Chapter 3 Motion in Three Dimensions 87-88

If the world were one-dimensional and all objects moved in straight line paths all the time, you would now know almost everything there is to know about motion. But such is not the case. The physical world is three-dimensional and objects move most generally in three-dimensional paths. For the physicist a formalism is needed to describe motion in more than one dimension, and it is this formalism that comes under study in this chapter.

The mathematical entity most useful in describing motion in two or three dimensions is the vector, for the vector can give us information not only about magnitude of quantities but also about their direction. In one dimension, there are only two possible directions in which an object can move - forward or backward. Thus positive and negative numbers suffice to describe motion in one direction. But in two or three dimensions there are an infinite number of directions in which a body can move and a new mathematical machinery (vectors) must be brought to bear on the problem.

Actually, we will not be working in three-dimensions in this chapter, but rather in two-dimensions. This is okay, because all the conceptual problems which arise in three-dimensions arise with much less fuss in two dimensions as well. In many cases, three-dimensional problems can be made into two-dimensiona problems, and there is the added advantage that all two-dimensional problems can be diagrammed on a piece of paper without the need for three-dimensional models.

We will also study a particular kind of motion, namely the case in which the path of an object traces out a perfect circle. We will investigate the conditions necessary for circular motion with emphasis on the forces which must act on a body to cause it to move in a circle.

Finally we will see that Newton's Law is useful when two or more forces act on an object in different directions.

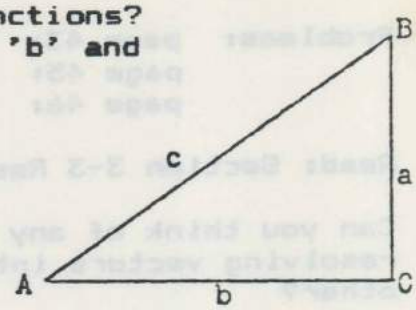
Performance Objectives

Upon completion of this chapter, you should:

1. Be able to express position in a plane by the use of a fixed reference point and a vector.
2. Be able to deduce the vector which represents the displacement between any two positions.
3. Be able to construct a vector equal to the sum of two or more given vectors.
4. Be able to construct a vector equal to the difference between two given vectors.
5. Given any vector in a plane, us both:
   a. Trigonometry, and
   b. a scale diagram to determine two mutually perpendicular components.
6. Be able to construct a vector equal to the product of a scalar quantity times a vector.
7. Be able to determine the change in velocity vector and the acceleration vector for the motion of a projectile.
8. Given the velocity at two different time intervals, be able to determine the change in velocity and the acceleration.
9. Be able to calculate the vector acceleration for an object moving in a circle.
10. Be able to state the frame of reference from which one analyzes a problem.
11. Be able to apply Newton's laws in situations where:
    a. The force is not in the direction of motion, and
    b. There are two or more forces acting in different directions.

CHAPTER 3 STUDY GUIDE -1-

1. Are you familiar with the basic trigonometry functions? If so, complete part 'a' only. If not, do part 'b' and then 'a'.



   a. For those in the know.

      (1) sin A = ______ (2) cos A = ______
      (3) sin B = ______ (4) cos B = ______
      (5) tan A = ______ (6) tan B = ______

      In summary:

      The sin of an angle is the ratio of ____________ to ____________.

      The cos of an angle is the ratio of ____________ to ____________.

      The tan of an angle is the ratio of ____________ to ____________.

   b. For those needing help.

      (1) Turn to pages 79 through 86 in T.T.& S. for help. and/or

      (2) Secure programmed instruction guide from your instructor. and/or

      (3) Secure Trigonometry in Science-An A.V. Learning Resource Prog.

2. Read: Section 3-1 Position, Distance, and Displacement page 1
   Section 3-1.a Vectors page 2
   Section 3-2 Graphical Methods of Addition and Subtraction of Displacement Vectors page 3

   Note...Your instructor has rewritten the above listed sections in hopes that it will provide needed information. Do inform your instructor of any areas that are unclear so that corrections can be made.

   a. What is a vector quantity? A scalar quantity?

   b. How many bits of information does each convey?

   c. When can vector quantities be considered equal?

   d. Why must we learn vector algebra, another form of mathematics?

   Note...Study Notes titled THE QUESTION OF DIRECTION: VECTORS gives information to help one understand the need for vector algebra.

3. Read items one through seven on sheet titled "VECTOR ALGEBRA". Once completed, ask instructor to clarify any areas of uncertainty.

4. Complete items A, B, C on sheet titled "VECTOR PRACTICE PROBLEMS". You need only work enough problems until you understand the procedure. When you do understand, go on to the next section. Be sure to make neat, precise drawings using a sharp pencil. Sloppiness will not be tolerated.

5. Problems: page 43: #1 #2 #3
page 45: #4 #6 #7
page 46: #9

6. Read: Section 3-3 Resolving Vectors Into Components page 5

Can you think of any reason why we will be especially interested in resolving vectors into two components, each at right angles to each other?

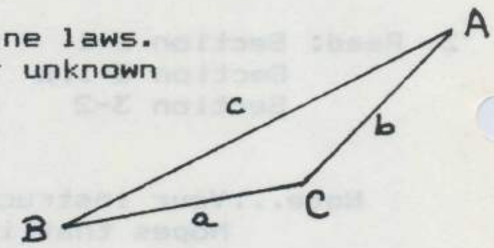
Ask your instructor to demonstrate how an object moving in a certain direction is affected by a force exerted perpendicular to the original motion.

7. There are numerous ways to find the resultant vector (the vector sum of two or more vectors).

(1) The method you will use here will be finding the x and y components of each vector, sum each component, and then find the resultant of these two components. Further information can be found in section 10 on the sheet titled "VECTOR ALGEBRA".

Section D on the sheet titled "VECTOR PRACTICE PROBLEMS" provide some problems in resolving a vector into its x and y components. Section E provides practice problems dealing with adding 3 vectors using this method.

(2) Another method is using the cosine and sine laws. You may remember using these to solve for unknown quantities in non-right angle triangles.



Sine Law: $a/\sin A = b/\sin B = c/\sin C$

Cosine Law: $c^2 = a^2 + b^2 - 2ab \cos C$

(3) A third method involves the use of a calculator. If your calculator can convert polar coordinate notation to rectangular coordinate notation, ask your instructor for assistance in how this is accomplished (if you have never done it before).

(4) Complete enclosed work sheet titled VECTOR ADDITION USING X-Y COMPONENTS.

8. Problems: page 46: #10 #11 #12
page 62: #37

9. A hunter leaves camp and travels 10-km south, then 10-km west, and then returns to camp by traveling 10-km north. (Yes the hunter is back in camp.) Upon arriving back in camp, the hunter shoots an animal.

a. What animal(s) was/were shot?

b. Explain.

10. Read: Section 3-4 Multiplying Vectors by Scalars p-46 (in text)
Section 3-5 Vector Changes & Constant Acceleration page 7

11. Experiment: MOTION OF A PROJECTILE (Experimental notes provided.)

12. Problems: page 49: #13 #14 #15 #16
page 62: #38

13. Read: Section 3-6 Changing Acceleration page 50 (in text)
Section 3-7 Circular Motion page 52 (in text)

a. Why is it more desirable to use the equation $v = 2\pi R/T$ rather than $v = \Delta d/\Delta t$ when analyzing uniform circular motion?

b. It is stated that there is a change in velocity. For this to be the case, there must be a change in displacement. What displacement is changing and how is it changing?

c. Justify that $a = 2\pi v/T$ follows from $v = 2\pi R/T$.
Trouble? Ask instructor for assistance.

14. Problems: page 52: #17 #18
page 54: #19 #20 #21 #22
page 62: #39 #40 #41 #43

15. Read: Section 3-8 Frames of Reference page 54

16. A motor propels a boat 'across' a river at 8.0 km/hr (i.e., the boat is always kept headed at right angles to the stream). The river is flowing at 6.0 km/hr with the distance across the river being 0.20 km. (You must assume that the speed of the current is the same from bank to bank.)

You may if you so desire, draw accurate scale diagrams and determine all answers from them. Or, you may make sketches and determine the results using algebra and/or trigonometry.

a. What is the resultant velocity of the boat?

b. How long did it take to cross the river?

c. How far did the boat land downstream from the starting point?

d. How long would it take to cross the river if there was no current?

e. What direction would you head the boat if you wish to go directly across the river?

f. How long would it take to cross the river if you went straight across?

g. Why is your answer in 'f' different than that of 'b'? It should be!

h. If you were swimming in a river and were having trouble making it to shore, in what direction would you head to reach shore? Why?

i. Now that you have completed this exercise, show your instructor your neat, organized work.

17. Problems: page 63: #46 #47 #48 #49

18. Read Section 3-9: How Forces Add: The Net Force page 57

19. Problems: page 58: #26 #27 #28 #29

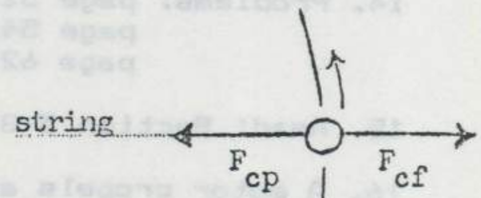
20. Investigation: CENTRIPETAL FORCE (Investigation notes provided.

