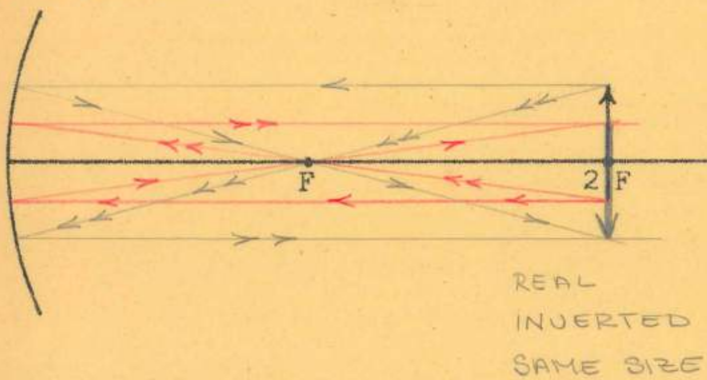
1. Object beyond 2F



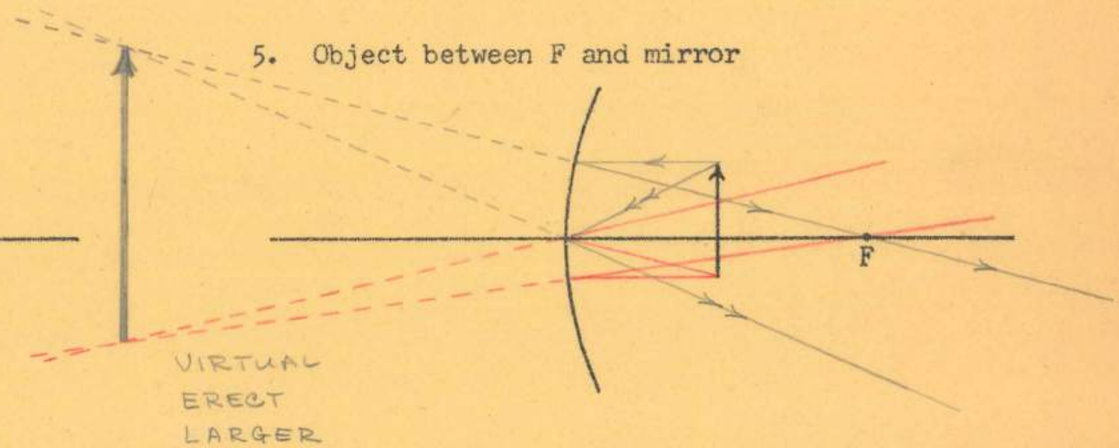
4. Object at F



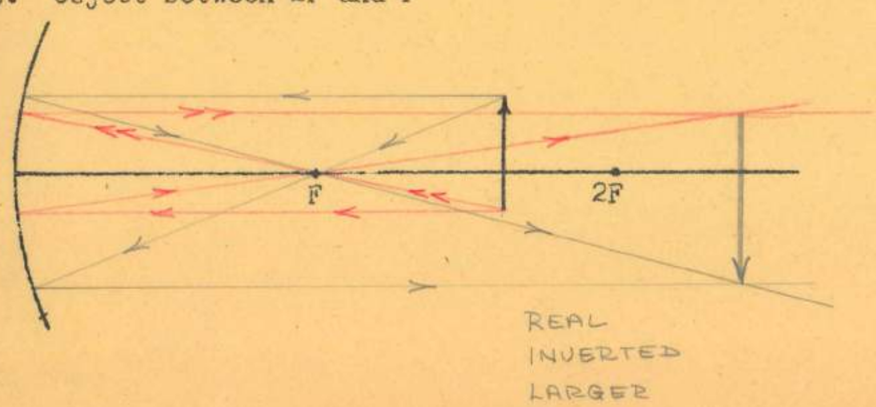
2. Object at 2F



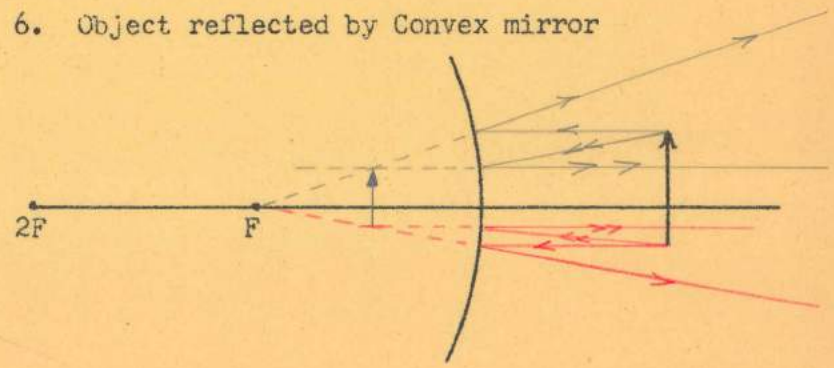
5. Object between F and mirror



3. Object between 2F and F

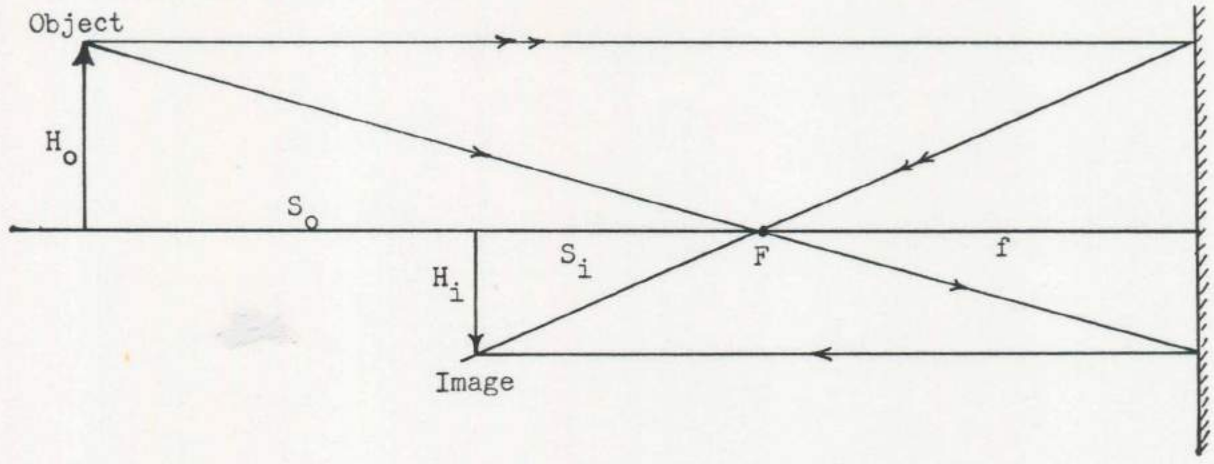


6. Object reflected by Convex mirror

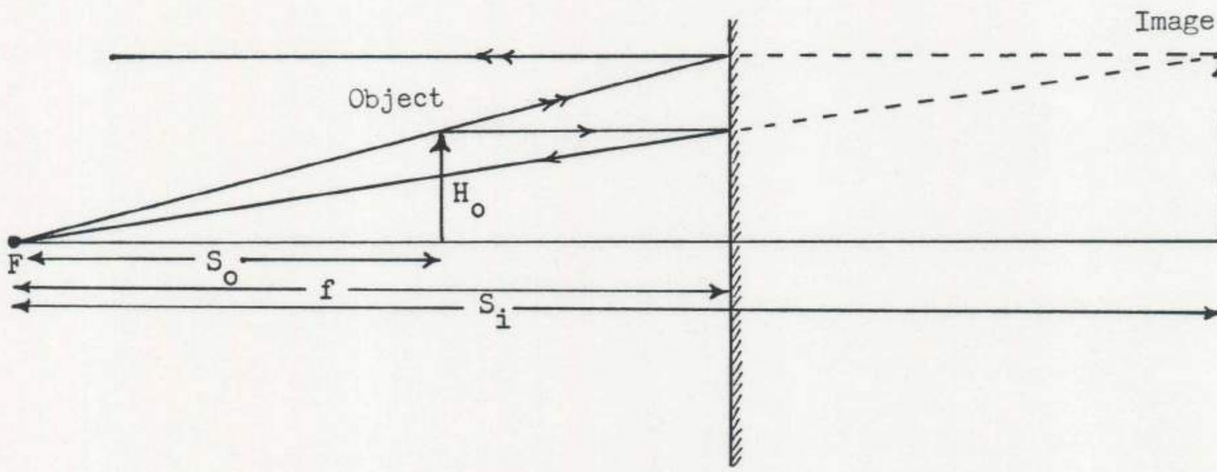


Chapter 18 Verification of Lens Formulae

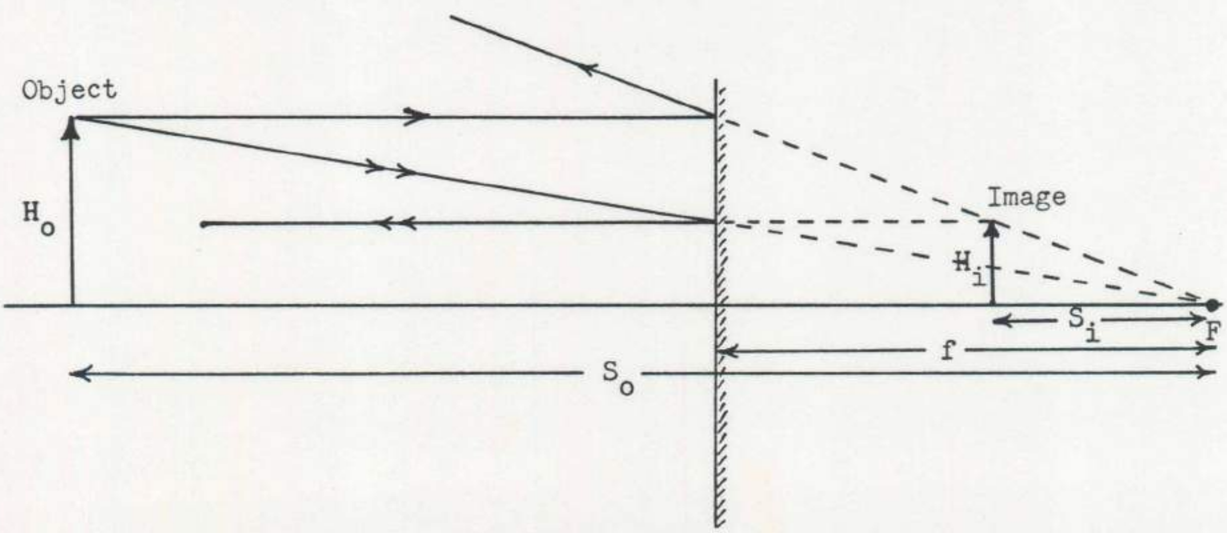
1. Concave Mirror (Image is real)



2. Concave Mirror (Image is virtual)



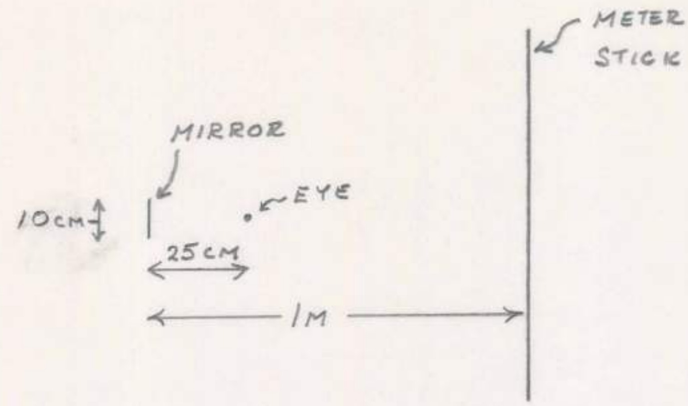
3. Convex Mirror (All situations)



 Name____________________

. If you move your eyes to a position 25 cm from a 10 cm wide mirror, what length (in cm) of a meter stick can you see behind you? The stick is 1 meter behind the mirror, as shown in the diagram. (5)

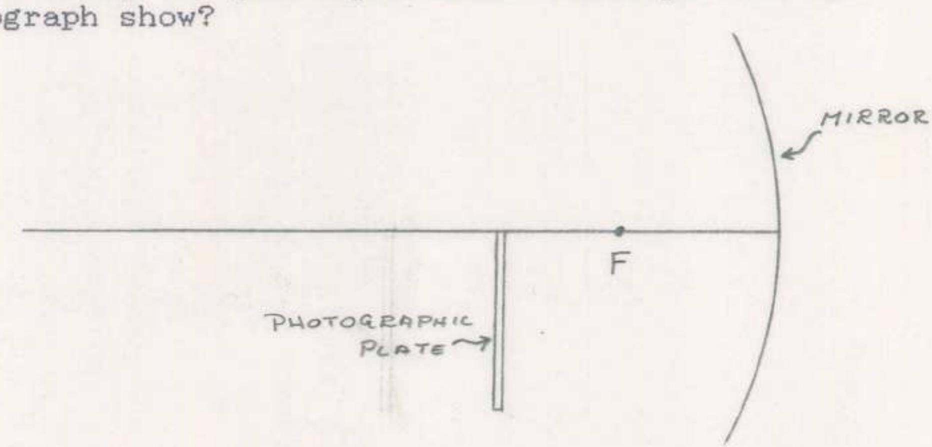
scale: 5 cm = 1 meter



