87-88

CHAPTER 5 UNIVERSAL GRAVITATION AND THE SOLAR SYSTEM

The force of gravity is introduced along with its magnitude near the surface of the earth. Next the difference between mass and weight are discussed including the reason objects are 'lighter' on the moon than on the earth. The FIELD concept is first introduced here. The notion of a FIELD is very profound with important ramifications in other areas of physics.

We then extend our vision to objects in space by extending the fundamental laws of gravity and motion developed by Newton. These laws lead directly to the observed motions of the planets which Kepler had so painstakingly classified a half-century before. The effect of this discovery on the 17th century world was extremely profound, spelling doom to the notion that the earth was the center of the solar system and universe. It is not hard to imagine the magnitude of the philosophical and religious overtones of such a radical demotion of the earth in the astronomical hierarchy.

The reading of this chapter takes a historical approach if you follow the order suggested, starting with early descriptions of the solar system and then outlining the steps in which the mystery of planetary motion was eventually solved. The real value of this approach is that it clearly outlines the manner in which the scientific method operates. First came the accurate observations of planetary motion by Tycho Brahe. This in turn led to the empirical laws of planetary motion derived by Kepler and finally to Newton's connection of planetary motion with the simple and elegant fundamental law of gravity. Thus, OBSERVATION, DESCRIPTION, and GENERALIZATION were the steps in which the problem was solved, and this is the classic manner in which science operates.

CHAPTER 5 PERFORMANCE OBJECTIVES

Upon completion of this chapter, you should:

1. be able to explain/define the difference between mass and weight using the concept of gravity.

2. be able to explain/define gravitational force field.

3. be able to find the acceleration of the moon knowing its period and radius of orbit and compare the value to the acceleration at the earth's surface.

4. be able to recognize some of the early attempts to understand our solar system.

5. be able to contrast the heliocentric & geocentric reference models.

6. be able to plot the retrograde motion of Mars given its position at various time intervals from photographs.

7. be able to state Kepler's Laws of Motion.

8. recognize that the orbits of the planets can be approximated as circular.

9. be able to state and use Newton's Universal Law of Gravitation.

1. Read: Section 5-1 The Gravitational Field Near the Earth page 87

2. The gravitational force field constant is identified as 'g'.

   a. Write a brief summary as to what 'gravitational force field' means to you. Share it with your partner(s) and then with your teacher.

   b. What is the value for Cleveland?

   c. What factor(s) influence the value of 'g'?

3. Investigation: FINDING THE ACCELERATION OF A BALL ROLLING DOWN A RAMP

   Your task is to determine the acceleration of the ball rolling down the ramp provided. You are to present a proper write-up including all essential components.

4. Problems: page 88: #2

5. Read: Section 5-7 From the Greeks to Kepler: A Brief Sketch page 98

   Here we encounter a very brief summary of how people struggled to explain the motions of objects in the heaven. By choice we will skip a lot of this struggle and proceed from the point where the true motions were about to be discovered.

   Optional...Project Physics Reader #2 titled MOTION IN THE HEAVENS contains 26 articles. A brief summary of each article is included in this packet. You may check out one of the books if you wish to do some reading at home.

6. Problems: page 106: #24 #25

   A must...Show your instructor the drawing you made from (25-c) as soon as you complete it.

7. The subject of retrograde motion came up in problem 25. It is important that you know what retrograde motion is and how it came about. Four different activities are given below. Do only enough so that you understand what retrograde motion is.

   a. Obtain a series of 15 photographs taken between 29 May and 15 October 1971 of Mars in Capricornus.

      1. Place the plastic sheet over one of the photographs. Mark the position of Mars (the brightest) and a few of the reference stars with a wax pencil. Record the date which is on the back of the photograph.

      2. Place the plastic sheet over another photograph in such a manner that the reference stars of this photograph are directly over the reference stars of the previous photograph. Locate Mars and label the date. Repeat this for the remaining photographs.

      3. By drawing a continuous line through the points you should see a plot like that shown in Figure 21-2 page 359 (red text). By estimating the dates of the beginning and end of the retrograde motion, you can determine the duration of the retrograde motion.

b. Film Loop-11 RETROGRADE MOTION - GEOCENTRIC MODEL

This film loop illustrates the motion of a planet such as Mars as seen from the earth. It was made using a large Epicycle Machine as a model of the Ptolemaic system. Directions for analysis are included in this packet.

c. Film Loop-12 RETROGRADE MOTION - HELIOCENTRIC MODEL

This film loop illustrates the motion of a planet such as Mars and the earth as seen from the sun. Directions are also included.

d. Transparency T-15 RETROGRADE MOTION

This transparency can be used to help understand the heliocentric explanation of an outer planet's apparent retrograde motion. Some directions are included with the transparency. Have your instructor give further explanation if needed.

8. Read: Section 5-2 Earth Satellites page 88
   Section 5-3 The Moon's Motion page 90

9. The following NASA Educational Briefs are included for your reading enjoyment.

   a. ORBIT OF THE MOON
   b. EARTH - MOON - SUN RELATIONSHIP

10. Problems: page 90: #4 #5 #6 #7

11. Read Section 5-4 Kepler's Laws page 91

12. Film Loop-16 KEPLER'S LAWS (Film Loop Notes included)

    a. You may make your own data point sheet as you watch the points being generated or you may request a copy of one that has already been prepared.

    b. Analyze the data points as per directions in the notes.

    c. Provide your instructor with sufficient evidence that you have verified Kepler's three laws using the data sheet.
    Note...You may think that these are not measurements in the true sense of the word; after all, didn't the computer 'know' about Kepler's Laws and display the orbits accordingly? Not so - the computer 'knew' (through the program given it) only Newton's laws of motion and the inverse-square law of gravitation. What we are measuring here is whether these laws of mechanics have as their consequence the Kepler laws which describe, but do not explain, the planetary orbits. This is exactly what Newton did, but without the aid of a computer to do the routine work. Our procedure in its essential is the same as Newton's, and our results are as strong as his.

13. Problems:: page 92: #8 #9 #10 #11
    page 106: #27 #28

14. Optional...Film NEWTON: THE MIND THAT FOUND THE FUTURE (30 min)

This film dramatizes the career of Isaac Newton, and highlights the discoveries that revolutionized science in the 17th century and are the basis for scientific work in our century, including the space program. Newton's friend and colleague, Edward Halley, the discoverer of "Halley's Comet", introduces the irascible genius, sitting peacefully in his orchard. A falling apple, a great mental leap, and the laws of universal gravity are discovered. Twenty years later, Halley convinces his friend to publish his findings and in eighteen months the PRINCIPIA is completed. Already a respected mathematician, Newton now becomes world famous. Active in a dozen different fields, Newton goes on to write a major work on mechanics, to create differential calculus, to make known his work on optics, and to invent the first revolving telescope, among other projects. Newton's eccentric personality, his originality of mind, and his place in the history of science are vividly illustrated by Halley. The laws and definitions that he contributed to physics as well as an experimental method on which to base future discoveries, are presented as the beginnings of the modern age of science.

15. Film: UNIVERSAL GRAVITATION (31 min) (Film notes provided.)

See study Notes I and II which are in this packet.

16. Much discussion in Section 5-5 and in the film goes into how Newton discovered his Universal Law of Gravitation. Unless you are so motivated to study the ideas in detail, you may proceed if you understand the following:

a. What is 'G'? What value do we use for it? Does this value change? If so, where?

b. How was 'G' determined?

c. Your instructor prefers that you use small 'm' and capital 'M' rather than m_1 and m_2 as shown in the text.

d. Newton said that F=ma governs motion anywhere. If this is true, then one can equate it to $F = GmM/R^2$. Equate these two equations and solve for 'a'. What conclusion(s) are you able to draw?

17. Equate $F=mv^2/R$ to $F=GmM/R^2$. Then equate $F = m4\pi^2R/T^2$ to $F=GmM/R^2$. What conclusion(s) are you able to draw in each case?

18. Problems: page 95: #12 #13 #14 (Table 3) #15 #16
page 106: #31 #32 #33 #34 #35 #36 #40

19. Optional...You might wish to read Chapter 7 in Vol 1 of THE FEYNMAN LECTURES ON PHYSICS titled "The Theory of Gravitation". See instructor for the book.

20. Optional...An article titled GENERAL RELATIVITY is included for your reading pleasure.

21. Complete the Written Exercise titled "Small World" which is enclosed.

Penny wise and 4.45 Newtons foolish.