21. Read: Section 3-10 The Vector Nature of Newton's Law page 58

a. In Section 3-7 we find that a = v^2/R and a = 4π^2R/T^2. Knowing that F = ma, then for uniform circular motion:

F_c = ---------- and F_c = ----------

22. CentriPetal (F_{cp}) force is defined as that force needed to make an object go in a circle as shown at the right. CentriFugal (F_{cf}) force has been defined as that force equal in value and opposite in direction to the centriPetal force and acts on the ball as shown. Your instructor does not agree with this. He thinks that the centriFugal force (as defined) is a Fictitious force as centriFugal force is spelled with an 'F'. He also thinks that centriPetal force (as defined) is a real force as centriPetal is spelled with a 'P' which represents the Pull on the string which makes the object go in a circle. What are the correct and what are the incorrect statements just listed and the rational to support your thinking.



23. Problems: page 60: #30 #31 #32 #33 #34 (Info for #32 in text.)

24. Complete written exercise. When completed, have it evaluataed.

Thought for the chapter: Texans wear 37.853 liter hat

ANSWERS Chapter 3

1. (a)(1) a/c (2) b/c (3) b/c (4) a/c (5) a/b (6) b/a
   Sine is the ratio of the side opposite to the hypotenuse.
   Cosine is the ratio of the adjacent side to the hypotenuse.
   Tangent is the ratio of the opposite side to the adjacent side.
2. (a) vector: represented by both magnitude (including units) & direction
   scalar: represented by magnitude (including units) only
   (b) vector - two, scalar - one
   (c) when both the magnitude and direction are equal
   (d) See study notes THE QUESTION OF DIRECTION: VECTORS
5. (1) 4 pairs
   (2) $2\sqrt{2}$ horizontal 45° up
   (3) 2 meters North
   (4) 2 blocks south
   (6) $\vec{v}_1 - \vec{v}_2 = -(\vec{v}_2 - \vec{v}_1)$
   (7) 7.8 meters S 11° W
   (9) 9.72 meters
7. (1)(a) 5m (b) 8.67m (2)(a) 5.66 m (b) -5.66 m (3) 10.67 m, 3.00m
   (4) 11.07 m at right 15.72° up
8. (10)(a) 7.1 km (b) 7.1 km (11) -86.6 km/hr N, -50 km/hr E (12) 0
   (37)(a) 2.12 cm N 3.3° W (b) 8.48 cm N 3.3° W (c) Ans. in b = 4x ans. in a
9. (b) Did you say a bear? What about a penguin?

12. (13) 0.6 cm on Fig. 3-12 which represents 6.7 cm, should be
    (14) Reverse velocity vector with tail at same point
    (15)(1) 0 (2) 1 m/s (3) 2 m/s (4) 3 m/s (5) 4 m/s (16) 10 m/s/s
    (38)(a) 0.59 m to right (b) 0.75 m down (c) 4.1 m/s hor. 68.5° down

13. (a) see instructor (b) R is changing in dir only, by 360° in time T

14. (17) 4.9 (18) S.A.B. (19) motion along a curved path
    (20)(a) 282 km/hr at 135° from orig. dir. (b) 400 km/hr at 180° from orig. dir.
    (21) nothing (22)(a) 4 times (b) divided by 2
    (39)(a) 1.0 m/s at 105° relative to $\vec{v}_1$ (b) 0.35 m/s at 75° deg relative to $\vec{v}_1$
    (40)(a) 0.21 cm/s (b) 0.21 cm/s right, 0.21 cm/s down (c) 0.30 cm/s down 45° left
    (41)(a) 83 m/s/s (b) 8.5 times larger (43) 0.04 m/s/s

16. (a) 10 km/hr, straight across 36.9° downstream
    (b) 0.025 hr (c) 0.15 km (d) 0.025 hr (e) across 48.6° upstream
    (f) 0.038 hr (g)(h) discuss with your instructor

17. (46)(a) 137 km N 141° clockwise, 344 km - same dir. (b) 411 km/hr N 141° clockwise
    (47)(a) 1 m/s 6.4 m/s 38.7° away from dir. of travel
    (b) 25.2 m 28.1° relative to ?
    (48)(a) E 4.5° N (b) 445 km/hr (49)(a) S 25° E
    (b) 960 km/hr (c) 240 km
19. (26) along direction of net force
    (27) 8 m/sec/sec to right
    (28) 3.7 Newton to right
    (29) 186 Newton perpendicular to canal on same side as F_2
21. $F = mv^2/R = m4\pi^2R/T^2$
23. (30) 24 Newton
    (31) speed does not change, direction of motion changes toward direction of force
    (32) 0.52 m/sec to left
    (33) 0.62 m/sec towards center of circle
    (34) (a) string cut - no centripetal force (b) yes

THE QUESTION OF DIRECTION: VECTORS

As soon as we move from one to two-dimensional motion, the problem of direction becomes important. In most cases of motion along a line, pairs of terms, such as right or left, toward or away from, up or down, were entirely adequate to our analysis. We saw in motion along a straight line path in chapter 1 and 2 that it was convenient to call one direction positive and the other negative, for in doing so the kinematics equations could be made to take direction into account in a consistent and useful way.

But the matter of direction is a little more troublesome for an object moving along a two-dimensional path. For example, if you look from the side at a person riding on a ferris wheel, you will see that he successively moves through every combination of right-left and up-down motion. Something more, therefore, is needed than the plus and minus sign convention to allow us to deal with such cases easily.

The first step is to be sure from now on that whenever we encounter a physical quantity - speed, force, energy, or whatever - we immediately try to find out whether or not it involves direction. Those quantities that involve direction are called VECTORS, while nondirectional quantities are called SCALARS. A scalar quantity can be expressed with a single number. Of the quantities we have used, distance and speed are examples of scalars. In response to the questions, "How far did you travel?" and "What was your average speed?", the answers, "320 miles" and "250 kilometers are correct answers. But 275 kilometers is not a complete answer to the question, "At the end of the first day of driving, where were you from home as the crow flies?" The kind of response needed in this case is something like "250 miles northwest". Direction (northwest) as well as magnitude (250 kilometers) must be given for vector quantities, and this always takes two things (northwest: means the compass direction of 315 degrees).

To our arsenal of concepts - distance, speed, and acceleration - we must now add two more: "distance traveled in a given direction", and "speed in a given direction". There are special names for these two vector quantities. The first is called DISPLACEMENT; the second, VELOCITY. Perhaps the best way to grasp the meaning of these is by looking at examples.

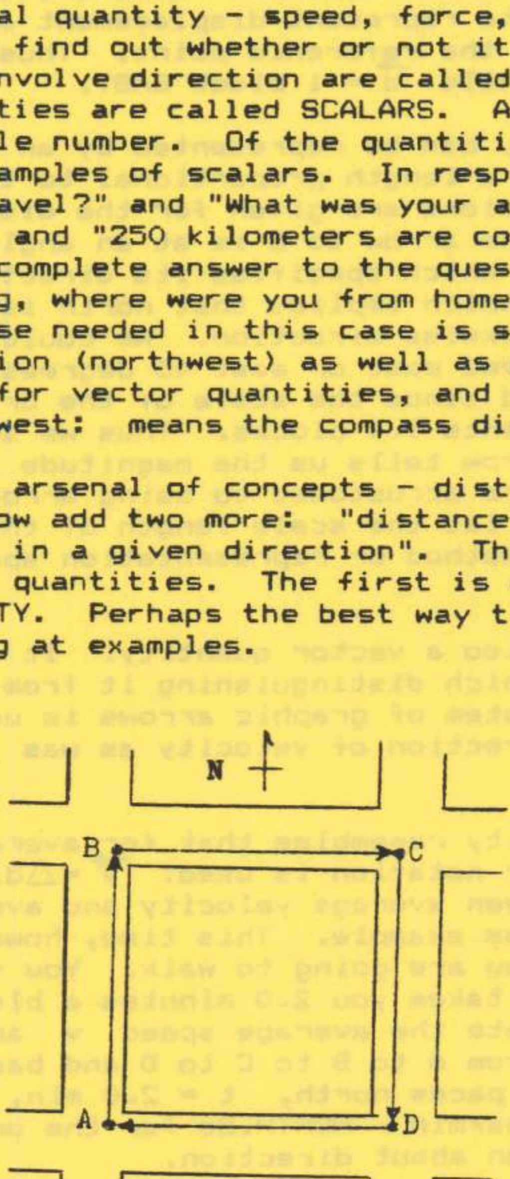


Fig. 1

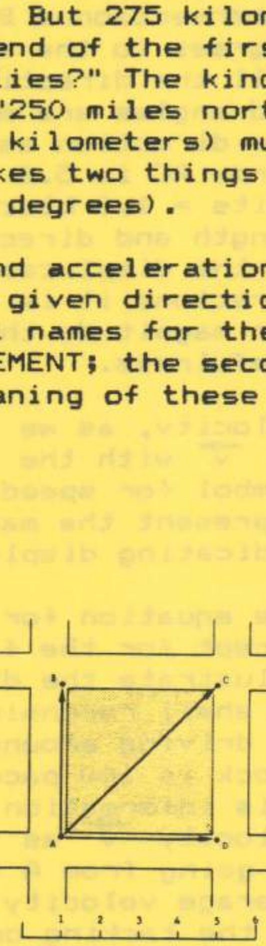


Fig.2 Vector displacements- scale: 4.0 units = 1.0 blocks.