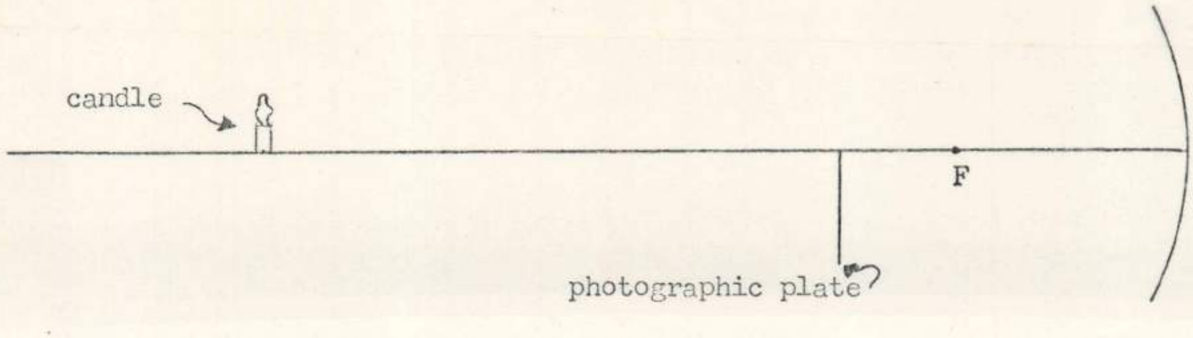
2. A photographer wants to take a picture of herself using a parabolic mirror of focal length 0.7 meter. If she puts a photographic plate of height 0.8 meter a distance of 0.5 meter away form the focal point and below the optical axis of the mirror as shown.
   a. How far from the focal point must she stand for a sharp image to be formed?
   b. She is 1.8 meters tall and stands so that her waist (0.8 meter from the top of her head) is along the principle axis. What part of her anatomy will the photograph show?

a. ____________ (3)

b. ____________ (3)



3. A candle is placed 3.0 meters from the focal point of a parabolic mirror. The base of the candle rests on the principle axis of the mirror. The mirror has a focal length of 1.0 meter. If a photographic plate is placed 0.5 meter from the focal point (see diagram) will it register a sharp image? Either way, explain your answer. (5)