Answers Chapter 5

2. (b) See page 3405 of Phy & Chem Handbook (c) latitude, altitude

4. (2) no change to two significant figures

6. (24)(a) 27.3 days (b) does not revolve (c) 29.5 days
   (25)(a) once every 29.5 days (b) 7.6×10^{10} m (c) see teacher
   (d) would not $v_e = 30$ km/sec $v_m = 1$ km/sec

10. (4) 0.36
    (5) a, v, T are all equal $F_L = 2 F_s$
    (6)(a) 5.9×10^{-3} m/s/s (earth toward sun) (b) 0.46
    (7) speed decreases, 'T' increases, 'a' decreases

13. (8) 2.2×10^{15} m^2/sec
    (9) 3.35×10^{18} m^3/s^2
    (10) 23 times larger
    (11) Mercury 23^o Venus 46^o
    (27) closest to sun in winter
    (28) 5.3×10^{12} meter

16. (a) Universal Gravitational Constant is -6.67×10^{-11} N m^2/kg^2 does not change
    (b) $G = F R^2/m M$ - see instructor
    (c) 'm' circles more around 'M' than 'M' around 'm'
    (d) $a = G M/R^2$ - can be used to find acceleration on surface of any object or in space around the object

18. (12) 3.6×10^3 times greater
    (13) 0.65
    (14) (a) $K_e = 9.9 \times 10^{12}$ m^3/sec^2 (b) 3.0×10^{-6}
    (15) 0.41 R_e
    (16) Equal in value, opposite in direction
    (31) $3\pi/G = 1.41 \times 10^{11}$ kg sec^2/m^3
    (32) S.A.B.
    (35)(a) 1.01×10^7 sec (about 21 days) (b) 0.075 mm/sec
    (36)(a) $M = 4\pi^2R^3/GT^2$ (c) $ma = 4\pi^2/T^2$ (d) $400\pi^2R/T^2$
    (40)(a) 3×10^{41} kg (b) about 1×10^{11}

Questions from "The Flying Circus of Physical Phenomena" by Jearl Walker

1. I've heard that my wristwatch will run faster on a mountain top than here. Why should it? Could you measure this effect to make it more believable? Would my watch run faster if I were in a jet plane at a high altitude?

2. When you see the moon low in the sky, it may appear to be larger than when you see it otherwise. Why?

READER ARTICLES

1. THE BLACK CLOUD by Fred Hoyle 1957

   In this introductory chapter to his science fiction novel, the noted astronomer Fred Hoyle gives a realistic picture of what goes on within an astronomical laboratory. The emphasis is on experimental astronomy.

2. ROLL CALL by Isaac Asimov 1963

   This pleasant introduction to the planets and the solar system is by a writer well-known as a scientist, a popularizer of science and a writer of science fiction. Asimov approaches the solar system historically, briefly considering the discovery of some of the planets.

3. A NIGHT AT THE OBSERVATORY by Henry S. F. Cooper, Jr. 1967

   What is it like to work at a major observatory? A reporter spends a night on Mt. Palomar talking about astronomy with Dr. Jesse L. Greenstein as he photographs star spectra with the 200-inch telescope.

4. PREFACE TO DE REVOLUTIONIBUS by Nicolaus Copernicus

   Copernicus addresses this preface of his revolutionary book on the solar system to Pope Paul III.

5. THE STARRY MESSENGER by Galileo Galilei 1610 .

   The introduction to Galileo's Starry Messenger not only summarizies his discoveries, but also conveys Galileo's excitement about the new use of the telescope for astronomical purposes.

6. KEPLER'S CELESTIAL MUSIC by I. Bernard Cohen 1960

   The end of this summary of Kepler's work in mechanics shows how seriously Kepler took the idea of the harmony of the spheres.

7. KEPLER by Gerald Holton 1960

   This brief sketch of Johannes Kepler's life and work was initially written as a review of Max Caspar's definitive biography of Kepler.

8. KEPLER ON MARS by Johannes Kepler 1609

   Kepler's description of how he came to take up the study of Mars, from his greatest book, The New Astronomy. Kepler records in a personal way everything as it occured to him, not merely the final results.

9. NEWTON AND THE PRINCIPIA by C. C. Gillispie 1960

   This article describes briefly the events which transpired immediately before the writing of the Principia.

10. THE LAWS OF MOTION AND PROPOSITION ONE by Isaac Newton 1687

    The Latin original of Newton's statement of the three laws of motion and the proof of proposition one is followed here by the English translation by Andrew Motte and Florian Cajori.

11. THE GARDEN OF EPICURUS by Anatole France 1920

    Anatole France is best known as the writer of novels such as Penguin Island. This brief passage shows that he, along with many writers, is interested in science.

12. UNIVERSAL GRAVITATION by Richard P. Feynman, Robert Leighton, Matthew Sands 1964

    A physical concept, such as gravitation, can be a powerful tool, illuminating many areas outside of that in which it was initially developed. As the authors show, physicists can be deeply involved when writing about their field.

13. AN APPRECIATION OF THE EARTH by Stephen H. Dole 1964

    The earth, with all its faults, is a rather pleasant habitation for man. If things were only slightly different, our planet might not suit man nearly as well as it now does.

14. Mariners 6 and 7 TELEVISION PICTURES: PRELIMINARY ANALYSIS by R. B. Leighton

    Televized close-up photographs of Mars show large creaters and plains like on the moon.

15. THE BOY WHO REDEEMED HIS FATHER'S NAME by Terry Morris 1966

    A dramatized account of the boyhood of the Japanese astronomer who discovered a recent comet. The same comet, Ikeya-Seke, is described also in the article by Owen Gingerich.

16. THE GREAT COMET OF 1965 by Owen Gingerich 1966

    The director of the Central Bureau for Astronomical Telegrams describes the excitement generated by a recent comet, and reviews current knowledge of comets.

17. GRAVITY EXPERIMENTS by R.H. Dicke, P.G. Roll, J. Weber 1966

    The delicate modern version of the Eotvos experiment described here shows that the values of inertial mass and gravitational mass of an object are equal to within one ten-billionth of a percent. Such precision is seldom attainable in any area of science.

18. SPACE THE UNCONQUERABLE by Arthur C. Clarke 1962

    Arthur Clarke began to think seriously about space travel before almost anyone else. His conclusions, as seen in the article's very first sentence, are somewhat more pessimistic than is now fashionable.

19. IS THERE INTELLIGENT LIFE BEYOND THE EARTH? by I.S. Shklovskii, C. Sagan 1966

    Many scientists have argued recently that intelligent life may be quite common in the universe.

20. THE STARS WITHIN 22 LIGHT YEARS THAT COULD HAVE HABITABLE PLANETS by S. Doyle 1964

    This table lists only those stars within 22 light years of the earth that might have planets which could support human life.

21. SCIENTIFIC STUDY OF UNIDENTIFIED FLYING OBJECTS (from Condon Report) 1968
22. THE LIFE-STORY OF A GALAXY by Margaret Burbidge 1962
23. NEGATIVE MASS by Banesh Hoffman 1965
24. EXPANSION OF THE UNIVERSE by Hermann Bondi 1960
25. THREE POETIC FRAGMENTS ABOUT ASTRONOMY
26. THE DYSON SPHERE I.S. Shklovskii and Carl Sagan 1966

Film Loop - 10 Retrograde Motion - Geocentric Model

Using a specially constructed large "epicycle machine" as a model of the Ptolemaic system, the film shows the motion around the earth of a planet such as Mars.

First we see the motion from above, with the characteristic retrograde motion during the "loop" when the planet is closest to the earth. It was to explain this loop that Ptolemy devised the epicycle system. When the studio lights go up, we see how the motion is created by the combination of two circular motions.

The earth is then replaced by a camera which points in a fixed direction in space from the center of the machine. (This means that we are ignoring the rotation of the earth on its axis, and are concentrating on the motion of the planet relative to the fixed stars.) For an observer viewing the stars and planets from a stationary earth, this would be equivalent to looking always toward one constellation of the zodiac (ecliptic); for instance, toward Sagittarius or toward Taurus. With the camera located at the center of motion the planet, represented by a white globe, is seen along the plane of motion. A planet's retrograde motion does not always occur at the same place in the sky, so not all retrograde motions are visible in any chosen direction.

Several examples of retrograde motion are shown. In interpreting these scenes, imagine that you are facing south, looking upward toward the selected constellation. East is on your left, and west is on your right. The direct motion of the planet, relative to the fixed stars, is eastward, i.e., toward the left. First we see a retrograde motion which occurs at the selected direction (this is the direction in which the camera points). Then we see a series of three retrograde motions; smaller bulbs and slower speeds are used to simulate the effect of viewing from greater distances.

Note the change in apparent brightness and angular size of the globe as it sweeps close to the camera. While the actual planets show no disk to the unaided eye and appear as points of light, certainly a marked change in brightness would be expected. This was, however, not considered in the Ptolemaic system, which focussed only upon the time-table of the angular motions and positions in the sky.