Consider the relationship between distance and displacement as you drive a car around a square city block (vainly looking for a parking place). Starting at intersection A (Fig 1) you drive north to intersection B, east to C, south to D, back to A, and then around again. At point A, the start of your first trip around the block, you had not yet moved, so both distance and displacement were zero. In moving from A to B, the distance traveled was one block and the displacement was one block NORTH. In going from A to C, the distance you traveled was two blocks, your distance away from A was $\sqrt{2}$ blocks, but your displacement was $\sqrt{2}$ blocks NORTHEAST. In otherwords, displacement describes how far and in what direction an object is from a specified reference point, not how far the object may have moved to get there, nor even just how far away it is from the reference point. In this case your displacement at C would have been the same, even if after leaving A you had driven nine blocks north, one block east, and eight blocks south - in any order. You will find that after driving a TOTAL DISTANCE of four blocks, your DISPLACEMENT is ZERO.

A symbol that conveys the idea of direction in most cultures is the arrow. We use the symbol $\vec{d}$ to represent displacement and symbol d to represent the distance from the reference point. Thus at intersection D: d = 1 block while $\vec{d}$ = 1 block EAST.

Displacement, like all vectors, can be represented by an arrow pointing in the proper direction and of a length proportional to the distance. In Figure 2, vector representations are given for the displacement to intersectionns B, C, and D. The arrow to C is at an angle of 45 degrees to the east-west line, which specifies its direction. We could call the direction 45 degrees which implies that north is zero degrees and angles are measured in clockwise direction. We could also identify the direction as north 45 degrees east or east 45 degrees north. The arrow AC is 5.6 units long, and since the scale of the drawing is 4.0 units = 1.0 block, this represents 1.4 blocks. Thus we see that the length and direction of the arrow tells us the magnitude and direction of the displacement $\vec{d}$. We are accustomed to using arrows to suggest directon; it is in agreeing to let the scale length of the arrow stand for magnitude that gives this method or representation special usefulness.

Velocity, as we have seen is also a vector quantity. It is symbolized by $\vec{v}$ with the arrow on top which distinguishing it from v, the symbol for speed. The same system of graphic arrows is used to represent the magnitude and direction of velocity as was just used for indicating displacement.

The equation for average velocity resembles that for average speed except for the fact that vector notation is used: $\vec{v} = \Delta \vec{d} / \Delta t$. To illustrate the difference between average velocity and average speed, we shall reconsider our previous example. This time, however, instead of driving aroung the block, you are going to walk. You find that each block is 160 paces and that it takes you 2.0 minutes a block. With this information we can calculate the average speed v and the average velocity $\vec{v}$ as you progress from A to B to C to D and back to A again. In going from A to B: d = 160 paces north, t = 2.0 min, and so the average velocity $\vec{v}$ is 80 paces/min - NORTH.So far the only difference is the tacking on of information about direction.

But now the situation changes as you move from B to C. Considering the route ABC, we find that since both the distance and the time doubles (320 paces and 4.0 min) the average speed remains unchanged. However, the displacement $\vec{d}$ does not double since it is given by the straight line distance from A to C. We can find the value of AC in two ways. The first is to rely upon the knowledge that the diagonal of a square is $\sqrt{2}$ times the length of one side: in this case 160 $\sqrt{2}$, or 226 paces N 45 degrees E. The other method is to add together graphically the arrows representing the displacement from A to B and from B to C. The advantage of this approach is that it can be used even in those cases where the geometry does not permit an easy solution by equation and where there are more than 2 vectors. Note that you first place the vectors to be added, head to tail, and then you draw a line from the tail of the first arrow to the head of the last one. The length of this resulting arrow (called the RESULTANT) gives the magnitude of the vector, and the orientation of the resultant gives the direction. Thus $\vec{d}$ = 226 paces N 45 degrees E.

Now we can find the average velocity from A to C. Substituting the values $\vec{d}$ = 226 paces N 45 degrees E and t = 4.0 min into $v = \triangle d/\triangle t$, we obtainn 56 paces/min N 45 degrees E. This compares to an average speed along the path of 80 paces/min.

Before moving on, two points should be made clear: (1) Acceleration, about which we have said little, is also a vector quantity. As we shall soon see, the direction of acceleration of moving objects is important information. (2) Velocity and accelration, and any other vector quantity, can be handled in the same way as displacement. That is, velocity vectors can be added to velocity vectors, and so on, but vectors cannot be mixed, (i.e. velocity cannot be added to displacement any more than speed can be added to distance).

SOME PRACTICE WITH VECTOR ALGEBRA

BASIC POSTULATE: Two vector quantities are equal if they have the same magnitude and units, are parallel to each other, and point in the same direction.

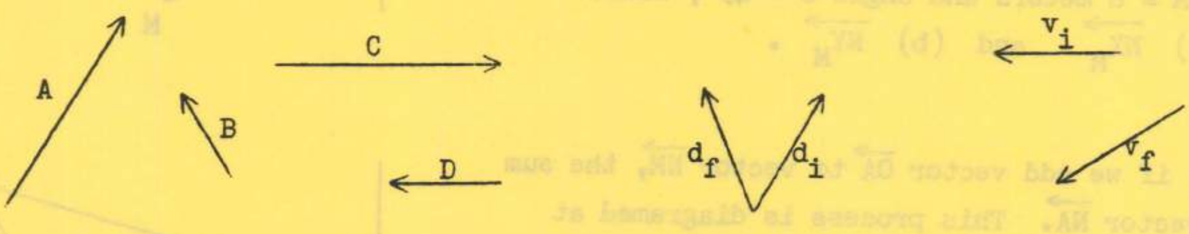
This postulate makes it possible for you to move the arrow representing a vector anywhere you wish in space, PROVIDING YOU ALWAYS KEEP IT PARALLEL TO ITSELF.

ADDITION OF VECTORS [Use which ever is more convenient.]

1. To add vector $\vec{B}$ to vector $\vec{A}$, connect the tail of $\vec{B}$ to ______________. The sum or resultant $\vec{R}$ $(\vec{R} = \vec{A} + \vec{B})$ is the vector which goes from ______________ to ______________.
2. If the tail of $\vec{B}$ is connected to the tail of $\vec{A}$, complete a parallelogram which has $\vec{A}$ and $\vec{B}$ as two of its sides. The resultant $\vec{R}$ is then drawn from ______________ to ______________.

SUBTRACTION OF VECTORS [Use which ever is more convenient.]

1. To subtract $\vec{B}$ from $\vec{A}$, change $\vec{B}$ to $-\vec{B}$ by reversing its direction, and add $-\vec{B}$ to $\vec{A}$ by one of the rules of addition.
2. To subtract $\vec{B}$ from $\vec{A}$, connect the tail of $\vec{B}$ to ______________. (This is used when the tail of $\vec{B}$ is connected to the tail of $\vec{A}$.) The difference $\overrightarrow{\Delta d}$ $(\overrightarrow{\Delta d} = \vec{A} - \vec{B})$ is the vector which goes from __________ to __________.



Find $\vec{A} + \vec{B}$	Find $\vec{B} + \vec{A}$	Find $\vec{A} - \vec{B}$	Find $\vec{B} - \vec{A}$
	Find $\vec{C} + \vec{D}$		Find $\vec{D} - \vec{C}$
Find $\vec{A} + \vec{C}$		Find $\overrightarrow{\Delta d}$	
	Find $\vec{A} + \overrightarrow{(C + D)}$		
Find $\overrightarrow{(A + C)} + \vec{D}$	Conclusions:	Find Δv	If $\vec{A} + \vec{x} = \vec{D}$, Find $\vec{x}$

1. DEFINITION OF AN ELEMENT OF THE SET OF VECTORS:

A vector is any quantity that has:

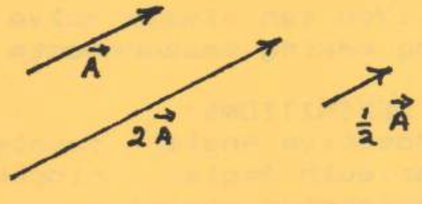
a. magnitude b. direction c. (is commutative under addition)

** A vector is physically represented by an arrow. **

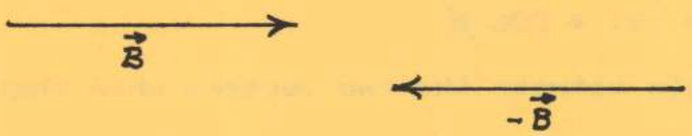
2. EQUALITY: Two vectors are equal if and only if they have the same magnitude and the same direction. Thus a vector can be displaced parallel to itself.

3. MULTIPLICATION OF A VECTOR BY A SCALAR:

If a vector is multiplied by a scalar, the result is another vector whose direction is the same as the given vector and whose length is altered by the factor of the scalar.



4. Multiplication of a vector by '-1' will turn the vector around.

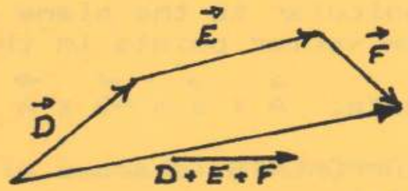


Note: strange - but multiplying a vector by $\sqrt{-1}$ will rotate it 90 deg.

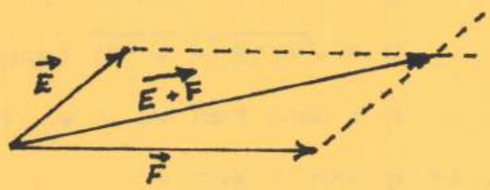
5. The zero vector is a vector of no length and every direction.

6. VECTOR ADDITION:

Any number of vectors can be added by lining them up head to tail (like elephants in a parade). The sum vector is from the tail of the first to the head of the last.



TWO vectors can also be added using the parallelogram rule. Put vectors to be added tail to tail - complete parallelogram and draw diagonal to get vector sum.