4. Describe precisely, the image you see of yourself in a concave mirror as you move towards the mirror beginning at a distance much greater than twice the focal length from the mirror. (15)

 Name____________________

5. A nail 7.0 cm high stands in front of a concave mirror at a distance of 12 cm from the principle focus. The focal length of the mirror is 20 cm.
   a. What is the size of the image? (4)
   b. Where relative to the mirror is the image(s) located? (7)
   c. Describe the image(s). (6)

6. The image of a candle is 50 cm from the center of a concave mirror. The candle is 12 cm long and its image is 15 cm long. What is the focal length of the mirror? (5)

____

7. What effect does tripling the focal length of a mirror have on the size and location of the image if the size of the object stays the same size and the distance the object is from the principle focus is the same? (6)

8. How large an image of the sun will be formed by the Palomar telescope, whose focal length is 30 meter? The sun's diameter is about 1.5 x ^{20} meter, and it is 1.6 x 10^{11} meter away. (4)

____

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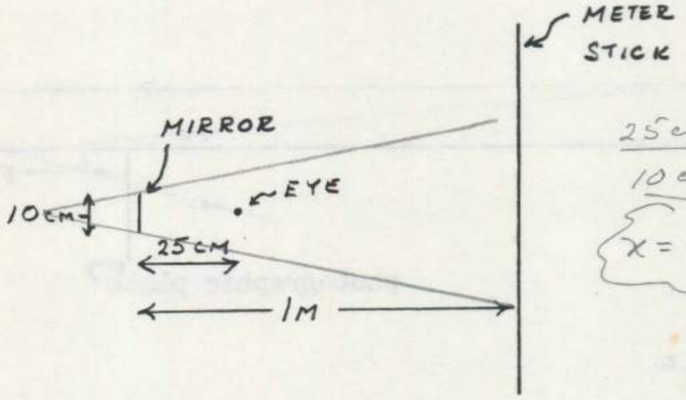
TEST - Chapter 18 -1- Name Key

1. If you move your eyes to a position 25 cm from a 10 cm wide mirror, what length (in cm) of a meter stick can you see behind you? The stick is 1 meter behind the mirror, as shown in the diagram. (5)

5

scale: 5 cm = 1 meter



$\frac{25\ cm}{10\ cm} = \frac{1.25\ M}{x}$

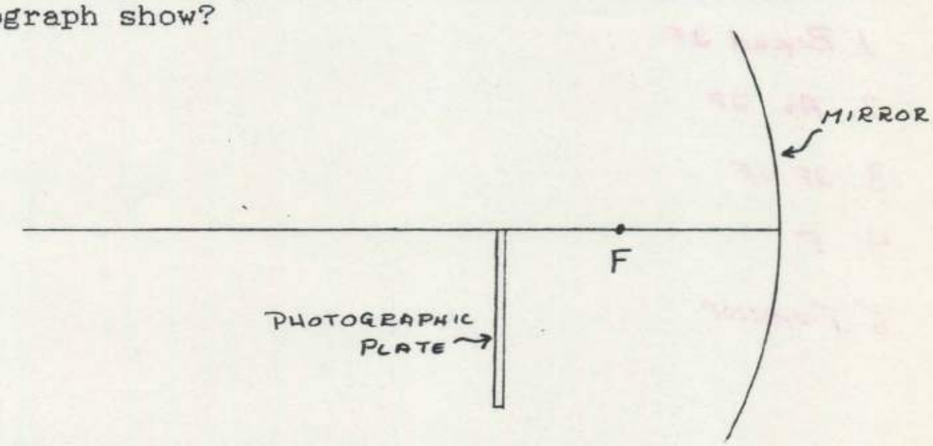
$x = 0.5\ M$

50 = 5
45-55 = 4
40-60 = 3

2. A photographer wants to take a picture of herself using a parabolic mirror of focal length 0.7 meter. If she puts a photographic plate of height 0.8 meter a distance of 0.5 meter away form the focal point and below the optical axis of the mirror as shown.
a. How far from the focal point must she stand for a sharp image to be formed?
b. She is 1.8 meters tall and stands so that her waist (0.8 meter from the top of her head) is along the principle axis. What part of her anatomy will the photograph show?

3 a. $S_o = 0.98 M$ (3)

3 b. Waist up (3)



$f = 0.7 M$

$S_i = 0.5 M$

$S_o = ?$

$S_o = \frac{f^2}{S_i} = \frac{(0.7 M)^2}{0.5 M} = 0.98 M$

$H_i = \frac{S_i \times H_o}{f}$

$\frac{0.5 M \times 0.8 M}{0.7 M}$

$H_i = 0.57 M$

11

 Name Key

5. A nail 7.0 cm high stands in front of a concave mirror at a distance of 12 cm from the principle focus. The focal length of the mirror is 20 cm.
   a. What is the size of the image? (4)
17 b. Where relative to the mirror is the image(s) located? (7)
   c. Describe the image(s). (6)

$H_o = 7.0$ cm
$S_o = 12$ cm
$f = 20$ cm

(a) $H_i = \frac{H_o f}{S_o} = \frac{7 cm \times 20 cm}{12 cm}$

$H_i = 11.7$ cm

(b) $S_i = \frac{f^2}{S_o}$

$S_i = 33.3$ cm

∴ 53.3 cm front → larger, real, beyond 2F, inverted

AND

13.3 cm behind → larger, virtual, erect

6. The image of a candle is 50 cm from the center of a concave mirror. The candle is 12 cm long and its image is 15 cm long. What is the focal length of the mirror? (5)

5 ______________

$S_i + f = 50$ cm
$H_o = 12$ cm
$H_i = 15$ cm
$f =$

$\frac{H_i}{H_o} = \frac{S_i}{f}$ $f = \frac{S_i H_o}{H_i} = (50_m - f) H_o = f H_i$

$50 H_o - f H_o = f H_i \Rightarrow 50 H_o = f H_i + f H_o$

$f = \frac{50 cm\ H_o}{H_i + H_o}$ OR

$\frac{50 cm \times 12 cm}{27 cm}$

$f = 22.2$ cm

7. What effect does tripling the focal length of a mirror have on the size and location of the image if the size of the object stays the same size
6 and the distance the object is from the principle focus is the same? (6)

$f = f$ $f' = 3f$
H_i H_i'
S_i S_i'
$H_o = H_o'$
$S_o = S_o'$

NEED: $f, H_i\ H_o\ S_o$

$\frac{H_i}{H_o} = \frac{f}{S_o} \Rightarrow H_i = \frac{f H_o}{S_o} \Rightarrow H_i \propto f$ ∴ $H_i' = 3 H_i$

NEED $f, S_i\ S_o$

$S_i S_o = f^2$ $S_i = \frac{f^2}{S_o}$ ∴ $S_i \propto f^2$ $S_i' = 9 S_i$

8. How large an image of the sun will be formed by the Palomar telescope, whose focal length is 30 meter? The sun's diameter is about 1.5 x 10[20] (Make 9) meter, and it is 1.6 x 10^{11} meter away. (4)

4 ______________

$H_i =$
$f = 30$ M
$H_o = 1.5 \times 10^{20}$ M
$S_o = 1.6 \times 10^{11}$ M

$H_i = \frac{f H_o}{S_o} = \frac{30 M \times 1.5 \times 10^{20} M}{1.6 \times 10^{11} M}$

$H_i = 2.8 \times 10^{10}$ M

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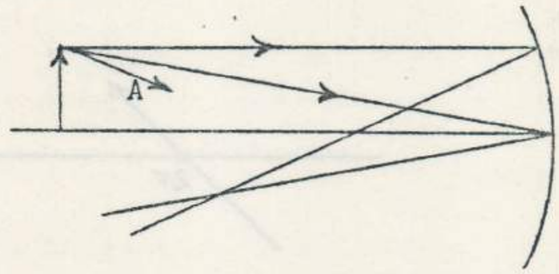
Chapter 18 WRITTEN EXERCISE

1. Using the following diagram:

   a. Label the position of the principal focus;

   b. Continue the ray A;

   c. Draw in the image.