Film Loop - 11 Retrograde Motion - Heliocentric Model

The machine used in Loop 10 was reassembled to give a heliocentric model with the earth and the planet moving in concentric circles around the sun. The earth (represented by a light blue globe) is seen to pass inside a slower moving outer planet such as Mars (represented by a white globe). The sun is represented by a yellow globe.

Then the earth is replaced by a camera, having a field about 25° wide, which points in a fixed direction in space. The arrow attached to the camera shows this fixed direction. (As in Loop 10, we are ignoring the daily rotation of the earth on its axis and are concentrating on the motion of the planet relative to the sun and the fixed stars.)

Several scenes are shown. Each scene is viewed first from above, then viewed along the plane of motion. Retrograde motion occurs whenever Mars is in opposition; this means that Mars is opposite the sun as viewed from the earth. But not all these oppositions take place when Mars is in the sector toward which the camera points.

1. Mars is in opposition; retrograde motion takes place.

2. The time between oppositions averages about 2.1 years. The film shows that the earth moves about 2.1 times around its orbit (2.1 years) between one opposition and the next one. You can, if you wish, calculate this value, using the length of the year (sidereal period) which is 365 days for the earth and 687 days for Mars.

In one day the earth moves 1/365 of 360°, Mars moves 1/687 of 360° and the motion of the earth relative to Mars is (1/365 - 1/687 of 360°. Thus 1/365 - 1/687 =

0.00274 - 0.00146 = 0.00128 = 1/780. Thus in one day the earth gets ahead of Mars by 1/780 of 360°; it will take 780 days for the earth to catch up to Mars again. The "phase period" of Mars is, therefore, 780 days, or 2.14 years. This is an avg. value.

3. The view from the moving earth is shown for a period of time greater than 1 year. First the sun is seen in direct motion, then Mars comes to opposition and undergoes a retrograde motion loop, and finally we see the sun again in direct motion.

Note the changes in apparent size and brightness of the globe representing the planet when it is nearest the earth (in opposition). Viewed with the naked eye, Mars does in fact show a large variation in brightness (ratio of 50:1). The angular size also varies as predicted by the model, although the disk of Mars, like that of all the planets, can be seen only with telescopic aid. The heliocentric model illustrated in this film is simpler than the geocentric model of Ptolemy, and it does give the main features observed for Mars and the other planets: retrograde motion and variation in brightness. However, detailed numerical agreement between theory and observation can't be obtained using circular orbits. With the proposal by Kepler of elliptical orbits, better agreement with observations was finally obtained, using a modified heliocentric system.

Film Loop-16 Kepler's Laws for Two Planets (Computer Program)

According to the computer program, each planet was acted on at equal time intervals by an impulsive force of blows of equal duration, directed toward a center (the sun). The force exerted by the two planets on each other is ignored in this program. In using the program, the operator selected a force law in which the force varied inversely as the square of the distance from the sun (Newton's law of universal gravitation). For clarity, the forces are not shown in this loop. The initial positions and initial velocities for the planets were selected, and the positions of the planets were shown on the face of the cathode-ray tube at regular intervals. (Only representative points are shown, although many more points were calculated in between those that were shown. The film is spliced into an endless loop, each planet's motion being represented in-definitely.

You can check all three of Kepler's laws by projecting this film on a sheet of paper and marking the position of each planet at each of the displayed orbit points. The law of areas is checked immediately, by drawing suitable triangles and measuring their areas. For example, you can check the constancy of the areas swept over three places; near perihelion, near aphelion and at a point approximately midway between.

To check Kepler's law of periods (third law), use a ruler to measure the distances of perihelion and aphelion for each orbit. To measure the periods of revolution, use a clock or watch with a sweep second hand; an alternative method is to count the number of plotted points in each orbit.

To check the first law, you must see if the orbit is an ellipse with the sun at one focus. Perhaps as good a way as any would be to use a string and two thumb tacks to draw an ellipse. First locate the empty focus which is symmetrical with the sun's position. Tie a piece of string in a loop which will just extend from one focus to a point on the ellipse, around the other focus and back to the original focus. Then put your pencil point in the loop and draw the ellipse, always keeping the string taut. How well does this ellipse (drawn assuming Kepler's first law) match the observed orbit of the planet?

EDUCATIONAL BRIEF

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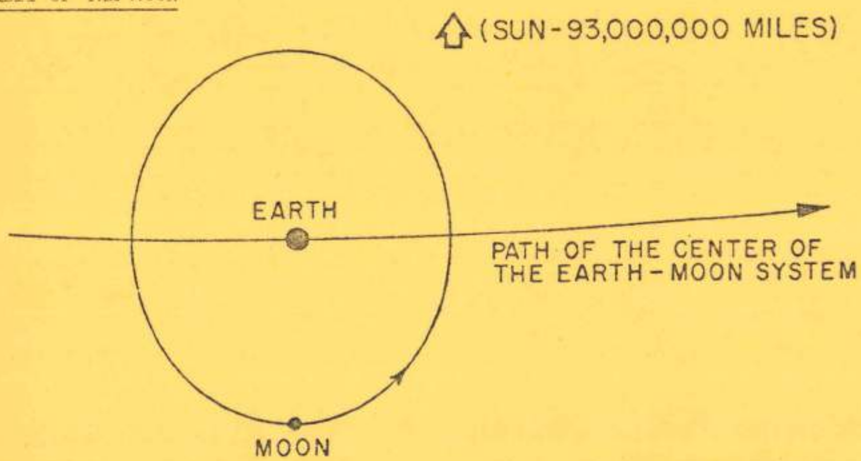
ORBIT OF THE MOON



SELENOLOGY: (Selene - Greek for the Moon) The branch of astronomy dealing with the Moon.

1. The Moon moves in an orbit about the center of the Earth-Moon system describing what is called the lunar orbit plane.

2. The orbit of the Moon is elliptical but it very nearly approaches a circle.

   a. The farthermost point from the Earth, or apogee, is 252,700 miles.

   b. The nearest point to the Earth, or perigee, is 221,000 miles.

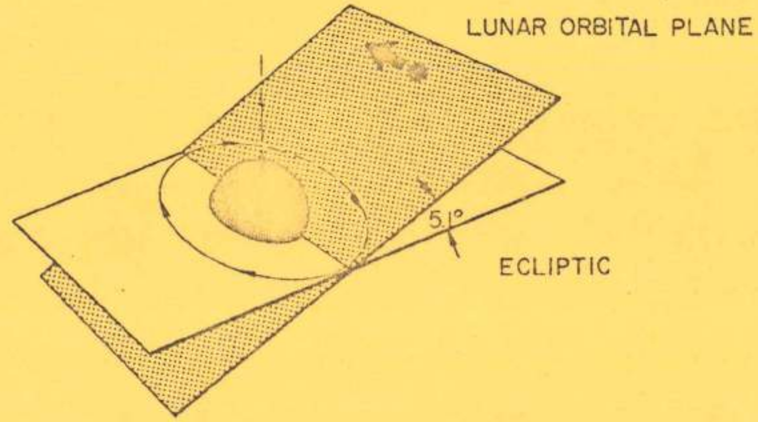
   c. The average distance from the Earth is 238,900 miles or 60.268 times the Earth's equatorial radius.

3. The time required for the Moon to complete one revolution is 27 days 7 hours 43 minutes 11 seconds.

4. The revolution of the Moon in its orbit about the Earth is in the same direction as that of the Earth about the Sun (counter clockwise).

(Over)

MOTION OF THE LUNAR ORBIT PLANE



1. The intersection of the lunar orbit plane with the ecliptic plane produces a line known as the line of nodes.

2. Due to the perturbation * produced by the Sun, the lunar orbit plane rotates about the ecliptic axis. This causes the line of nodes to revolve, completing one full revolution in 18.6 years or at a rate of 19.4 degrees annually.

3. The direction of rotation of the line of nodes is in a retrograde sense or motion in a clockwise direction with respect to the Earth's orbit.

* Perturbations - Deviations of a mass from the normal path of motion it would experience if it were one of two point masses which move subject only to their mutual gravitational attraction. These deviations are produced by forces external to the two body system itself.