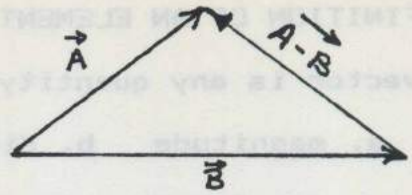
a. vector addition is associative: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

b. vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

c. multiplication by a SCALAR is distributive over vector addition

7. VECTOR SUBTRACTION:

Two vectors can be subtracted by putting their tails together and going from the head of the vector being subtracted to the head of the vector being subtracted from.



This rule makes vector subtraction the same as addition of a negative vector.
i.e. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Note...You can always solve vector addition or subtraction by scale drawing making measurements to determine answers.

8. ANGLE DEFINITIONS:
   a. Positive Angle: counterclockwise from positive x-axis
   b. azimuth Angle: clockwise from north
   c. Reference Angle:

9. MULTIPLICATION OF TWO VECTORS:

   a. The dot product of two vectors is a SCALAR whose value is given by:

   $\vec{A} \cdot \vec{B} = |A| * |B| * \cos \emptyset$

   Where $\emptyset$ is the angle between the two vectors when their tails are together.

   b. The cross product of two vectors is ANTHER VECTOR whose magnitude is given by:
   $\vec{A} \times \vec{B} = |A| * |B| * \sin \emptyset$

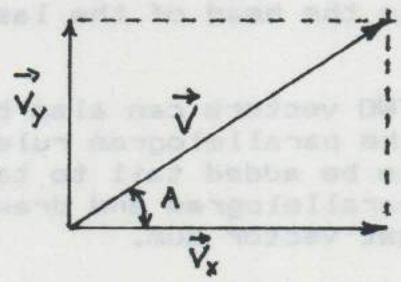
   where $\emptyset$ is the same as above. The direction of the new vector is perpendicular to the plane of the two vectors being multiplied. The new vector points in the direction given by the right hand rule.
   Note: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

10. VECTOR COMPONENTS (shadows along x and y axis)
   a. If given: $\vec{v}_x$ and $\vec{v}_y$, find $\vec{v}$

   $v = \sqrt{v_x\text{^}2 + v_y\text{^}2}$ (magnitude of v)

   $A = \arc\tan v_y / v_x$ (direction of v)

   b. if given A and v $\quad v_x = v \cos A$
   $v_y = v \sin A$



11. ADDITION OF VECTORS BY COMPONENT METHOD

   a. Place all vectors with tails together on the origin of a coordinate system
   b. Resolve (break down each vector into its x and y components).
   c. Add all x-components to find net-x and all y-components to find net-y.
   d. Add the net-x and net-y VECTORS to find the sum of all vectors.

A. Do Problems #1, #2, and #3 by three methods:

a. Head to tail method using scale drawing
b. Parallelogram method using scale drawing
c. Mathematical computation (i.e. not by scale drawing)

Note: All angles are measured counterclockwise from positive x-axis

1. Add the two vectors $\vec{A}$ and $\vec{B}$

$\vec{A}$ = (10 cm, 90 deg) $\vec{B}$ = (12 cm, 180 deg)

2. Add the two vectors $\vec{C}$ and $\vec{D}$

$\vec{C}$ = (8 cm, 0 deg) $\vec{D}$ = (6 cm, 90 deg)

3. Add the two vectors $\vec{E}$ and $\vec{F}$

$\vec{E}$ = (6 cm, 30 deg) $\vec{F}$ = (10 cm, 80 deg)

B. Do Problems #4 and # 5 by tail rule using scale drawing.

4. Add the three vectors $\vec{G}$, $\vec{H}$, $\vec{I}$

$\vec{G}$ = (3 cm, 90 deg) $\vec{H}$ = (4 cm, 180 deg) $\vec{I}$ = (6 cm, 270 deg)

5. Add the three vectors $\vec{E}$, $\vec{F}$, and $\vec{B}$.

C. Do vector subtraction by the two methods:

a. Tail to tail using scale drawing.
b. Mathematical computation (not scale drawing).

6. Find value of $\vec{A} - \vec{B}$

$\vec{A}$ = (6 cm, 90 deg) $\vec{B}$ = (12 cm, 180 deg)

7. Find value of $\vec{E} - \vec{F}$

$\vec{E}$ = (6 cm, 30 deg) $\vec{F}$ = (10 cm, 80 deg)

D. Find the x and y components of the following vectors by three methods.

a. Scale drawing (shadow method)
b. Trigonometric calculation
c. Calculator using polar to rectangular conversion (if able)

8. Find components of $\vec{E}$ = (6 cm, 30 deg)

9. Find components of $\vec{J}$ = (12 cm, 180 deg)

10. Find components of $\vec{K}$ = (8 cm, 45 deg)

11. Find components of $\vec{L}$ = (9 cm, 70 deg)

E. Problems 12, 13, 14, and 15 are to be added by using the following procedure:

a. Scale drawing as a check.
b. Component method of addition of vectors.

12. $\vec{M}$ = (8 cm, 30 deg) $\vec{N}$ = (10 cm, 90 deg) $\vec{O}$ = (5 cm, 180 deg)

13. $\vec{R}$ = (5 cm, 300 deg) $\vec{S}$ = (2 cm, 90 deg) $\vec{T}$ = (3 cm, 150 deg)

14. $\vec{U}$ = (6 cm, 70 deg) $\vec{V}$ = (4 cm, 200 deg) $\vec{W}$ = (3 cm, 290 deg)

15. $\vec{X}$ = (6 cm, 40 deg) $\vec{Y}$ = (8 cm, 155 deg) $\vec{Z}$ = (4 cm, 250 deg)

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ANSWERS

1. 15.6 cm, 140.2 deg
2. 10.0 cm, 36.9 deg
3. 14.6 cm, 61.7 deg
4. 5.0 cm, 216.9 deg
5. 13.8 cm, 111.5 deg
6. 13.4 cm, 26.6 deg
7. 7.7 cm, 296.8 deg
8. 5.2 cm, 3.0 cm
9. -12.0 cm, 0 cm
10. 5.7 cm, 5.7 cm
11. 3.1 cm, 8.5 cm
12. 14.1 cm, 82.2 deg
13. 0.8 cm, 263.3 deg
14. 1.6 cm, 115.1 deg
15. 5.3 cm, 139.2 deg

3-1 POSITION, DISTANCE, AND DISPLACEMENT

To mathematically describe motion, we must indicate the position of an object at a particular instant of time. This is usually done by giving the coordinates of the object once the coordinate lines are chosen. That is, we must choose a particular location to be the origin of the coordinates. In the case of Cartesian coordinates, we must choose particular directions of the x and y axes as in Figure 10-1. Here we see that the position on the plane is given by two coordinate numbers: the x coordinate is stated first, the y coordinate second. The description becomes complete when we choose a particular instant when the time t equals zero.

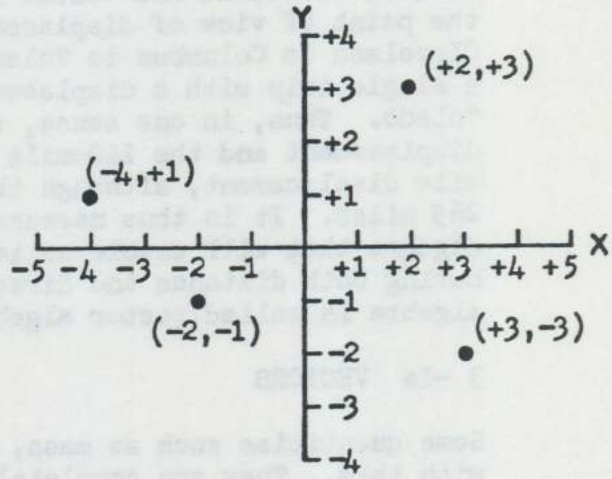


Figure 10-1

When an object moves from position A to position B it is said to have been displaced. If we follow the motion of the object along the path that it travels from A to B, we can measure the distance it traveled. We can, if we so desire, ignore the actual path and fix our attention on the change in position. This change in position is called the displacement from A to B as in Figure 10-2. That is, displacement is the net change in position from the initial position A to the final position B. It is represented by an arrow running from A to B. The displacement from A to B will thus be the same no matter what path is actually traversed.

The most significant difference between distance and displacement is that distance is expressed as a number in certain units while displacement is a number of those units plus a direction. Consider, for example, an automobile trip from Cleveland to Detroit. The distance for this trip is a certain number of miles, a number that could be obtained from the odometer of the automobile. To describe the displacement one must look at the initial point (Cleveland) and the end point (Detroit) and describe the difference between these positions. The simplest way to do this is to draw a straight line from Cleveland to Detroit on a map. The line is characterized by its length and its direction. Thus the distance from Cleveland to Detroit is some number of miles, which depends on the particular roads chosen for the trip, whereas the displacement is a specific number of miles in a particular direction.