2. An object 3 mm high forms a real image 5 mm high, 25 cm from a concave mirror.

   a. How far from the mirror is the object?

   b. What is the focal length of the mirror?

3. How far from a parabolic mirror of 20 cm focal length must an object be placed to give an image:

   a. magnified two (2) times,

   b. unchanged in size,

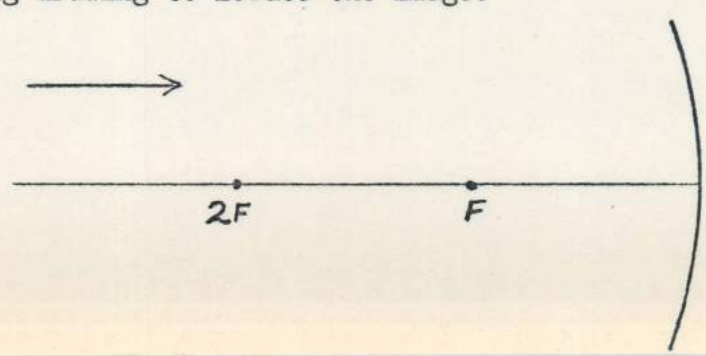
   c. reduced to 1/2 its size.

   d. What will be the location of the image(s) for each of the three situations?

   e. What type of image(s) will be produced?

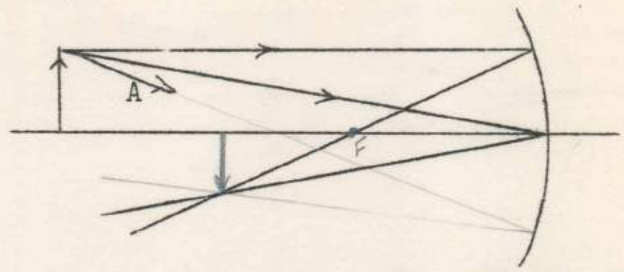
	Object Location	Image Location	Type of Image
a.	______	______	______
b.	______	______	______
c.	______	______	______

4. Complete the following drawing to locate the image.



Chapter ~~2~~ 18 WRITTEN EXERCISE

1. Using the following diagram:
   a. Label the position of the principal focus;
   b. Continue the ray A;
   c. Draw in the image.



2. An object 3 mm high forms a real image 5 mm high, 25 cm from a concave mirror.
   a. How far from the mirror is the object?
   b. What is the focal length of the mirror?

$H_o = 3$ mm, $H_i = 5$ mm, $S_i = 25$ cm $- f$

(b) $\frac{H_i}{H_o} = \frac{S_i}{f}$; $\frac{5\text{ mm}}{3\text{ mm}} = \frac{25\text{ cm} - f}{f}$; $75\text{ cm} - 3f = 5f$; $75\text{ cm} = 8f$; $9.375\text{ cm} = f$

(a) $S_i S_o = f^2$; $S_o = \frac{f^2}{S_i}$; $S_o = \frac{(9.375\text{ cm})^2}{25\text{ cm} - 9.375\text{ cm}}$; $S_o = 5.625$ cm; $9.375\text{ cm} + 5.625\text{ cm} = 15\text{ cm}$

OR → $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{f}$; $\frac{S_o}{S_i} = \frac{D_o}{D_i}$; $S_o = 3$ mm, $S_i = 5$ mm, $D_i = 25$ cm; $\frac{3\text{ mm}}{5\text{ mm}} = \frac{D_o}{25\text{ cm}}$, $D_o = 15$ cm; $\frac{1}{25\text{ cm}} + \frac{1}{15\text{ cm}} = \frac{1}{f}$; $f = 9.375$ cm

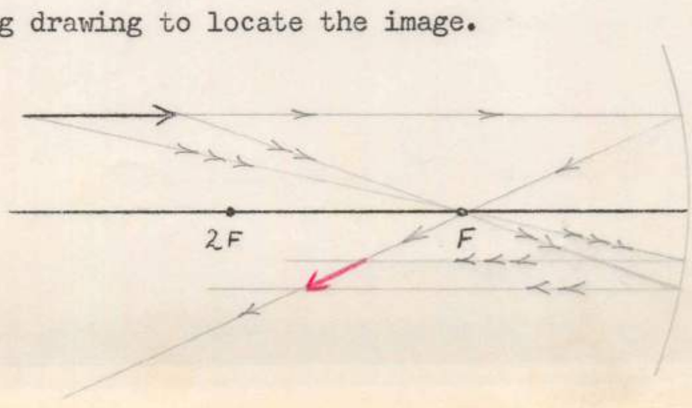
3. How far from a parabolic mirror of 20 cm focal length must an object be placed to give an image:
   a. magnified two (2) times,
   b. unchanged in size,
   c. reduced to 1/2 its size.
   d. What will be the location of the image(s) for each of the three situations?
   e. What type of image(s) will be produced?

Magnification $= \frac{H_i}{H_o} = \frac{f}{S_o} = \frac{S_i}{f}$

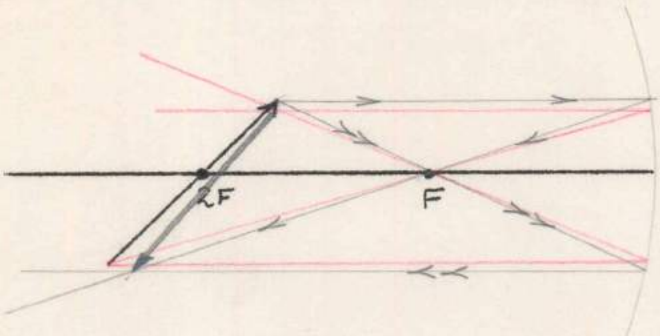
(a) $S_o = f\left(\frac{H_o}{H_i}\right) = 20\text{ cm} \times \frac{1}{2} = 10$ cm
(b) $S_o = f \times 1 = 20$ cm
(c) $S_o = f \times 2 = 40$ cm

	Object Location	Image Location	Type of Image
a. ($S_o = 10$ cm, $S_i = 40$ cm)	30 cm, 10 cm	60 cm in front, 20 behind	real, virtual
b. ($S_o = 20$ cm, $S_i = 20$ cm)	40 cm, 0 cm	40 cm in front, 0 cm	real, virtual
c. ($S_o = 40$ cm, $S_i = 10$ cm)	60 cm	30 cm in front	real