REFERENCES

1. Alter, Pictorial Guide to the Moon, Crowell Co. New York, 1963

2. Baldwin, The Measure of the Moon, University of Chicago, 1963

3. Berdner & Odishaw, Science in Space, McGraw-Hill Co., Inc., 1961

4. Danby, J.M.A., Fundamentals of Celestial Mechanics, Macmillan Co., New York, 1962

EDUCATIONAL BRIEF

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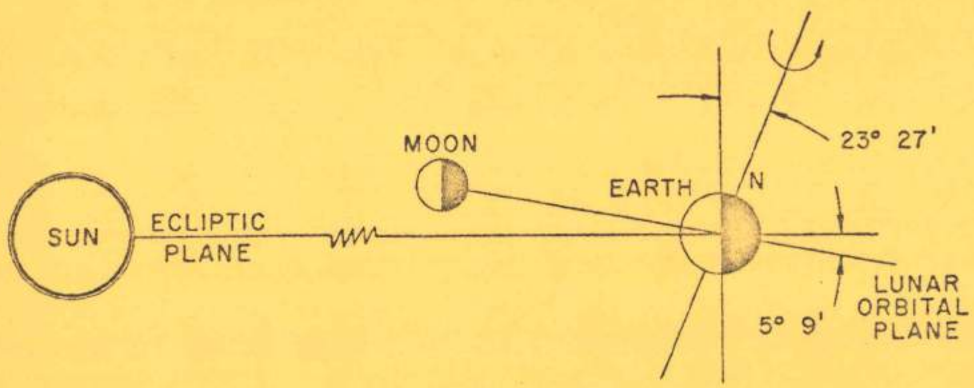
\# 2002

EARTH-MOON-SUN RELATIONS



1. An orientation of the Earth-Moon system to the Sun is made in the diagram above as viewed from within the Earth orbital plane.

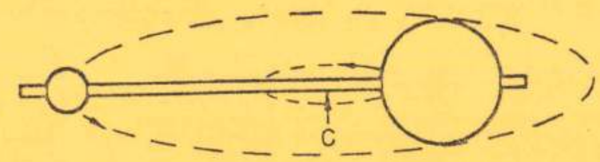
2. The orbital plane described by the Earth about the Sun is called the ecliptic plane.

3. The polar axis of the Earth is inclined 23 degrees 27 minutes to the ecliptic axis, thereby causing the Earth's equatorial plane to be permanently inclined to the ecliptic at the same angle.

4. The lunar orbital plane is the plane described by the Moon about the Earth. This plane is inclined about 5.1 degrees to the ecliptic plane.

(Over)

CENTER OF THE EARTH-MOON SYSTEM



1. The Earth and Moon are linked together by a force of mutual attraction called gravity; thus, behave as a single system.

2. This system could be compared to a dumbbell with a heavy weight on one end and a lighter weight on the other end (illustrated above). A dumbbell weighted in this manner would have a center of balance (or center of gravity) closer to the heavier end. For the unbalanced dumbbell to spin smoothly, it must be rotated about the center of balance.

3. The Earth-Moon system resembles the unbalanced dumbbell. The Earth compares to the heavy weight and the Moon to the lighter weight. Gravitational force replaces the connecting bar.

4. Like the unbalanced dumbbell, the center of balance of the Earth-Moon system is closer to the heavy weight, the Earth. In fact, this center is about 2,900 miles from the center of the Earth along a line to the center of the Moon.

5. The center of the Earth-Moon system remains inside the Earth, but is not located at the center of the Earth.

6. The center of balance for the Earth-Moon system is called the barycenter. It is this center which describes the path of the system about the Sun. Both the Earth and the Moon revolve about the barycenter.

7. The average position of the barycenter can be calculated from the relationship: $m_1 r_1 = m_2 r_2$

   r_1 = distance of the barycenter from the center of the Earth

   r_2 = distance of the barycenter from the center of the Moon

   m_1 = mass of the Earth (5.975×10^{24} kilograms)

   m_2 = mass of the Moon (7.3492×10^{22} kilograms)

   $r_1 + r_2$ = 238,900 miles

MEASURE GRAVITATIONAL DISTANCES FROM THE CENTERS OF THE ATTRACTING BODIES

In using Newton's Law of Universal Gravitation, be sure to measure the distances between attracting bodies from the centers of the bodies, and NOT from their surfaces. To explain in detail why this is so would require a lengthy essay, so we'll just hint at the reasons here. It turns out that all bodies acted upon by gravitational forces behave as though all their mass were concentrated at a single point. This point is called the center-of-mass. Thus, large gravitating bodies may be analyzed as point-particles with a mass equal to the total mass of the body and with the hypothetical "particle" located at the center of mass of the large body.

These results hold for all bodies, but things become especially convenient when the attracting bodies are spherical and uniform. (By uniform we mean that the object's mass is spread evenly throughout.) When the body is both spherical and uniform, the center of mass is exactly at the geometric center of the body. For the purposes of this course you may consider all gravitating bodies to be spherical and uniform, which in the case of planets and their satellites turns out to be a very good approximation.

Thus, if we wish to compute the gravitational force between the earth and a satellite orbiting the earth at an altitude of 300 km, we consider the earth to be a "particle" with mass equal to the mass of the earth and which is located at the center of the earth. We must then calculate the distance from the satellite to the center of the earth (which is 300 km plus the radius of the earth) and plug this number, along with the masses of the satellite and earth, into Newton's law of Universal Gravitation.

II. "g" ON THE EARTH

In this note, we will show why the magnitude of the acceleration of gravity at the earth's surface is 9.8 m/sec^2.

The gravitational force between a planet and a relatively small body on or near the surface of the planet is what we usually call the weight of the body. Using the universal gravitational law, the weight can be expressed as:

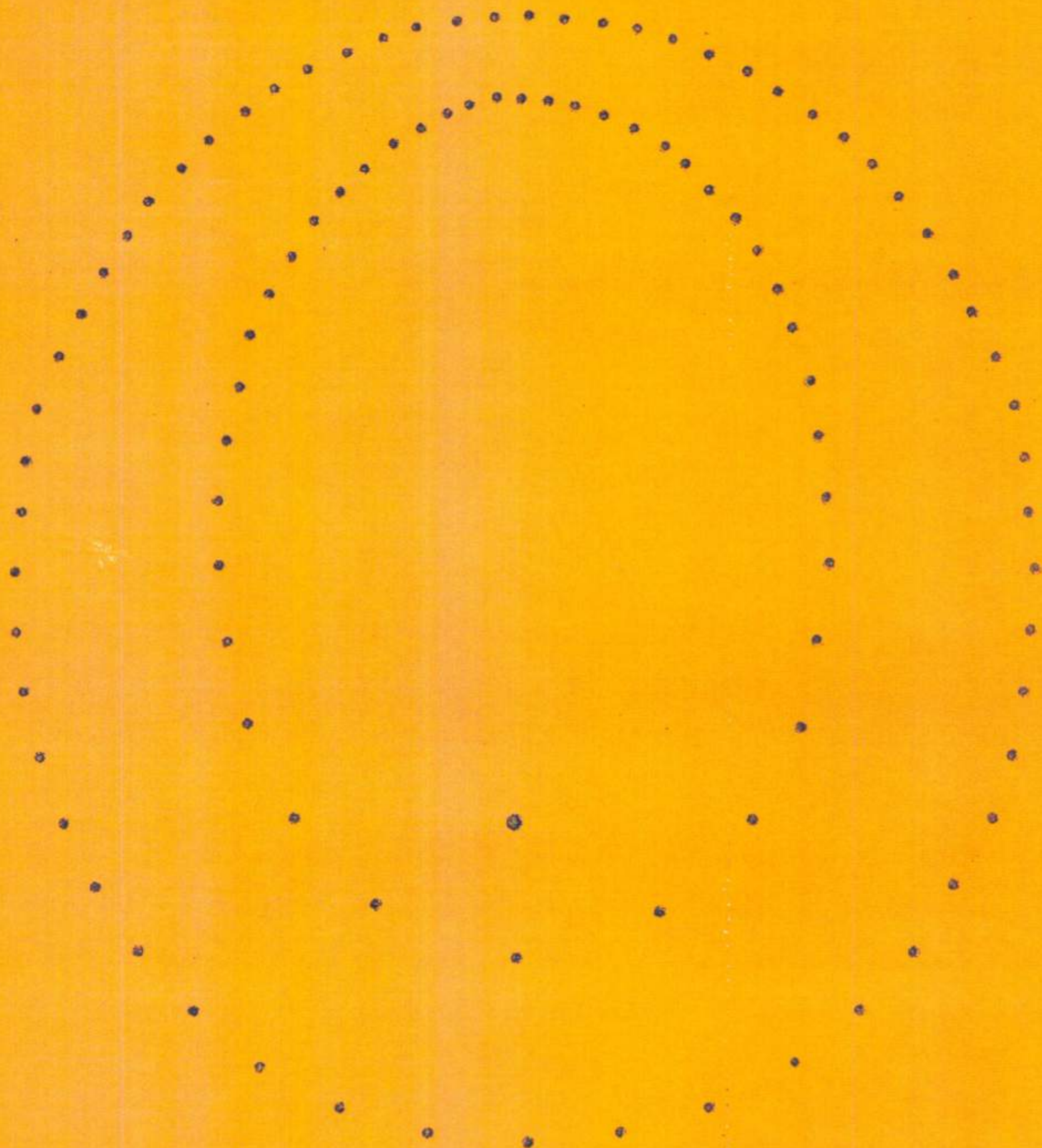
$$\text{weight} = \text{force of gravity at surface} = G \cdot m_p \cdot m \,/\, R_o^{\,2}$$

where m_p is the huge mass of the planet and R_o is the distance from the center of the planet to the surface. From Newton's law of motion, we can also write the force of gravity as: $F = mg$. where g is just the acceleration due to the pull of gravity at the surface of the planet. Equating these two relations for the gravitational force of a body at the surface of a planet we get:

$$mg = G \cdot m_p \cdot m/R_o^{\,2} \qquad g = G \cdot m_p/R_o^{\,2}$$

The mass of the small body can be cancelled (as shown) from both sides of the equation so that g, the acceleration due to gravity at the surface, depends on only the mass and radius of the planet. Using the values of the earth's mass and radius from the text in the above equation, find g which should = 9.8 m/sec^2

Because the universal law of gravitation says that masses exert a mutual force on each other, you exert a force on the earth - in addition to the earth exerting a force on you. The force of gravity on the average person standing on the earth's surface is about 650 Nt. (150 lbs) So the average person "pulls on" the earth with a force of 650 Nt. However, since the mass of the earth is so much larger than your mass, the effect of the force you exert on the earth can be neglected.