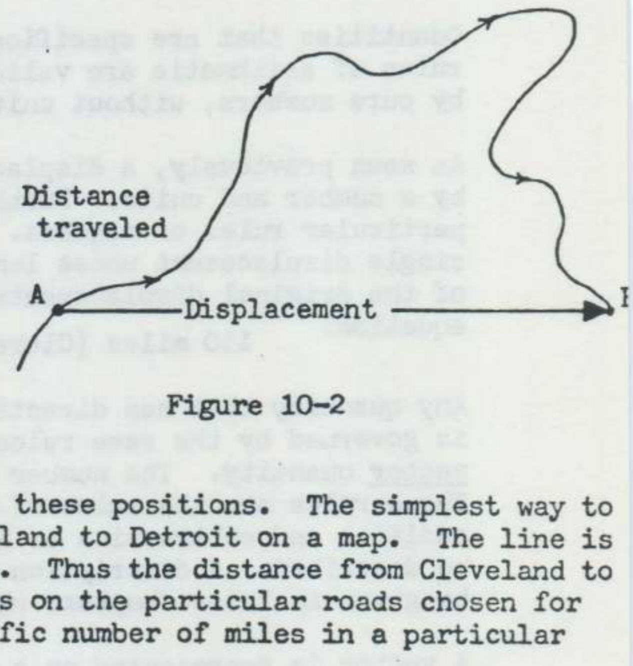
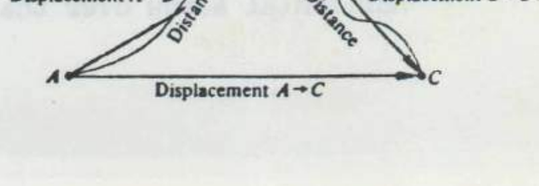


Figure 10-2

Consider now, two successive motions of an object. Suppose it moves from A to B and then moves from B to C (Figure 10-3). The distance traveled from A to C

Figure 10-3



is the sum of the distances traveled from A to B and from B to C. The length of the displacement from A to C is not necessarily the sum of the length of the displacement from A to B and the length of the displacement from B to C. Consider a trip From Cleveland to Columbus followed by a trip from Columbus to Toledo (see Figure 10-4). The first trip is about 125 miles and the second is about 140 miles, while a straight line distance between Cleveland and Toledo is about 110 miles. From the point of view of displacement, the trip from Cleveland to Columbus to Toledo can be thought of as a single trip with a displacement from Cleveland to Toledo. Thus, in one sense, the sum of the 125-mile displacement and the 140-mile displacement is a 110-mile displacement, although the distance traveled is 265 miles. It is thus necessary to develop a type of algebra that will enable us to deal with quantities having both distance and direction. This type of algebra is called vector algebra.

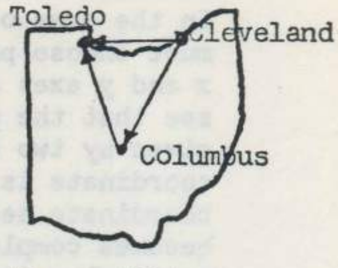


Figure 10-4

3 -1a VECTORS

Some quantities such as mass, time, and temperature do not have direction associated with them. They are completely specified by a number accompanied by appropriate units. For such quantities, the ordinary rules of arithmetic are applicable. For example, a single object of mass 5-kg is equivalent to two objects, one of mass 2-kg and the other of mass 3-kg as far as the property of mass is concerned. It is true that one object (of mass 5-kg) is not identical to the two objects (of masses 2-kg and 3-kg). But if all we are interested in is the property of mass, then we can always replace one object by two of the same total mass. Thus we can write the equation:

$$5 \text{ kg} = 2 \text{ kg} + 3 \text{ kg}$$

Quantities that are specified by a number with units and for which the ordinary rules of arithmetic are valid are called <u>scalar</u> quantities. Quantities specified by pure numbers, without units, are also scalars.

As seen previously, a displacement is a quantity that has direction and is specified by a number and units. Further, we also saw that displacements are governed by particular rules of algebra. Two successive displacements are equivalent to a single displacement whose length may be very diffenent from the sum of the lenghts of the original displacements. We can describe the Cleveland to Toledo by the equation:

110 miles (Cleve-Tol) = 125 miles (Cleve-Col) + 140 miles (Col-Tol)

Any quantity that has direction, specified by a number accompanied by its units and is governed by the same rules of algebta that govern displacements, is called a <u>vector</u> quantity. The number (with units) is called the <u>magnitude</u> of the vector. These rules are the rules of vector algebra. In the following sections vector addition and subtraction as well as multiplication of a vector times a scalar will be described. A description of the multiplication of a vector times a vector will be given in later chapters when it will be used in solving problems.

A vector is represented on a diagram by drawing an arrow. The length of the arrow is made proportional to the magnitude of the vector by choosing an appropriate scale. The direction of the vector is given by the direction of the arrow. In print, symbols representing vector quantities are represented by drawing a short horizontal arrow over the symbol.

3-2 GRAPHICAL METHODS OF ADDITION AND SUBTRACTION OF DISPLACEMENT VECTORS

Since we have defined vectors in terms of the rules of algebra of displacements, we will first consider the addition of two displacements.

In Figure 10-5 we have two displacements, $\vec{A}$ and $\vec{B}$. If a body starts at a point, called O, and undergoes displacement $\vec{A}$ first, followed by displacement $\vec{B}$, its motion is described by Figure 10-5b. Here the object will proceed from point O to point P, and then go to point Q, where the line OP represents the displacement $\vec{A}$ and the line PQ represents the displacement $\vec{B}$. The total displacement, from the initial point O to the final point Q is a vector we can call $\vec{C}$. Thus, $\vec{C}$ is equivalent to $\vec{A}$ followed by $\vec{B}$, or $\vec{C} = \vec{A} + \vec{B}$

[Note...Figure 10-5 is a scale drawing where 1-cm on the drawing represents 2-meters of displacement. The displacement $\vec{A}$ therefore represents 5-meters, $\vec{B}$ represents 3.0-meters, and the vector sum $\vec{C}$ has a magnitude of 5.8-meters.]

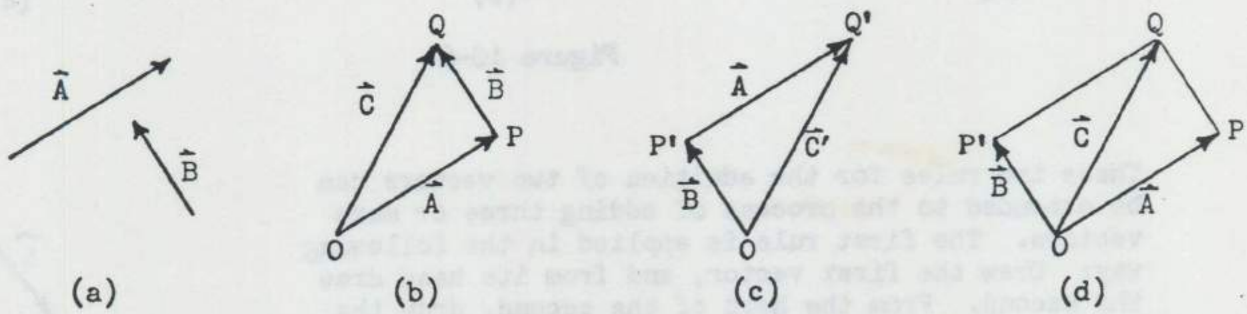


Figure 10-5

If the object undergoes the displacement $\vec{B}$ first, and then $\vec{A}$, the motion is described by Figure 10-5c. Here the object goes from O to P' to Q'. The total displacement is the vector $\vec{C'}$. That is, the displacement $\vec{C'}$ is equivalent to the displacement $\vec{B}$ followed by displacement $\vec{A}$, or

$$\vec{C'} = \vec{B} + \vec{A}$$

In Figure 10-5d, vector $\vec{A}$ and vector $\vec{B}$ have been positioned such that their tails are at a common point. The parallelogram is completed by drawing PQ parallel to OP' and then drawing P'Q parallel to OP. A line drawin from O to Q represents the vector sum of vectors $\vec{A}$ and $\vec{B}$ which we have called vector $\vec{C}$. It is clear that this parallelogram is composed of the two triangles that are in Figure 10-5b and Figure 10-5c. Therefore, the displacements $\vec{C}$ and $\vec{C'}$ are equal and:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

This means that the sum of two displacements, or vectors, is independent of the order in which they are taken. Further we have two rules for adding, either of which may be used.

1. To add two vectors, first draw either of the two vectors. Then starting at the head of the first, draw the second vector. The sum of the two vectors is the vector drawn from the tail of the first vector to the head of the second. This is shown in Figure 10-6b.

2. To add two vectors, first draw either of the vectors. Then, starting at the tail of the first, draw the other. With these two lines as two of the sides, complete the parallelogram. The sum of the vectors is the vector formed by the diagonal from the point where the tails of the vectors meet to the opposite corner of the parallelogram. This method is shown in Figure 10-6c. The sum of the two vectors is called the <u>resultant</u>.

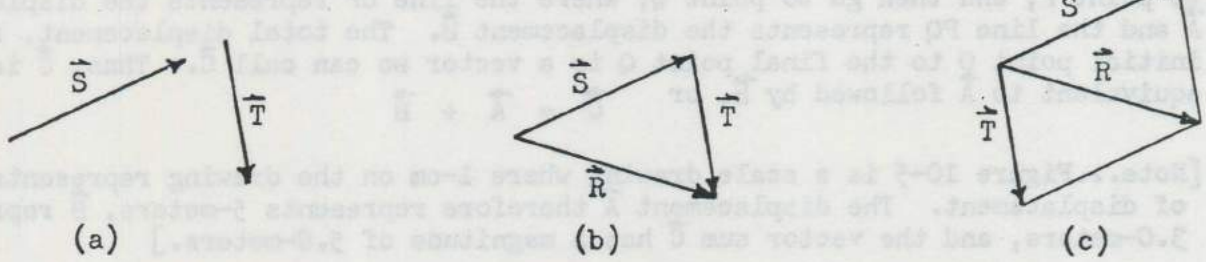


Figure 10-6

These two rules for the addition of two vectors can be extended to the process of adding three or more vectors. The first rule is applied in the following way: Draw the first vector, and from its head draw the second. From the head of the second, draw the third, and from its head, draw the fourth. Continue the process until all the vectors have been drawn. The sum is the vector drawn from the tail of the first vector to the head of the last (see Figure 10-7). The second rule can be applied to more than two v vectors by first getting the sum of any two of the vectors and then adding this sum to the third vector. This sum can be added to the fourth and so on.