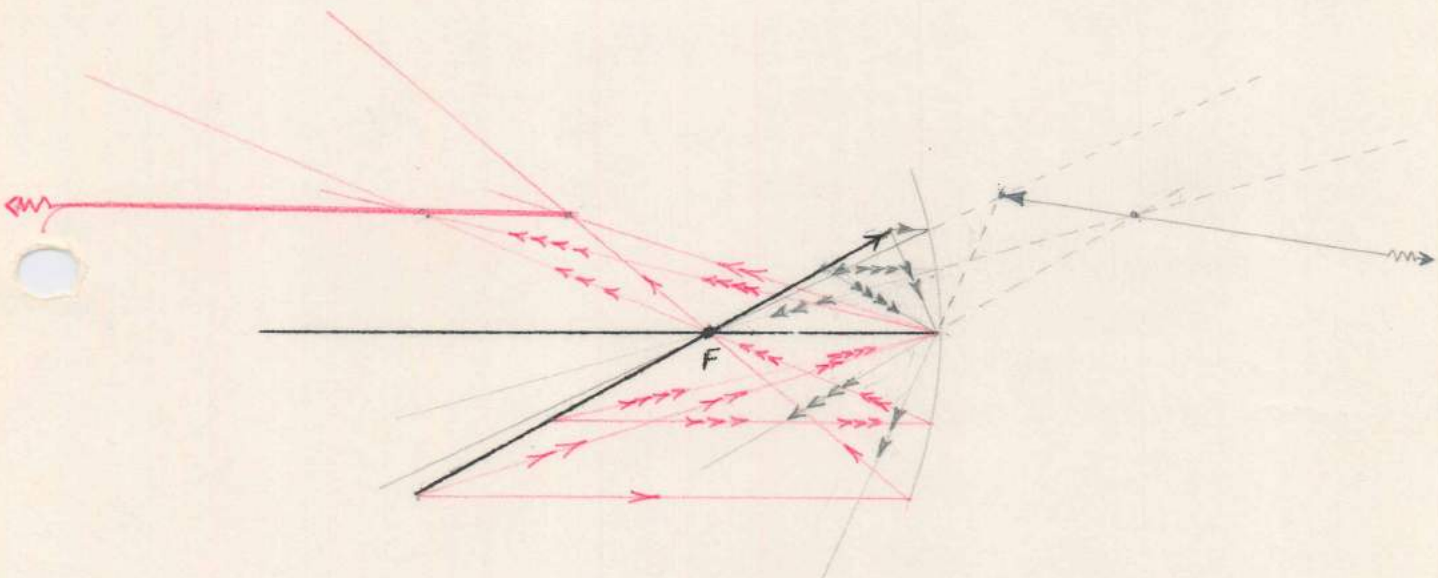
4. Complete the following drawing to locate the image.



5. Complete the following drawing and locate the image.



6. Complete the following drawing and locate the image.



PSSC ASSIGNMENT #7

SUPPLEMENTARY PROBLEMS ON REFLECTION OF LIGHT

1. A flagpole at the edge of a still pond rises 18 feet 4 inches above ahe surface of the pond. An observer standing 10 feet from the base of the flagpole sees its image reflected in the water. If the eyes of the observer are 5.0 feet 8 inches above the surface of the water, how far away from his eyes does the image of the top of the flagpole appear to be? Draw a diagram to illustrate your answer.

2. A spherical concave mirror forms a real image 18 centimeters from the mirror. If the magnification is two, what is the position of the object and the radius of curvature of the mirror? (draw a ray diagram)

3. A concave spherical mirror has a radius of curvature of 24 centimeters. A small object is placed on the principal axis of the mirror. The distance from the mirror to the object is 18 centimeters. What is the position of the image and the magnification? (draw a ray diagram)

4. How far from a concave spherical mirror with a focal length of 10 centimenters must a small object be placed to form an inverted image with a magnification of two? (draw a diagram)

5. An object 7.0 millimeters high is place on the axis of a concave spherical mirror with a focal length of 12 cemtimenters. If the object is 15 centimeters from the mirror, what is the position and size of the image? (draw a diagram)

6. A small object 1.0 centimeters high is placed 8.0 centimeters from a convex spherical mirror with a focal length of 12 centimeters. What is the position and size of the image formed?

7. An object 1.0 centimeters high is placed on the axis of a convex spherical mirror at a distance of 4.0 centimeters from the mirror. If the radius of curvature of the mirror is 10 centimeters, what is the position and height of the image formed?

8. A convex spherical mirror has a radius of curvature of 10 centimeters. A pin 1.0 centimeter high is placed on its principal axis at a point 3.0 centimeters from the mirror. What is the position and height of the image of the pin?

A FISH'S VIEW OF ITSELF IN A TANK WITH CONCAVE MIRROR ENDS

by

Robert C. Good, Jr.
Penncrest High School

Many people have noticed that there are many images of an object in a barber shop. This effect is produced by the nearly parallel plane mirrors on the opposite walls. One mirror forms a virtual image which acts as a virtual object for the opposite mirror. This object is farther away from the opposite mirror so that its image will also be farther away. Thus, the images successively recede to produce multiple images.

It occured to the author that a set of mirrors could be devised in which the images would superimpose to nullify the multiple image effect. This paper derives the mirror arrangement for this case.

A pair of identical concave mirrors of radius, R, face each other. The problem is to find the separation, H, of the mirrors for which there will be one object and one image regardless of the position of the object.

One notes that if a tiny fish is at the center of a truly spherical bowl, that there will be one object and one image. The inverted image of the fish is formed just beneath the fish which acts like an object that forms an image on the real fish. In this case H = 2R. See Figure 1.

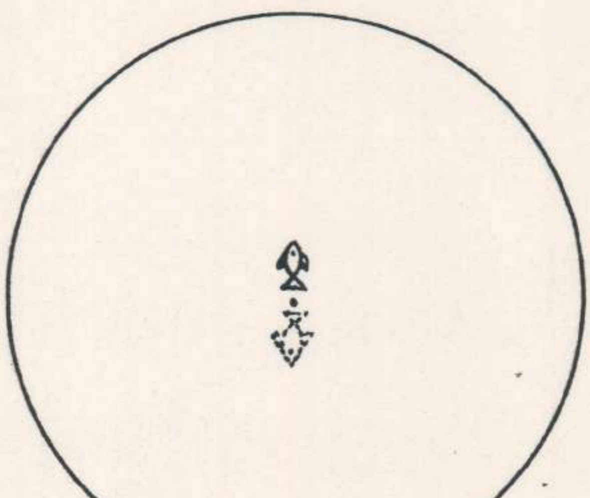


FIGURE 1. SPHERICAL FISH BOWL WITH FISH AT ITS CENTER

Other mirror separations may be found. The following analysis uses the elementary mirror equation:

$$2/R = 1/P + 1/Q \tag{1}$$

where R is the radius of curvature of the mirror, P is the objects distance and Q is the image distance both measured along the principle axis from the mirror vertex. The normal sign convention holds: the light strikes the mirror and both P and Q are positive on the concave side. Figure 2. is a sketch of the mirrors, object and image.