GENERAL RELATIVITY

According to the basic ideas of the dynamics of Galileo and Newton, the laws of motion are valid only in an "inertial frame," that is, a system which is at rest or moving with constant velocity. These two possible states are indistinguishable, because forces are detected only when a body is accelerated. Newtonian dynamics begins to break down for bodies moving with speeds near that of light, but Einstein's theory of special relativity has satisfactorily extended the basic notions of classical dynamics to the range of all possible speeds.

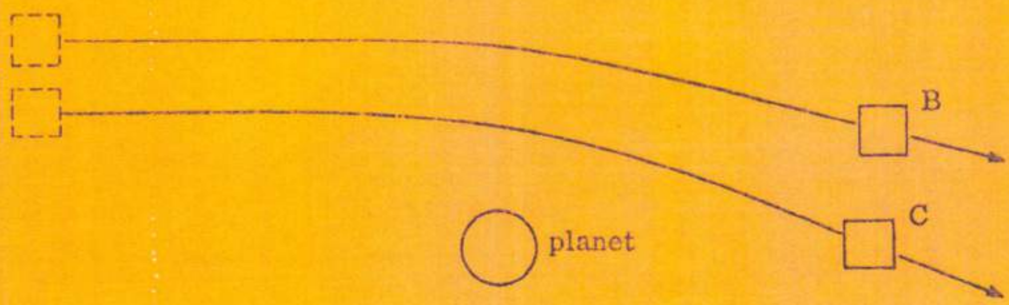
The existence of gravity, however, complicates matters considerably. We know that the surface of the earth is not an inertial frame in the Newtonian sense. If we project a body upward, it does not continue to move up but reverses direction and falls back to the floor. We generally describe the situation by saying that we are in an inertial frame, but that there is a force — the "force of gravity" — also acting on the body. Indeed, it is impossible to isolate any body from the gravitational forces exerted by the mass of the rest of the universe. Thus, the definition of an inertial frame in terms of a state of "rest" or "uniform motion" is only an approximation in a universe which contains mass, since all bodies in the universe interact with one another to a certain extent.

Einstein, in his theory of general relativity, attempts to restate the laws of physics in a manner which takes into account the equivalence of gravitation and acceleration. This theory is, as is well known, of enormous mathematical complexity, and it is hardly to be recommended as an exercise for high school students. However, the basic concepts of the theory are simple enough to be illustrated by a few examples.

Suppose we were drifting along in space in a rocket ship in the "weightless" condition which every modern student understands. In this state we could perform dynamical experiments on ourselves and the surrounding objects floating in our room and quickly decide that we were in a true inertial frame, since the laws of motion would be obeyed exactly. If we jumped up from the floor, we would most certainly hit the ceiling, no matter how gently we jumped. Now imagine, during the course of these experiments, that this "weightless" interlude suddenly comes to an end. We would find ourselves standing on the floor, feeling our own weight. All of the surrounding objects which a moment earlier had been floating around the room would suddenly fall to the floor. What could we conclude had happened?

Without looking out the window of the rocket ship (this, incidentally, wouldn't really help), we could conclude that one of two things must have happened. Either our rocket ship had suddenly accelerated upward or we have just *come to rest* (with the floor "down") on the surface of some planet. In either event, it would be obvious that the laws of physics had suddenly changed and we were no longer in an "inertial frame." We would be hard put to devise an experiment to tell whether we had in fact been accelerated or had entered a gravitational field, since either occurrence would have had the same effect on our observations. This is precisely what is meant when we say that the "inertial mass" of a body and its "gravitational mass" are inextricably interrelated. It is this "principle of equivalence" which lies at the heart of general relativity — that is, that the effects of an acceleration *are in principle* indistinguishable from the effects of a gravitational field. But what about our inertial frame? We had a perfectly good way of measuring this earlier, simply by observing the validity of the laws of motion. Evidently, during this time interval, when our rocket ship was an "inertial frame," we must have been in uniform motion in the absence of a gravitational field (impossible in our universe), or *in "free fall" in a gravitational field.* Thus, the true inertial frame is not one which is in uniform motion in our space, but rather one which is in free fall.

General relativity, then, is simply the description of nature as viewed by an observer in a true inertial frame — patiently doing experiments in a freely falling box. To this observer, the laws of dynamics would be exact within his box, but he would quickly conclude that the space outside his box had some very peculiar properties. To illustrate, imagine that we are in free fall in such a box, moving at a fairly high speed through space. Through our window we observe another person moving along parallel to us at the same speed and also doing dynamical experiments. We could compare results by flashing messages back and forth and agree that our results were identical and both of us were in inertial frames. If we then arrived in the vicinity of a large planet (in such a



manner that we didn't conclude the experiment by hitting the planet), both boxes would be deflected by the gravitational field, as shown above.

Our box, labeled *B* in the figure, would be deflected somewhat, and the other box, labeled *C*, would be deflected rather more since it is closer to the planet. Since both we and the observer in *C* are in free fall, within the confines of our own boxes neither of us would feel or be able directly to measure any effects due to the acceleration. We are both still in inertial frames as far as our internal measurements can tell us. However, since we are now obviously drifting apart, we would have to conclude that the distance between us suddenly started to increase without either of us apparently having accelerated. Instead of describing what has happened in terms of some "force" which we could not detect, we might rather describe the occurrence in terms of a distortion in space and time which we could observe.

This is the essence of the theory of general relativity. The gravitational field, according to this viewpoint, causes a "warping" of space and time in the vicinity of a mass. Gravity then enters not as a "force" but as a property of space and time.

This whole procedure would be rather fruitless if it did not permit the extension of the laws of physics and the prediction of certain observable events. In fact, general relativity predicts certain facts which are not correctly explained either by Newtonian dynamics or special relativity. One of these involves the curvature of light in a gravitational field. In general relativity, the "shortest distance between two points" is not a straight line but is a "geodesic," the path which light would follow in traveling between the two points, bent by the gravitational field. According to special relativity, light has energy, and since mass and energy are equivalent, light will be deflected by a gravitational field. However, the amount of deflection is not given correctly by this simple argument because the force of gravity on a moving object depends on its speed. We can actually observe the bending of light which travels from distant stars and passes very close to the sun during a solar eclipse. The effect is very small, about two seconds of arc, but it is accurately measurable, and agrees precisely with the predictions of general relativity.

The theory of general relativity is of considerable importance in physics, but is more the working tool of the cosmologist in his study of the universe. The cosmologist is concerned with such questions as "Is the universe finite?" — that is, would a geodesic directed outward from the known universe go on forever; or traveling always out, return to its starting point? Much of the effort of modern astronomers is directed toward answering this question. At present, we do not know whether the space we know is finite or infinite, but it is certainly "warped," in the relativistic sense.

UNIVERSAL GRAVITATION (31 min.)

J. N. P. Hume and D. G. Ivey, University of Toronto

Using artificial satellites, this film shows how one could be led to the law of universal gravitation in an imaginary solar system consisting only of a sun and a moonless planet.

Summary:

In the one-planet solar system can one determine whether the planet revolves about the sun, or vice versa? It is shown experimentally (with Dry Ice pucks) that, for a two-body system of this sort, the larger the mass of one body relative to the other, the more nearly the fixed center of rotation for both lies at the position of the larger mass. Hence, a reference frame attached to the larger mass will be very nearly an inertial frame.

The orbits of various satellites are displayed on an oscilloscope face (actually the oscilloscope output of a computer). When a single satellite is launched from the planet, the planet provides an inertial frame of reference. The satellite moves in an elliptical path with the planet at one of the foci of the ellipse, and the radius vector from planet to satellite sweeps out equal areas in equal times. This equal area law shows that the gravitational force on the satellite is directed toward the planet, and this together with the observed orbit indicates that it varies inversely with the square of the separation of planet and satellite.