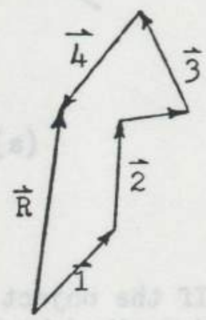


Figure 10-7

We can extend the process of addition of displacements to include subtraction by defining the negative of a displacement vector, and then asserting that subtraction is the same as the addition of the negative of a displacement vector. In Chapter 9, we used the negative sign to indicate a displacement opposite to that of positive. Here we will define the negative of vector $\vec{a}$ to be a vector $\overrightarrow{-a}$, which is equal in magnitude, but opposite in direction to $\vec{a}$. With this meaning of the negative of a vector, we can define the process of subtraction as the process of adding the negative of a vector as shown in Figure 10-8b. Thus:

$$\vec{a} - \vec{b} = \vec{a} + (\overrightarrow{-b})$$

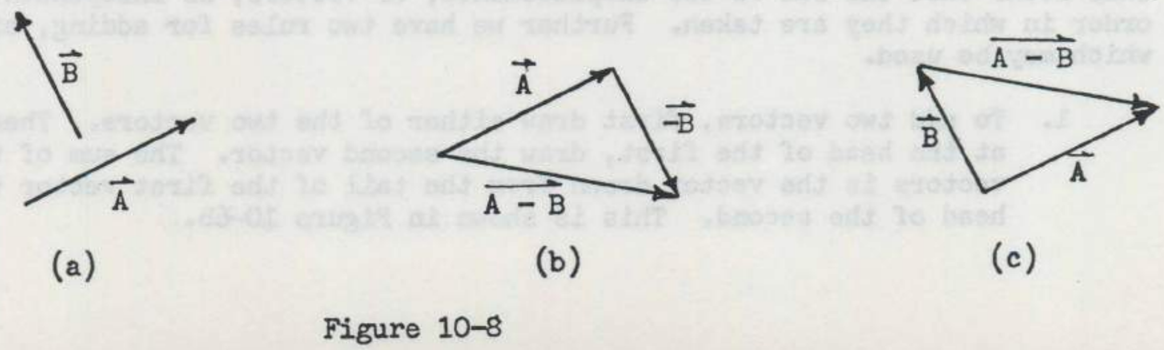


Figure 10-8

There is also a second way to subtract one displacement from another. To do this place the two displacement vectors tail-to-tail and draw a line connecting their heads. The direction of this displacement is from the displacement being subtracted to the other displacement (Figure 10-8c). As we can see, the difference is the same as in Figure 10-8b. Thus these two methods of vector subtraction are equivalent.

Note...Further analysis of Figure 10-8 reveals that $\vec{B} - \vec{A}$ equals the negative of $\vec{A} - \vec{B}$ as the only difference is the direction of the vector difference.

3-3 RESOLVING VECTORS INTO COMPONENTS

We have been adding two or more vectors to find their resultant. The two vectors combined in this way are called <u>components</u>. Frequently, we will be interested in the opposite process, that of beginning with a single vector and then finding its components in certain directions. This process of finding the magnitudes of the component vectors along specified directions is called <u>resolution</u> of a vector into its components.

The method of resolving vectors into specific components is opposite to that of combining two component vectors to find their resultant. We will start with the resultant and then find its two components as follows.

1. Draw the given vector $\vec{A}$ to scale and in the given direction (Figure 10-9a).
2. From the tail of $\vec{A}$, construct two lines OX and OY in the direction that the two component vectors will point (Figure 10-9b).
3. From the head of $\vec{A}$, draw two lines PS and PT parallel to lines OX and OY (Figure 10-9c). Note that vector $\vec{A}$ is a diagonal of a parallelogram.
4. On lines OX and OY, construct arrows from O to S and O to T (Figure 10-9d).

The two arrows labeled $\vec{B}$ and $\vec{C}$ (Figure 10-9d) represent the two component vectors of the resultant vector $\vec{A}$. The magnitude of these two component vectors can be found using the scale from which vector $\vec{A}$ was constructed.

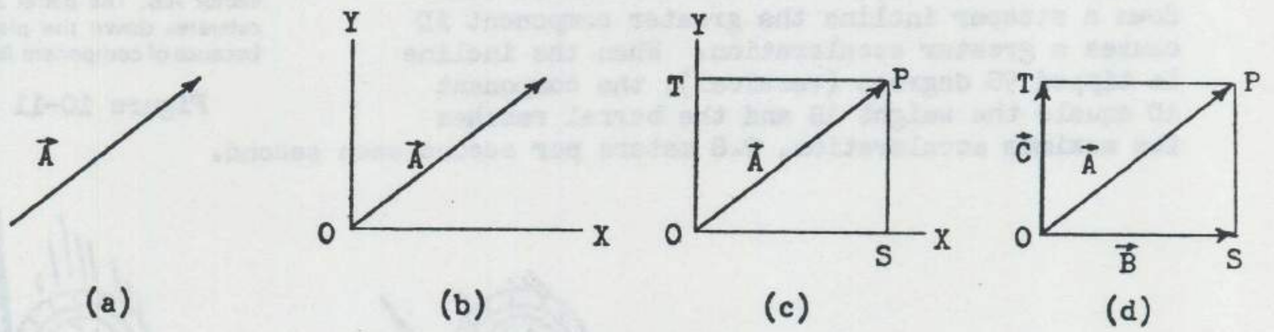


Figure 10-9

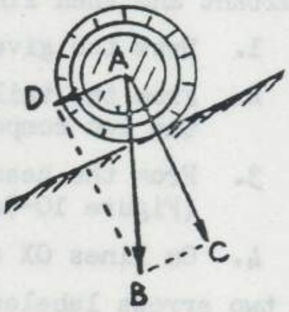
Consider a barrel that can be rolled up an inclined plane with a force less than that required to lift it. Why this is so can be understood by considering the components of its weight. (Since weight is a vector quantity, we can treat it like a displacement vector.) We can represent the weight of a barrel on an incline as a vector that can be separated into components. Figure 10-10.

The vector $\vec{AB}$ represents the weight of the barrel; the vector $\vec{AC}$ the component of force that is equal to the force the barrel exerts against the incline; the vector $\vec{AD}$ represents the component of the weight parallel to the incline. When

Figure 10-10

It is easier to roll the barrel up the incline than it is to lift it vertically. Why?

the barrel pushes down on the incline, the incline pushes back with a force exactly equal but opposite to component AC. Therefore the component AC is neutralized and has no tendency to produce motion. It is the component AD that accelerates the barrel down the incline. If we wish to prevent the barrel from rolling down the incline, we must apply a force equal and opposite to the force AD. If we wish to roll the barrel up the incline, we must push with an initial force somewhat greater than AD to accelerate it from rest, but once it is moving a force equal and opposite to AD is all that is required to maintain constant velocity. We can see from Figure 10-11 that the vector AD is shorter than the vector AB; hence the force necessary to roll the barrel up the incline is less than the force necessary to lift the barrel vertically. Note that the steeper the incline, the greater is the magnitude of AD. Figure 10-12. We would have to push harder to roll a barrel up a steep incline; similarly, if we let the barrel roll down a steeper incline the greater component AD causes a greater acceleration. When the incline is tipped 90 degrees (vertical), the component AD equals the weight AB and the barrel reaches its maximum acceleration, 9.8 meters per second each second.



The weight of the barrel is represented by vector **AB**. The barrel accelerates down the plane because of component **AD**.

Figure 10-11

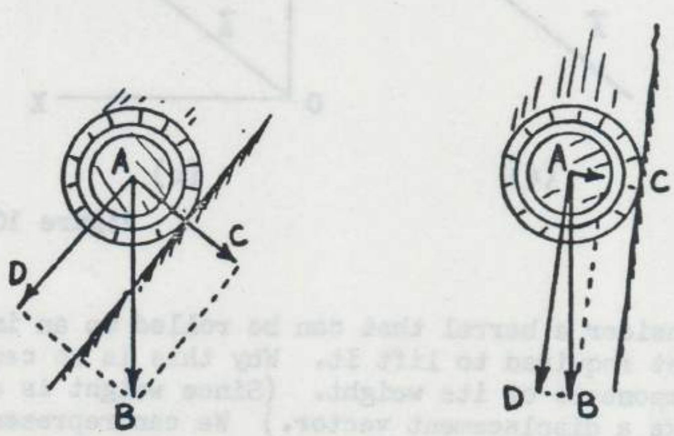


Figure 10-12

Component **AD** increases as the angle of incline increases.

Figure 10-12 is a drawing of a multiple-flash photograph of a ball projected horizontally with an initial velocity of 2.0 m/sec and photographed every 1/30 second as it falls. One finds, when looking at the vertical lines on the drawing, that during each interval, the horizontal component of the displacement remains constant. This indicates a constant horizontal velocity component. Inspection of the vertical displacement shows that during each interval the spacing increases. Thus the vertical velocity is increasing.