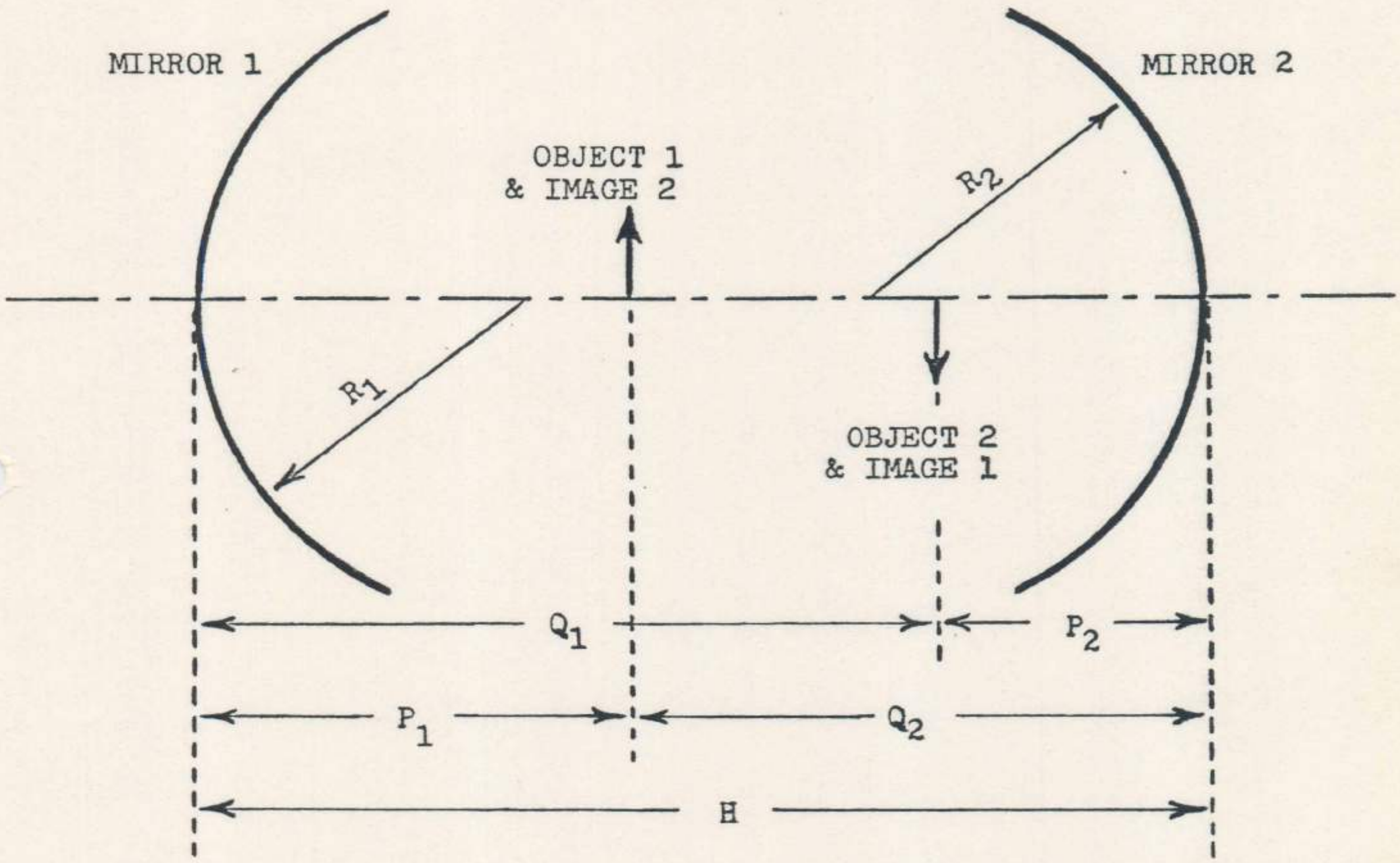


FIGURE 2. SKETCH OF MIRRORS, OBJECT, AND IMAGE

The mirror equations are:

$$2/R_1 = 1/P_1 + 1/Q_1 \quad \text{and} \quad 2/R_2 = 1/P_2 + 1/Q_2 \tag{2}$$

where the subscripts refer to mirrors 1 and 2. For identical mirrors:

$$R_1 = R_2 = R \tag{3}$$

The spacial equations are:

$$P_1 + Q_2 = H \quad \text{and} \quad P_2 + Q_1 = H$$

With these five equations, H may be found in terms of R and P_1. Straight forward substitution and elimination of the variables P_2, Q_1 and Q_2^1 leads to:

$$H^2(2P_1 - R) + H(R^2 - 2P_1R - 2P_1{}^2) + 2P_1{}^2R = 0 \qquad (5)$$

This quadratic equation has two solutions:

$$H = P_1/(1 - R/2P_1) \quad \text{and} \qquad (6)$$

$$H = R \qquad (7)$$

Equation (6) may be rearranged to the form:

$$(H/R) = (P_1/R)^2/(P_1/R - \tfrac{1}{2}) \qquad (8)$$

Figure 3. is a graph of equation (8). The lower portion where $(H/R) < 0$ shows an impossible solution because Figure 2. shows that H must be positive. The upper portion has a minimum at $(H/R) = 2$. This is the solution $H = 2R$ discussed earlier. The remainder of the curve shows that for any value of $(P_1/R) > \frac{1}{2}$, there is only one value for (H/R) which produces the single image condition. For example, at $(P_1/R) = 5.5$, $(H/R) = 6.05$, the first image will be at 0.55 and the second image will be back on the object at 5.5. With this separation of 6.05 there are only two object positions which will produce the single image. Such a solution is not the one sought.

Using equation (7), $H = R$, as the solution for H, there will always be only one image regardless of the value for P_1. Of course, P_1 must be less than H or $P_1 \leq R$; on fact, $P_1 \leq R/2$ because if P_1 is greater than R/2, the other mirror will reflect light first so that P_1 is less than R/2 again. When P_1 is less than R/2, the first image is virtual. This virtual object for the second mirror produces a real image on top of the object.

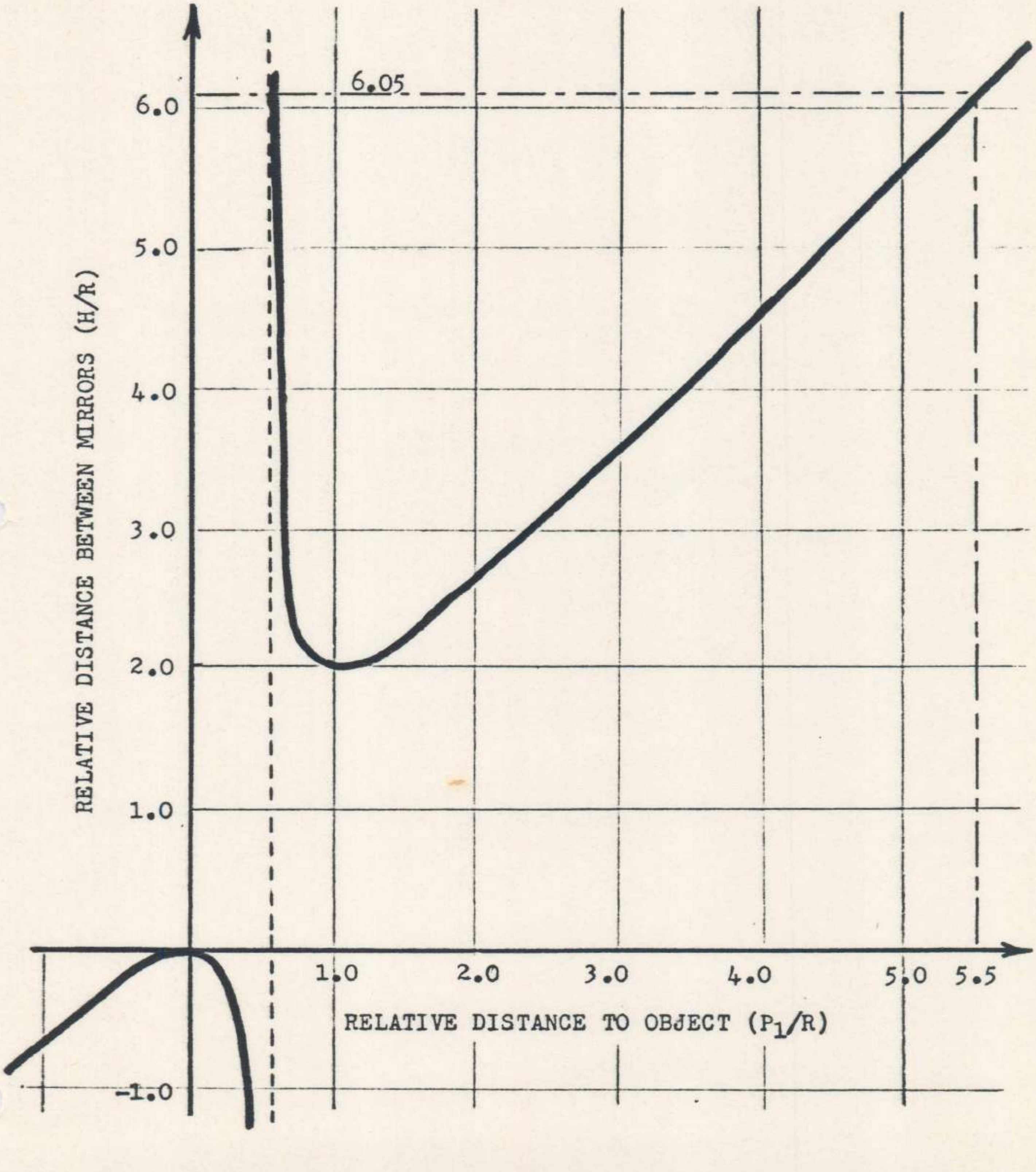
CONCLUSIONS AND RESULTS:

The solution $H = R$ (wherein the fish bowl looks like a football standing on its end) is the most general solution to the problem posed because there is only one image regardless of the position of the fish along the mirror axis. Yet as Figure 3. shows for any specific position of the fish such that P_1 is less than R/2, one can always find a separation, H, of the mirrors for which there will be a single image produced.