Then additional satellites are launched, each obeying the above laws, and it is also observed that the ratio of the cube of the average separation of planet and satellite to the square of the period of revolution has the same value for all the satellites. However, this ratio, calculated for the revolution of the sun about the planet, gives a value about 300,000 times as large.

Then a satellite was launched so far out that it orbited around the sun. Viewed from the planet, its orbit was complicated; from the sun, however, it was an ellipse. The value of R^3/T^2 for this satellite relative to the sun turned out to have the same large value as the R^3/T^2 of the original planet relative to the sun (or of the sun relative to the planet). A second planet has been added to the solar system.

From these observations, one is led to the gravitational law of attraction

$$F = K\frac{m}{R^2}$$

where K is a constant depending only on the attracting body. If K is then taken proportional to the mass of the attracting body, the proportionality constant G is then a universal constant, and $F = G\frac{m_1 m_2}{R^2}$.

Finally, it is pointed out that only ratios of masses can be obtained from a study of satellite motion and that a separate experiment, such as the Cavendish experiment, is needed to determine the numerical value of G and hence to allow mass determinations.

Points for Discussion and Amplification:

(a) This film provides an alternative to the actual historical development by which the law of universal gravitation was obtained. In general, it is wise to use this film to help understand the textbook argument after the students have studied Chapter 22 of the PSSC text.

(b) Note that on a planet with advanced technology, the problem of extracting the inverse-square law from the orbit is exceedingly simple compared to that faced by Newton, who had to develop both the laws of dynamics and much of the mathematics to effect a solution.

(c) The acceleration, $4\pi^2R/T^2$, of the sun relative to the earth (which is the same as that of the earth relative to the sun) is 0.006 m/sec^2 and not 0.003 m/sec^2 as stated in the film.

One may well point out here that the identification of the gravitational field strength $\vec{g}$ with the acceleration of a freely falling body, without regard to frames of reference, can be in error. For example, the distance from the earth to the sun is about 23,000 earth-radii. Hence, the value of $\vec{g}$ at the sun's distance from the earth is

$$\left(\frac{1}{23,000}\right)^2 \times 9.8\,\text{nt/kg or about } 2 \times 10^{-8}\,\text{nt/kg}.$$

Were the sun's mass small compared with that of the earth, so that the earth frame of reference were inertial for this two-body system, this value of $2 \times 10^{-8}\,m/sec^2$, instead of 0.006 m/sec^2 would indeed be correct for the acceleration of the sun towards the earth.

(d) The mass of a planet which has an orbiting satellite can be obtained from the motion of that satellite (assuming the motion is known independently). The mass of a moonless planet is obtained by more involved calculations - using, for example, the observed perturbations of its orbit by other planets.

(e) A statement occurs in the film that "Newton knew that the moon is not only attracted to the earth but also by the sun, so its orbit (about the earth) is perturbed slightly." This statement may be confusing to a student who realizes that the sun's attraction for the moon is a little more than double the earth's attraction for it. The perturbations in the moon's orbit, as seen from earth, are due to the relatively small variations in the moon's distance from the sun.

(f) In considering satellite orbits, the variation of $\vec{g}$ with altitude becomes of interest. An alternate relation to the PSSC text development $\left[g = G\frac{M_e}{(R+h)^2}\right]$ is $\Delta g = g_0 \times \frac{-2h}{R}$, which will give approximate results much more readily. This relation is developed as follows:

$$\Delta g = g - g_0 = \frac{GM_e}{(R+h)^2} - \frac{GM_e}{R^2} = \frac{GM_e}{R^2}\left[\frac{R^2}{(R+h)^2} - 1\right] = g_0\left[\frac{-2hR - h^2}{(R+h)^2}\right].$$

[OVER]

Universal Gravitation - (3)

For $h << R$, Δg then becomes

$$\Delta g \cong g_0 \times \frac{-2h}{R},$$

which holds to within 10% for altitudes up to about 300 miles or 1% up to 30 miles.

(g) An alternative summary to that in the film follows.

(1) From the observed equal area law, it follows that the force on a satellite is directed toward the attracting body.

(2) From the fact that the orbit is an ellipse with the attracting body at one focus, it follows that the force varies inversely with the square of the distance from attracting body to satellite.

(3) From the observation that for all satellites about the same attracting body R^3/T^2 has the same value (R the average distance and T the period), it follows that the force is proportional to the satellite mass m_s.

This can be shown simply for circular orbits.

$$F = \frac{c}{R^2} = m_s a = m_s \frac{4\pi^2}{T^2} R$$

from which $\frac{R^3}{T^2} = \frac{c}{4\pi^2 m_s}$.

Since R^3/T^2 does not depend on the satellite mass, the constant on the right-hand side of this equation must itself be proportional to m_s.

$$c = k m_s$$

therefore $$F = \frac{k m_s}{R^2} \qquad (1)$$

where k depends only on the attracting body.

(4) If we assume that the force exerted by the satellite on the attracting body is equal in magnitude to that exerted on the satellite by the attracting body, there follows

$$\frac{k_a m_s}{R^2} = \frac{k_s M_a}{R^2}$$

where the subscript a refers to the attracting body and s to the satellite.

From this $$\frac{k_a}{M_a} = \frac{k_s}{m_s}$$

Since the left-hand side depends only on the attracting body, and the right-hand side only on the satellite, this equality means that the ratio k/m doesn't depend on either body. In other words, it is a universal constant, let us say G.

Universal Gravitation - (4)

Using Eq. (1) in which the k is the same as k_a, there follows

$$F = G\frac{M_a m_s}{R^2}$$

and the law of universal gravitation follows as a generalization to any two masses and M_2 as

$$F = G\frac{M_1 M_2}{R^2}$$

Finally, the ratio of masses of sun to earth is given by

$$\frac{M_{sun}}{M_{earth}} = \frac{(R^3/T^2) \text{ for planets about the sun}}{(R^3/T^2) \text{ for satellites about the earth}} = \frac{3.3 \times 10^{18}}{1 \times 10^{13}} = \frac{1}{3} \times 10^6.$$

WRITTEN EXERCISE CHAPTER 5 THE SMALL WORLD

There has been discovered recently a new planet rotating in an orbit midway between the moon and the earth. Bowling planet, as it is called contains the two large island countries, Pinlania and Matchland. Their favorite pastime oscillates from water battles between countries, to sitting and watching their moon, Tennisson, revolving about them in space.

Planet, (Bowling)
mass = 6 kg
radius = 12 cm

Pin people
mass = 2 gm
height = 2.5 cm

Distance from center of planet (Bowling) to center of moon (Tennisson) = 3 meters

Moon (Tennisson)
mass = 150 gm
radius = 4 cm

Match people
mass = 4 gm
height = 4 cm

Answer the following questions concerning the planet. Put all answers in MKS units using scientific notation. Show all work in acceptable fashion. If work is not highly organized, it will not be accepted.

1. What is the acceleration due to gravity on the surface of the planet Bowling?

2. What is the attraction between a Pin man and his planet (Bowling)?

3. What is the gravitational attraction between the planet and its satellite?

4. What is the planet's gravity out in the moon's orbit?

5. What is Tennisson's period of revolution?

6. What is the centripetal acceleration of Tennison toward Bowling?

7. What is the velocity of Tennisson in its orbit?

8. What is the gravitational attraction (not militarily) between a Pin man and a Match man at 5 cm?

9. The escape velocity from Pinlania into outer space to the first approximation, follows the formula v^2 = 2gr. What is the velocity needed to travel to the earth? (r = 12 cm, the radius of the planet)

10. What would need to be the tangential velocity of Bowling in order for the people of Match land to always see their moon in the same position?