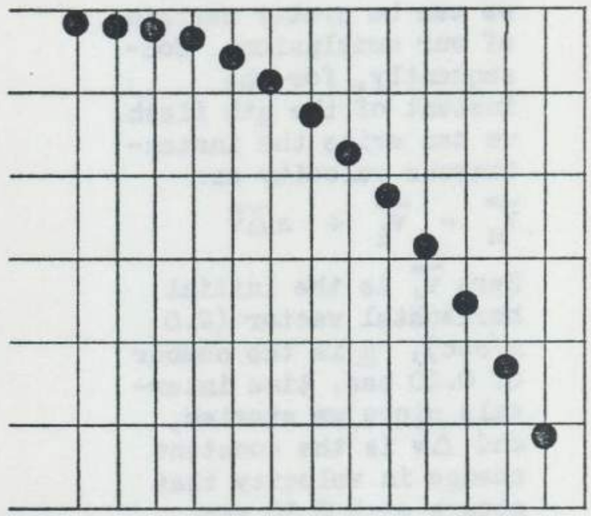


Figure 10-12

We will consider projectile motion as two motions occuring at the same time. The horizontal part of the motion is uniform motion in a straight line. The vertical part of the motion is uniform accelerated motion as its motion is governed by the laws relating to falling bodies that will be discussed in chapter 12.

We shall now find the instantaneous velocity vector every 0.10 second time interval. We know the instantaneous velocity vector always points in the direction of the path. In addition we know the horizontal component of this and any other instantaneous velocity vector to be 2.0 m/sec. To find the instantaneous velocity vector first draw a line indicating its direction which is in the direction of the path (see Figure 10-13a). Now draw the horizontal velocity vector to scale placing its tail at the upper left end of the line representing the instantaneous velocity vector (Figure 10-13b). Finally draw a line perpendicular from the head of this arrow until it crosses the line representing the instantaneous velocity vector (Figure 10-13c). The arrow along the direction of the path represents the instantaneous velocity vector at that time. Using this method we plotted the instantaneous velocity vectors every 0.10 second time interval (Figure 10-14).

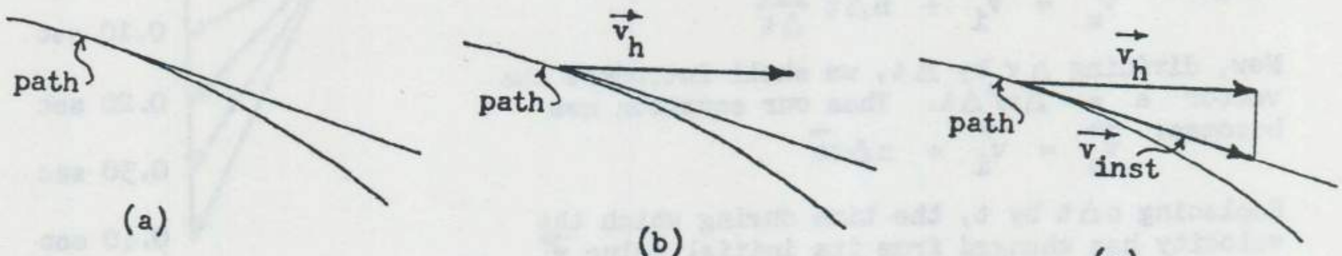


Figure 10-13

Figure 10-15 shows the instantaneous velocity vectors with their tails at one common point. Using the principle of vector subtraction we can see that the change in velocity of the instantaneous velocity vector is constant in magnitude and direction (downward). The average acceleration is therefore constant. Because the average acceleration is constant throughout the motion, we can be almost certain that the instantaneous acceleration is also constant throughout the motion. To confirm this, of course, we might want to take additional strobe photographs of the falling ball, using shorter and shorter time intervals to approach the limit as $\Delta t \to 0$. But even without this, we can be pretty certain of our conclusion. Consequently, for the instant of the nth flash, we can write the instantaneous velocity as:

$$\vec{v}_n = \vec{v}_i + n\overrightarrow{\Delta v}$$

Here $\vec{v}_i$ is the initial horizontal vector (2.0 m/sec), n is the number of 0.10 sec. time intervals since we started, and $\overrightarrow{\Delta v}$ is the constant change in velocity that occurs each 0.10 sec. time interval. By adding n of these changes to the original velocity, we get the velocity n intervals further along.

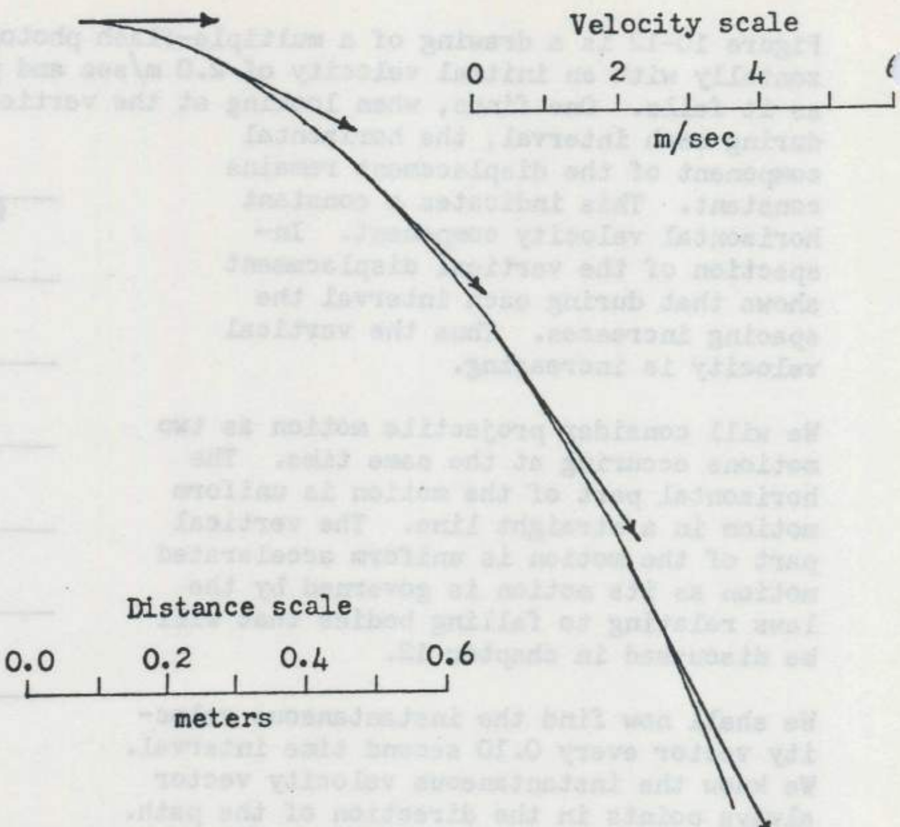


Figure 10-14

We can rewrite the last equation so that it more closely resembles the equations we developed for the description of motion along a straight-line path. (See Chapter 9, especially Section 9-7 and box). There we defined the acceleration along the path as:

$$a = \Delta v / \Delta t.$$

By both multiplying and dividing $n\overrightarrow{\Delta v}$ by Δt, we get:

$$\vec{v}_n = \vec{v}_i + n\Delta t\,\frac{\overrightarrow{\Delta v}}{\Delta t}$$

Now, dividing Δv by Δt, we shall introduce the vector $a = \Delta v/\Delta t$. Thus our equation now becomes:

$$\vec{v}_n = \vec{v}_i + n\Delta t\vec{a}$$

Replacing $n\Delta t$ by t, the time during which the velocity has changed from its initial value $\vec{v}_i$ to its value when the nth flash occurs, we can express the velocity at time t as:

$$\vec{v} = \vec{v}_i + \vec{a}t$$

Note that since $\vec{v}_i$ is in a horizontal direction and $\vec{a}$ in a downward direction, $\vec{v}$ will point more and more downward as time progresses.

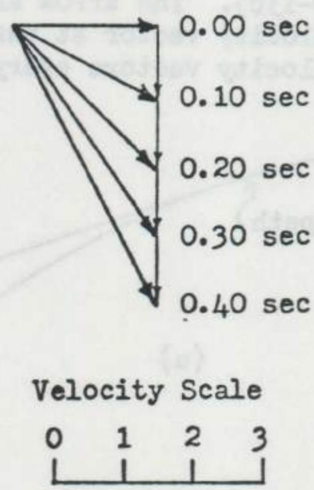


Figure 10-15

Proper analysis of the apparently complicated parabolic motion shown in Figure 10-12 is accomplished by resolving the motion into perpendicular components. The motion in the horizontal direction is uniform with $\vec{v}_h = 2.0$ m/sec. Since the motion is uniform, the horizontal displacement $\vec{d}_h$ when the object crosses the lowest white line is:

$$\vec{d}_h = \vec{v}_h t \quad = \quad 2.0 \text{ m/sec} \times 12(1/30 \text{ sec}) \quad = \quad 0.8 \text{ m}$$

The motion in the vertical direction is accelerated motion with the initial vertical velocity $\vec{v}_i$ equal to zero. (The initial velocity was in the horizontal direction only.) Here the vertical displacement is found by using the horizontal lines which are spaced 15 cm apart. The object falls a vertical displacement $\vec{d}_v$ of 75 cm or 0.75 m as it crosses the lowest white line.

The resultant displacement $\vec{d}_R$ is found as follows:

$$\vec{d}_R = \vec{d}_h + \vec{d}_v = 0.8 \text{ m (horiz)} + 0.75 \text{ m (vert)} = 1.1 \text{ m (Horiz } 43.1^\circ\text{-down)}$$

To determine the resultant velocity $\vec{v}_R$ as the object crosses the lowest white line we first determine the perpendicular components and then add them using vector addition.