Editor's Note:

We saw Bob demonstrate this effect at Philadelphia College of Textiles and Science, and we will attempt to persuade him to do this demonstration again at the SEPS/AAPT meeting on March 17, 1979.

FIGURE 3. RELATIVE SEPARATION OF CONCAVE MIRRORS TO FORM ONLY ONE IMAGE OF OBJECT BETWEEN THEM.



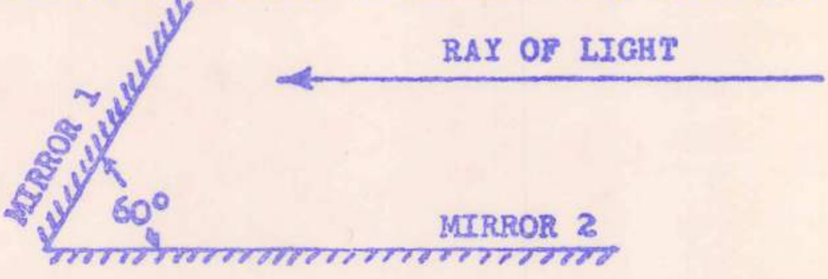
PSSC PHYSICS QUIZ #______ NAME______________

FLAT MIRRORS DATE______________

(Need protractor and Ruler)

Test Question

1. A ray of light is shown approaching Mirror #1. This ray of light is parallel to Mirror #2. Carefully draw the path that this light ray will follow until it is heading away from the Mirrors. (2Pts.)



2. On the diagram above, label the angles that are equal according to the second law of specular reflection. (1 Pt.)

3. What is the measure in degrees of the angle between the initial incident ray and the final reflected ray. (1 Pt.)

Test

4. A pin is placed between two mirrors at right angles (90 degrees) with the reflecting surfaces facing each other. On the back of this paper make a scale drawing showing the position of mirrors, object, and all possible images. (1 Pt.)

5. Draw a Human Eye on the diagram made for question # 4. Draw the path followed by one ray of light for each image that contributes to the formation of the image as seen by the owner of the eye. (2Pts.)

Test

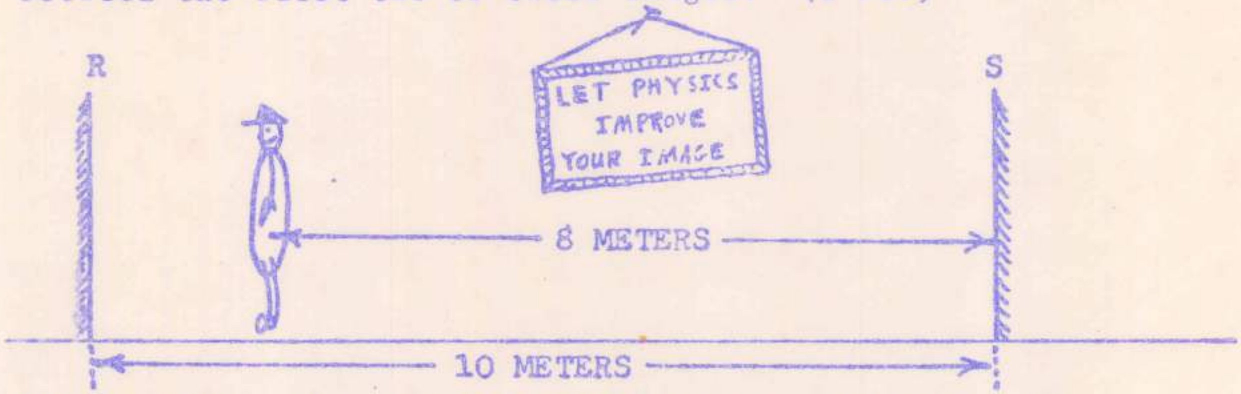
6. The image of your friend seems to just cover a mirror three centimeters high. When this mirror is held 0.75 meters from your eye. Your friend is 50 meters behind the observer. How high is the friend? (1 Pt.)

Test

7. Give an argument to support the idea that the image seen "in" a flat mirror is a virtual image and not a real image. (1 Pt.)

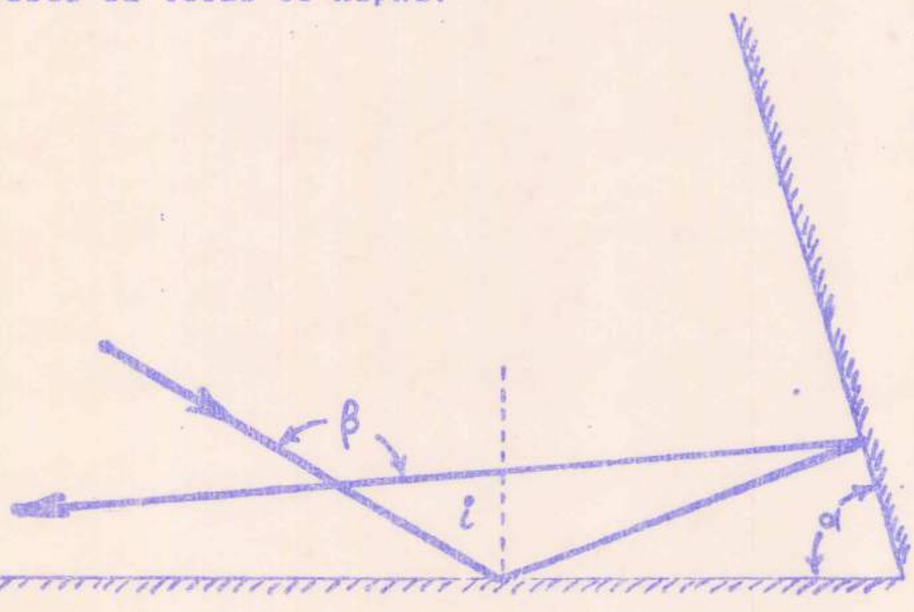
Bonus Question

8. Two full length mirrors (R & S) stand vertically as shown below. The mirrors are ten (10) meters apart. If you stand two (2) meters from mirror R you can see a long series of images of yourself in mirror R. What is the distance between the first two of these images. (1 Pt.)



Bonus Question ?

EXTRA POINT: The sketch below shows two mirrors at an angle Alpha (α). A ray of light strikes one mirror as shown, then strikes the other mirror and finally crosses the original ray at an angle Beta (β). Show that Beta is independent of the angle of incidence (i) and find its value in terms of Alpha.



PSSC PHYSICS QUIZ #3 NAME:....................

CURVED MIRRORS (need ruler) October 11, 1983

TEST

1. Assume an object is placed on the principal axis of a concave mirror. Where would the object be located if the image formed by the mirror is: NOTE: If impossible write, "impossible to form this type image."

a) real and enlarged...

b) virtual, erect and enlarged...

c) virtual and inverted...

d) real and diminished...

(2 points)

$H_o = 2cm$ $f = 5cm$ $S_o + f = 11cm$ $S_i + f = ?$ $H_i = ?$ when S_o so $M = 1$

Test

2. An arrow is two centimeters high and is located 11 centimeters in front of a concave spherical mirror with a ~~radius of curvature~~ f of 10 centimeters.

a) On the back of this paper make a full scale drawing using principal rays to locate the image formed. (2 points)

b) Calculate the position of the image. (2 points)

c) Calculate the size of the image. (2 points)

d) At what position(s) of the object will the magnification be one? (2 points)

Extra Point: An object arrow six centimeters in length is place one centimeter above and parallel to the principal axis of a concave spherical mirror. The focal length of the mirror is five centimeters. The arrow points toward the mirror with its head eleven centimeters from the mirror and its foot 17 centimeters from the mirror. Using principal rays construct a full scale drawing this situation and determine the magnification.