11. If a Matchland boy married a Pinlania girl, name the outstanding characteristics of their offspring?

problem 1

42 NAME KEY

TEST ITEMS CHAPTER 22

1. Fig. 22-10 shows the orbits of the planets about the sun. When does Pluto move (a) fastest? closest to sun

2 (b) slowest? FARTHEST FROM SUN

problem 10

3 2. If a small planet were discovered whose distance from the sun was eight times that of the earth, how many times longer would it take to circle the sun? 22.7 TIMES

$\frac{R^3}{T^2} = K \qquad \frac{R_e^3}{T_e^2} = \frac{R_p^3}{T_p^2} \qquad \frac{T_p^2}{T_e^2} = \frac{R_p^3}{R_E^3} \Rightarrow \frac{T_p}{T_E} = \sqrt{\frac{8^3 \cancel{R_e^3}}{\cancel{R_e^3}}} = 8^{3/2} = \sqrt{516}$

problem 13

3 3. The radius of the moon's orbit is 30 times greater than the radius of the earth. How many times greater is the acceleration of a falling body on the earth than the acceleration of the moon toward the earth? 900 TIMES Greater

$F = G\frac{\cancel{m}M}{R^2} = \cancel{m}g \qquad \frac{g_e}{g_m} = \frac{R_m^2}{R_e^2} \quad \frac{30^2 R_e^2}{\cancel{R_e^2}} =$

problem 15

3 4. At what height above the earth's surface will a rocket have one-fourth the force of gravitation on it that it would have at sea level? Express your answer in earth radii. 1 R_E above sea level

$\frac{R_?^2}{R_e^2} = \frac{g_e}{g_?} \qquad R_?^2 = \frac{g_e}{g_?} R_e^2 \Rightarrow R_? = \sqrt{\frac{4 g_e}{g_e}}\, R_e$

problem 17

3 5. Find the weight of a 200-kg man on Jupiter. 4890 ~~4890 NT~~ 4930

$F = G\frac{mM}{R^2}$

$= .667\times10^{-11}\frac{NT\,M^2}{Kg^2} \times \frac{200Kg \times 1.9\times10^{27}Kg}{(7.2\times10^7)^2 M^2} \qquad \frac{6.67\times2\times1.9\times\cancel{10^{27}}}{7.2\times7.2\times\cancel{10^{14}}} \qquad 10^3$

problem 19

3 6. A satellite circles the earth once every 98 minutes at a mean altitude of 500 km. Calculate the mass of the earth. (Since the answer to the problem may be found in the text, you must show how you arrived at the answer.) An answer without work receives zero credit. 5.5×10^{24} Kg

$F = \frac{\cancel{m}4\pi^2R}{T^2} = G\frac{\cancel{m}M}{R^2}$

$M_E = \frac{4\pi^2R^3}{GT^2} = \frac{4\pi^2\times(5\times10^5+6.4\times10^6 M)^3\ Kg\,sec^2}{6.67\times10^{-11}\ M^3 \times (5.88\times10^3)^2 sec^2}$

 KEY

7. problem 23 Assume the earth is perfectly round and has a radius of 6400 km.

3 (a) How much less does a man with a mass of 100 kg apparently weigh at the equator than at the poles because of the rotation of the earth? 3.4 NT LESS

$F = ma \Rightarrow w = mg \Rightarrow w = 100\,kg \times 9.8\,\frac{M}{sec^2} = 980\,NT$ (at N-pole)

$V = \frac{2\pi R}{T} \Rightarrow F_c = \frac{m\,4\pi^2 R}{T^2} = \frac{100\,kg \times 4\pi^2 \times 6.4 \times 10^6 M}{8.61 \times 8.61 \times 10^8\,sec^2} = \frac{4 \times 6.4 \times \pi^2}{8.61 \times 8.61} = 3.4\,NT$

3 (b) How fast would the earth have to spin in order that he would exert no force on a scale at the equator? 7.9×10^3 M/sec

$F = mg = w = \frac{mv^2}{R} \Rightarrow V^2 = \frac{wR}{m} = \frac{\cancel{m}gR}{\cancel{m}} \Rightarrow v = \sqrt{9.8\frac{M}{sec^2} \times 6.4 \times 10^6 M}$

$T = 1.4$ hrs

3 (c) How many times larger is the speed of rotation in (b) than the actual speed? 17 TIMES

$\frac{V}{V_{actual}} = \frac{T_{actual}}{T} = \frac{24}{1.4}$

8. problem 24 A 10,000-kilogram spaceship is drifting on a long mission toward the outer edge of the solar system. It has put out a small experimental satellite which revolves around it at a distance of 120 meters under their mutual gravitational attraction.

3 (a) What is the period of revolution of the satellite? 1.01×10^7 sec (120 days)

3 (b) What is the speed of the satellite? 7.5×10^{-5} M/sec

$\frac{G\cancel{m}M}{R^2} = \frac{\cancel{m}4\pi^2 R}{T^2}$

$T = 2\pi R\sqrt{\frac{R}{GM}}$

$6.28 \times 1.2 \times 10^2 \sqrt{\frac{120}{10^4 \times .667 \times 10^{-10}}}$

$= 7.54 \times 10^2 \times 1.34 \times 10^4$

$V = \frac{2\pi R}{T} = \frac{2\pi \times 1.2 \times 10^2 M}{1.01 \times 10^7 sec}$

9. problem 27 Astronomical observations indicate that the sun is describing a circular orbit around the center of our galaxy. The radius of the orbit is about 30,000 light-years ($=2.7 \times 10^{20}$m) and the period of one complete revolution is about 200 million years. In this motion the sun is acted on by the gravitational pull of the great quantity of stars lying inside its orbit.

4 (a) Calculate the total mass of these stars from the data given. ~~1.95×10^{31} Kg~~ 2.92×10^{41} Kg

$R = 2.7 \times 10^{20} M$

$T = 2 \times 10^8$ years

$\frac{m_s 4\pi^2 R}{T^2} = G\frac{m_s M_?}{R^2}$

$M_? = \frac{4\pi^2 R^3}{GT^2}$

$\frac{39.5 \times 2.7 \times 2.7 \times 2.7 \times 10^{60} M^3}{(2 \times 10^8\,year)^2} \times \left(\frac{1\,year}{3.16 \times 10^7 sec}\right)^2 = \frac{3.95 \times 2.7 \times 2.7 \times 2.7}{6.67 \times 10^{-11}\, 2 \times 2 \times 3.16 \times 3.16} \times 10^{61} \times 10^{30}\ \times 10^{31}$

$\frac{19.5 \times 10^{30}}{6.67 \times 10^{-11}}$

1.95×10^{31}

4 (b) How many stars of mass equal to the sun (2×10^{30} kg) does this represent? 1.46×10^{11} STARS

$\frac{2.92 \times 10^{41} \cancel{kg}}{2 \times 10^{30} \cancel{kg}/star}$ 1.46×10^{11} STARS

TEST ITEMS UNIT III CHAPTER 22

1. Between September 21 and March 21 there are three days fewer than between March 21 and September 21. These are the dates when day and night are of equal length, and between them the earth moves 180° around its orbit with respect to the sun. From this and Kepler's law of equal areas, explain how you can determine the part of the year during which the earth is closest to the sun.

3

Sept 21 - Mar 21 shorter => time of more rapid motion ∴ closest to sun then

Kepler - = areas in = times

2. A 10,000-kilogram space ship is drifting on a long mission toward the outer edge of the solar system. It has put out a small experimental satellite which revolves around it at a distance of 120 meters under their gravitational attraction. What is the period of revolution of the satellite? 1.01×10^7 sec (120 days) What is its speed in its orbit? 7.5×10^{-5} M/sec

4

$m_{ship} = 10^4$ kg

$R_{satellite} = 120$ M

$T = ?$

$G \frac{\cancel{m} M_E}{R^2} = \frac{\cancel{m} 4\pi^2 R}{T^2}$

$T^2 G M = 4\pi^2 R^3$

$T^2 = \frac{4\pi^2 R^3}{GM}$

$T = 2\pi R \sqrt{\frac{R}{GM}}$

$T = 6.28 \times 1.2 \times 10^2 \sqrt{\frac{120}{10^4 \times .667 \times 10^{-10}}}$ 10^2 10^{-6}

$T = 7.54 \times 10^2 \times \sqrt{\frac{1.2}{.667} \times 10^8}$

$T = 7.54 \times 10^2 \times 1.34 \times 10^4$

$T =$

$V = \frac{2\pi R}{T}$

$= \frac{6.28 \times 1.2 \times 10^2 M}{1.01 \times 10^7 sec}$

6

3. An earth satellite, A, moves in a circular orbit with radius four times the earth's radius. Compare its period of revolution with that of a second satellite, B, which is very close to the earth's surface.

(a) 8:1

1st

second

(b) What is the ratio of their speeds? 1 : 2

(c) What is the ratio of the centripetal force acting on the two satellites? 1:16

$R_E = x$

$R_A = 4x$

$R_B = x$

(a) $\frac{R_A^3}{T_A^2} = \frac{R_B^3}{T_B^2}$

$\frac{T_A}{T_B} = \sqrt{\frac{R_A^3}{R_B^3}}$ $\frac{64x^3}{x^3}$

b $v = \frac{2\pi R}{T}$

$\frac{V_A}{V_B} = \frac{R_A T_B}{T_A R_B}$

$\frac{4\cancel{x}}{\cancel{x}} \times \frac{1}{8}$

 13

4. Describe the motion of an artillery shell fired horizontally from the viewpoint of (a) an observer moving along in a jet plane which has a horizontal velocity equal to the horizontal component of velocity of the shell; (b) an observer in free fall (no air resistance).

4 (a) shell drops straight down $a = g$

(b) shell goes away in a straight line

5. The earth attracts a one-killogram mass at its surface with a force of 9.8 newton. The radius of the earth is 6.38 x 10^6 meters. Using Newton's law of universal gravitation, calculate the mass of the earth.
3 $G = 0.667 \times 10^{-10}$

5.98×10^{24} Kg

$$F = G\frac{M_1 M_2}{R_e^2} \Rightarrow M_E = \frac{F R_E^2}{G M} = \frac{9.8\,nt \times (6.4\times10^6)^2\,m^2\ Kg\ sec^2\ Kg\ nt}{.667\times10^{-10}\ m^3\ Kg\ nt\ sec^2}$$

$$\frac{9.8\times6.4\times6.4\times10^{12}}{.667\times10^{-10}}$$

6. An automobile weighing 3200 newtons is traveling around an unbanked circular trach which is one mile in circumference at a linear speed of 75 km/hr. Calculate
4 (a) the centri~~fugal~~ force exerted by the car, 534 NT
(b) the centripetal acceleration. $1.63\ M/sec^2$

$C = 2\pi R$

$R = \frac{C}{2\pi}$

$$F = ma = \frac{mv^2}{R} = \frac{3200\,nt}{9.8\,M}\ \frac{sec^2\ Kg\ M}{1\,nt\ sec^2} \times \frac{75\,Km}{hr} \times \frac{75\,Km}{hr} \times \frac{2\pi}{1\,mile} \frac{.6\,mile}{1\,Km} \frac{10^3 M}{1\,Km} \frac{1\,hr \times 1\,hr}{3.6\times3.6\times10^6\,sec^2}$$

$$2\pi \times \frac{3.2\times7.5\times7.5\times6\times10^7}{9.8\times3.6\times3.6\times10^6}$$

$$a = \frac{F}{m} = \frac{85\,nt}{3200\,nt} \times \frac{9.8\,M}{sec^2} =$$

7. A pendulum is 8 feet long. Calculate its period, letting the acceleration of gravity equal 32 ft/sec^2. π sec = 3.14 sec
2

$$T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{8}{32}} =$$

TP THE SUN IS OVERHEAD AT NOON AND WE ARE AWARE THAT THE SUN PULLS ON US, THEN COMPARED TO MIDNIGHT SHOULDN'T WE BE LIGHTER? IF SO, SHOULDN'T WE BE ABLE TO MEASURE IT ON BATHROOM SCALES?

TP WE GENERALLY MEASURE ACCELERATION AS BEING $9.8\ \mathrm{NT/KG}$ WHICH IS $9.8\ \mathrm{M/SEC^2}$. THIS IS REPRESENTS THE PULL OF THE EARTH ON A GIVEN MASS. HOW DOES THIS PULL OF THE SUN COMPARE? IS IT NOTICIBLE? IS IT A MEASURABLE EFFECT?

TP THINK OF THE SUN DIRECTLY OVERHEAD. HOW MUCH IS IT PULLING ON US? THE SUN IS ALSO PULLING ON THE EARTH WE ARE STANDING ON. WE KNOW IN FACT THAT THIS PULL CAUSES AN ACCELERATION TOWARDS THE SUN. THUS WE ARE IN AN ACCELERATED REFERENCE FRAME WITH THE BATHROOM SCALES PUSHING UP ON US. DOESN'T THAT MAKE US HEAVIER AT NOON? SO WHICH TALKS HARDER? IS IT THE FACT OF THE SUN PULLING US UP OFF THE BATHROOM SCALES OR IS IT THE FACT THAT THE SUN IS PULLING UP ON THE EARTH WHICH IS IN TURN PUSHING UP ON THE SCALES? WHICH IS THE STRONGER EFFECT?

TP TO FIND THE SUN'S EFFECT ON THE EARTH, YOU HAVE TO WORRY ABOUT INTREGATING THE SUN'S GRAVITATIONAL FIELD OVER THE WHOLE EARTH WHICH SOUNDS LIKE A COMPLICATED PROBLEM. BUT NEWTON SOLVED THAT ON FOR US THAT WE CAN APPROXIMATE, THAT WE CAN GET THE RIGHT ANSWER BY TAKING THE MASS OF THE EARTH CONCENTRATED AT THE CENTER. AT NOON WITH THE SUN DIRECTLY OVERHEAD WE ARE OBVIOUSLY CLOSER TO THE SUN THAN THE CENTER OF THE EARTH. THEREFORE THE SUN PULLS HARDER ON US THAN THE EARTH SO IN FACT WE ARE LIGHTER AT NOON THAN WE WOULD BE IF THE SUN WERN'T THERE.

TP BUT WHAT ABOUT AT NIGHT? AT NIGHT WE ARE FARTHER FROM THE SUN THAN THE CENTER OF THE EARTH. THUS THE EARTH IS BEING ACCELERATED TOWARDS THE SUN FASTER THAN WE ARE SO THE BATHROOM SCALES (IF FASTENED TO THE EARTH) IS BEING PULLED OUT FROM UNDER US. WE MUST THEN CONCLUDE THAT WE ARE ALSO LIGHTER AT MIDNIGHT THAN WE WOULD BE IF THE SUN WERN'T THERE. WE'RE LIGHTER BOTH AT NOON AND AT MIDNIGHT THAN WE WOULD BE AT 6 A.M. or 6. P.M. WHEN THE SUN PULLS US TO THE SIDE RATHER THAN UP OR DOWN. THEREFORE WE'RE ON A TWELVE HOUR CYCLE RATHER THAN A 24-HOUR CYCLE.

TP IS THE DIFFERENCE IN WEIGHT SOMETHING WE CAN SEE ON THE BATHROOM SCALE? LETS TRY SOME CALCULATIONS AND SEE. THE GRAVITATIONAL PULL OF THE SUN CAN BE FOUND USING ANY OF THE FOLLOWING:

$$F = ma = \frac{GmM}{R^2} = \frac{mv^2}{R} = m\left(\frac{2\pi}{T}\right)R$$

TO CALCULATE THE ACCELERATION DUE TO THE SUN WE USE:

$$a = \frac{GM}{R^2} = \frac{v^2}{R} = \left(\frac{2\pi}{T}\right)^2 R$$

$$a = 6.67 \times 10^{-11}\ \frac{\mathrm{NT\,M^2}}{\mathrm{KG^2}} \times \qquad = 6.\ \times 10^{-3}\ \frac{\mathrm{NT}}{\mathrm{KG}}$$

COMPARING THIS TO THE 9.8 $\frac{NT}{Kg}$ WE SEE THAT THIS FORCE IS VERY SMALL BUT WE MUST REMEMBER THIS IS NOT THE FORCE WE'RE GOING TO READ ON THE BATHROOM SCALES. WHAT WE'RE GOING TO READ IS THE DIFFERENCE BETWEEN THE FORCE AT THE SURFACE OF THE EARTH AND THE ONE AT THE CENTER OF THE EARTH. SO WE REALLY MUST DO A DIFFERENCE CALCULATION. THE FORCE PER MASS DIFFERENCE IS:

$$\frac{F}{m} = \frac{GM}{R-R_E} - \frac{GM}{R} = \qquad = 5\times10^{-7}\ \frac{NT}{Kg}$$

WHICH IS 1 PART IN 10 MILLION DOWN FROM 9.8 $\frac{NT}{Kg}$. WE MUST CONSIDER THIS A NEGLIGIBLE (sp) EFFECT AS WE DON'T READ OUR BATHROOM SCALES THAT WELL.

A. DOES THE MOON HAVE A SIMILAR EFFECT ON US? WE KNOW THE MOON IS ALOT SMALLER BUT IT ALSO IS ALOT CLOSER. WE MAKE SIMILAR CALCULATIONS AS BEFORE BEING CAREFUL THAT WE USE THE MASS OF THE MOON. AGAIN USING our calculator we get:

$$\frac{F}{m} = \frac{GM}{R^2} = \frac{v^2}{R} = \left(\frac{2\pi}{T}\right)^2 R = \qquad 3.2\times10^{-5}\ \frac{NT}{Kg}$$

Again comparing us to the 9.8 $\frac{NT}{Kg}$ THAT WE FEEL PULLING US DOWN, WE FIND THE EFFECT OF THE MOON VERY SMALL. AND AS BEFORE, THE EFFECT ON THE BATHROOM SCALES IS THE DIFFERENCE ON THE FIELD BETWEEN THE SURFACE AND CENTER OF THE EARTH

$$\frac{F}{m} = \frac{GM}{R-R_E} - \frac{GM}{R} = \qquad = 1\times10^{-6}\ \frac{NT}{Kg}$$

COMPARING THIS VALUE WITH THE VALUE OF THE SUN $5\times10^{-7}\ \frac{NT}{Kg}$ WE FIND THAT EFFECT OF THE MOON TO BE TWICE AS IMPORTANT AS THAT OF THE SUN.