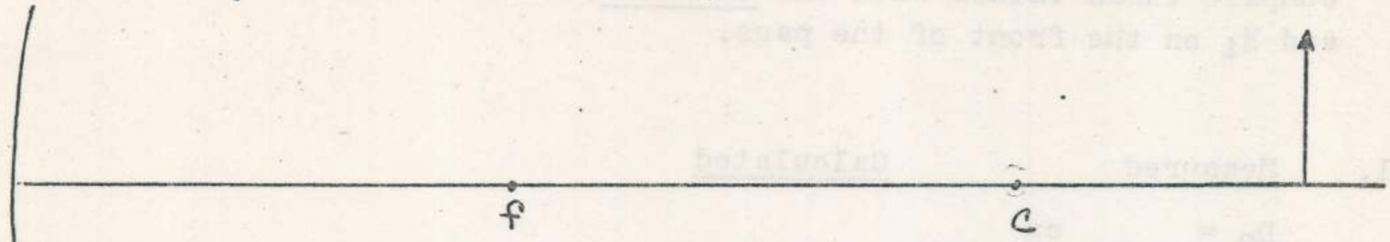
On Quiz

added to Quiz

MIRROR IMAGES - WORKSHEET NAME ____________________

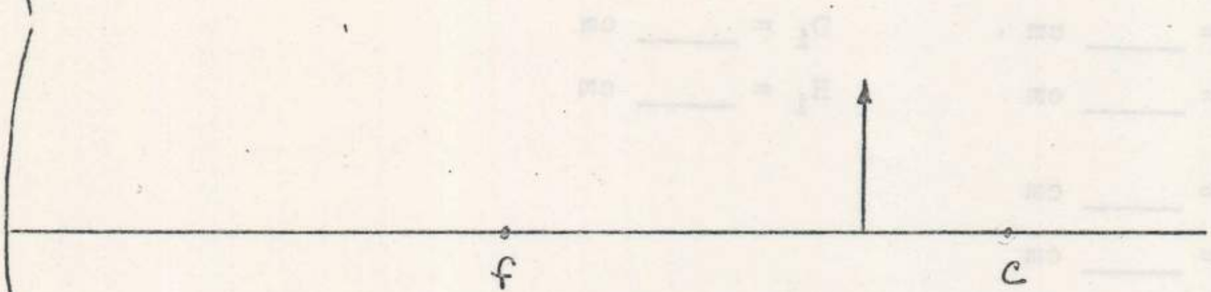
USING RAY DIAGRAMS, LOCATE THE IMAGE OF THESE 2 cm. OBJECT "ARROWS" - FIND THE HEIGHT OF EACH IMAGE.

①



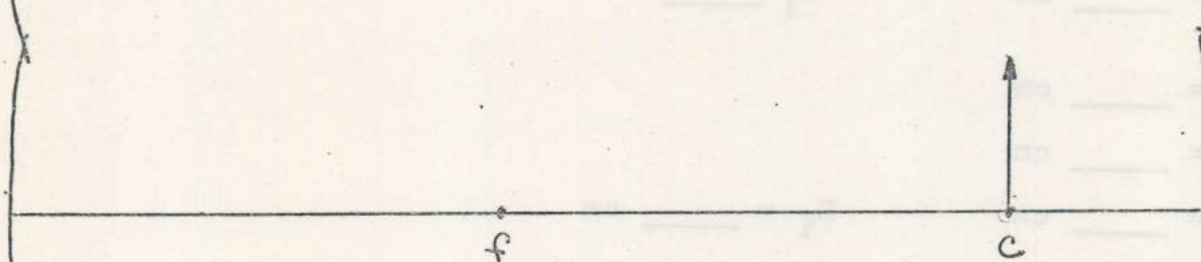
H_i = ____
D_i = ____

②



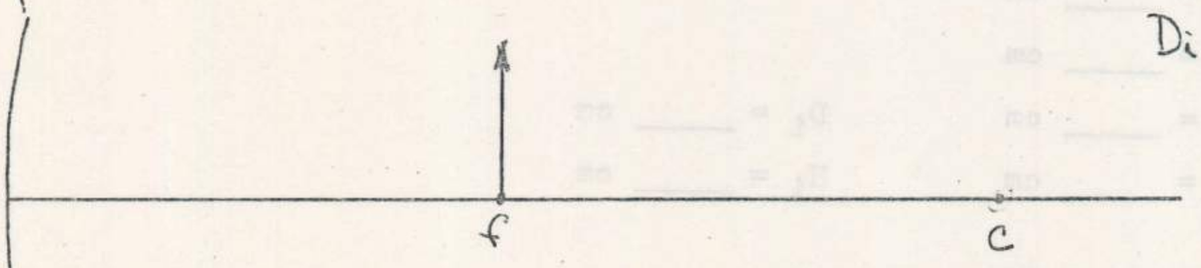
H_i = ____
D_i = ____

③



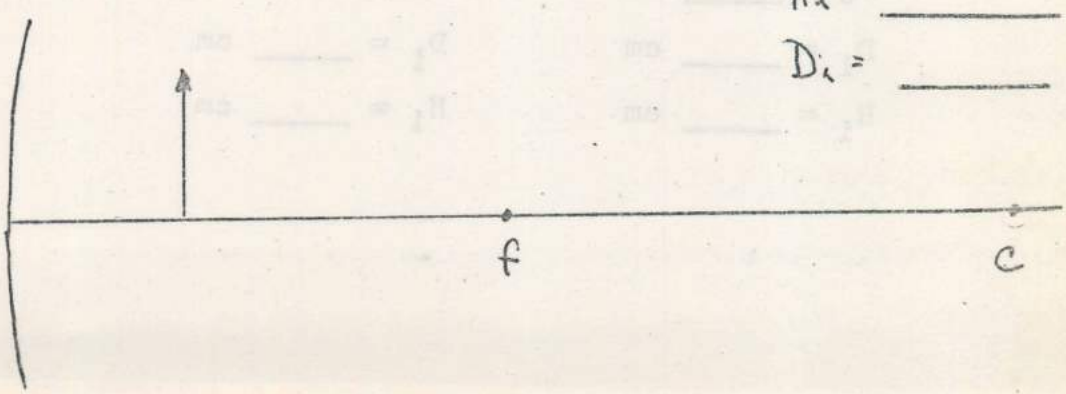
H_i = ____
D_i = ____

④



H_i = ____
D_i = ____

⑤



D_i = ____
H_i = ____

For each drawing on the front, measure D_o and H_o (these are the "given" in each drawing). Then calculate values for D_i and H_i and compare these values with the measured values you recorded for D_i and H_i on the front of the page.

	Measured		Calculated	
1.	D_o = ____	cm		
	H_o = ____	cm		
	D_i = ____	cm	D_i = ____	cm
	H_i = ____	cm	H_i = ____	cm
2.	D_o = ____	cm		
	H_o = ____	cm		
	D_i = ____	cm	D_i = ____	cm
	H_i = ____	cm	H_i = ____	cm
3.	D_o = ____	cm		
	H_o = ____	cm		
	D_i = ____	cm	D_i = ____	cm
	H_i = ____	cm	H_i = ____	cm
4.	D_o = ____	cm		
	H_o = ____	cm		
	D_i = ____	cm	D_i = ____	cm
	H_i = ____	cm	H_i = ____	cm
5.	D_o = ____	cm		
	H_o = ____	cm		
	D_i = ____	cm	D_i = ____	cm
	H_i = ____	cm	H_i = ____	cm