The vertical velocity $\vec{v}_v$ occurs after the object falls 0.75 m in 12/30 sec. It is found by:

$$d = \tfrac{1}{2}t(v_f + v_i) \quad \text{Thus:} \quad v_f = \frac{2d_v}{t} = \frac{2 \times 0.75 \text{ m}}{12/30 \text{ sec}} = 3.75 \text{ m/sec}$$

The resultant velocity $\vec{v}_R = \vec{v}_h + \vec{v}_v$

$$= 2.0 \text{ m/sec (horiz)} + 3.75 \text{ m/sec (vert)}$$

$$= 4.25 \text{ m/sec} \quad \text{Horiz } 61.9^\circ \text{ Down}$$

In summary the displacement and/or the velocity can be determined at any position by:

1. Find the horizontal and vertical components of displacement and/or velocity.
2. Combine the two components using proper vector addition.

(graph paper ruler and protractor needed)

1. By any method you wish, find the sum $(\vec{A} + \vec{B})$ and the difference $(\vec{A} - \vec{B})$ of the vectors listed below.

   $\vec{A}$ = (3, 45 deg) $\vec{B}$ = (5, 120 deg) (length in arbitrary units)

2. Show by scale drawing that the associated law of addition

   [i.e. $(\vec{E} + \vec{F}) + \vec{G} = \vec{E} + (\vec{F} + \vec{G})$]

   holds for the vectors below. Be careful you don't make any unstated assumptions.

   $\vec{E}$ = (10, 90 deg) $\vec{F}$ = (5, 135 deg) $\vec{G}$ = (6, 180 deg)

3. Find the x and y components of the vector, $\vec{V}$ = (11, 160 deg).

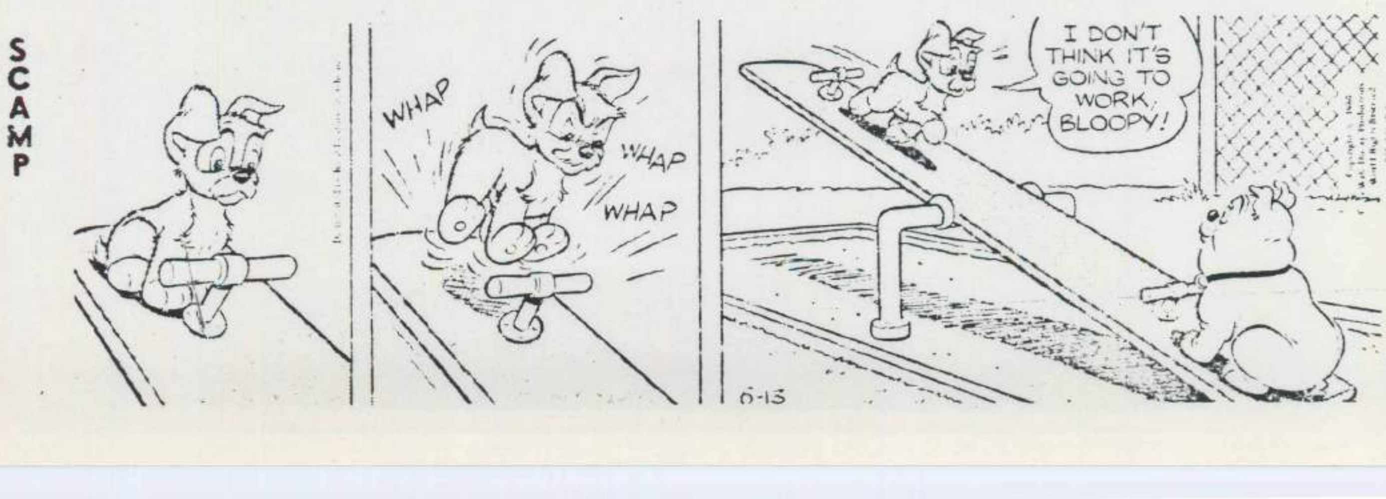
4. If the x-component of a vector is -6.0 and the y component of the same vector is -8.0, completely specify the vector that has these components.

5. What point is Banesh Haffmann making in the following:

The Curious Incident of the Vectoral Tribe

It is rumored that there was once a tribe of Indians who believed that arrows are vectors. To shoot a deer due northeast, they did not aim an arrow in the northeasterly direction; they sent two arrows simultaneously, one due north and the other due east, relying on the powerful resultant of the two arrows to kill the deer.

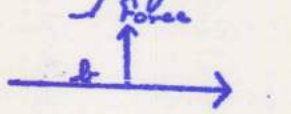
Skeptical scientists have doubted the truth of this rumor, pointing out that not the slightest trace of the tribe has ever been found. But the complete disappearance of the tribe through starvation is precisely what one would expect under the circumstances; and since the theory that the tribe existed confirms the NONVECTORAL BEHAVIOR OF ARROWS, it is surely not a theory to be dismissed lightly.



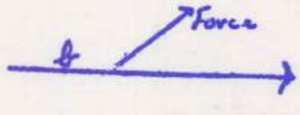
Experiment III-6. Centripetal Force.

A. Derivation - method 1

Since motion in a circle at constant speed is an accelerated motion, the acceleration is always perpendicular to the velocity and directed toward the center. A little common sense will verify this necessity for normality. If the velocity vector "$\vec{v}$" is to remain

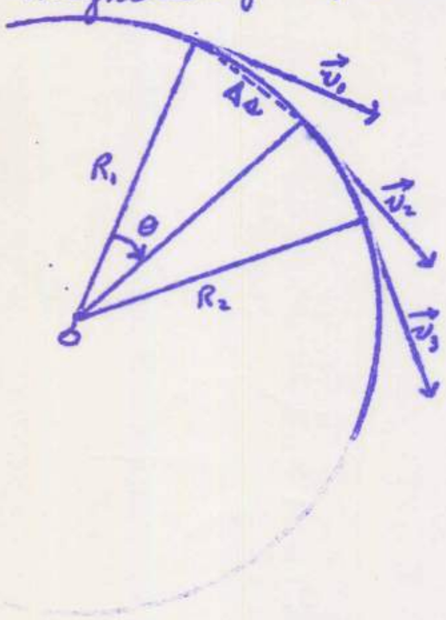


constant in magnitude acceleration has to be normal to it. If the acceleration is not normal



to the velocity vector, the magnitude has to change and therefore you do not have constant speed.

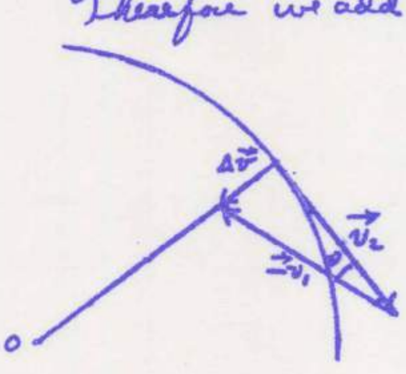
If the magnitude of the velocity is constant this means the magnitude of $\vec{v}_1$ = magnitude of $\vec{v}_2$ in the following diagram.



Let Δt be the time to go to Δs, as the velocity changes from $\vec{v}_1$ to $\vec{v}_2$.

The acceleration is $\frac{\Delta \vec{v}}{\Delta t}$, but since $\Delta \vec{v}$ is the change in velocity during time Δt, $\Delta \vec{v}$ is the vector difference of $\vec{v}_2$ and $\vec{v}_1$.

Therefore we add $\vec{v}_2 + (-\vec{v}_1) = \Delta \vec{v} = \vec{v}_2 - \vec{v}_1$



Now by inspection triangles with sides Δv, v_1, v_2 and Δs, R_1, R_2 are similar. (Both are isosceles and sides $\vec{v}_1$ and $\vec{v}_2$ are mutually perpendicular to sides R_1, R_2.)

If we use the magnitudes of the vectors, we find $\frac{\Delta v}{\Delta s} = \frac{v}{R}$, where $v_1 = v_2 = v_3 = v$.

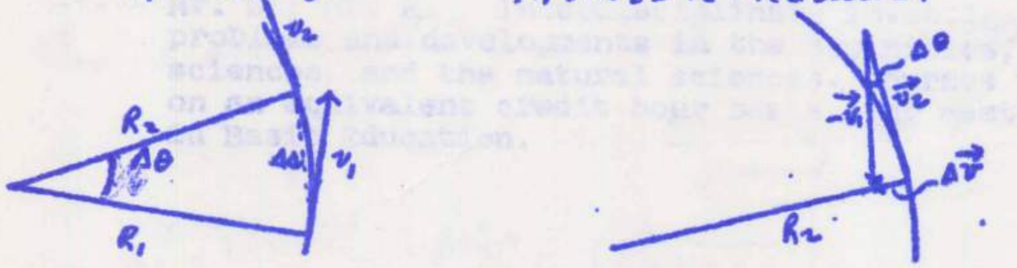
If $\Delta v = \frac{v}{R} \Delta s$ is divided by Δt, $\frac{\Delta v}{\Delta t} = \frac{v}{R} \frac{\Delta s}{\Delta t} = \frac{v}{R} v = \frac{v^2}{R}$

The quantity $\frac{\Delta v}{\Delta t}$ is the magnitude of the acceleration, (called centripetal acceleration), a_c. $\therefore$ $a_c = v^2/R$ this centripetal acc. gives rise to vector direction changes and is directed toward the center of the circle.

(Source: OREAR - Fund. of physics.
VANNAME - Analytical Mechanics)

B. Derivation - Method 2. (Source: SHORTLEY + WILLIAMS - Physics.)

Instantaneous velocity is always tangent to the circle and perpendicular to the radius.



$\Delta\vec{v}$ is $\perp$ to Δs and $\Delta\vec{v}$ therefore in the limit as $\Delta\theta \to 0$ is directed towards the center of the circle.

Now since $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$, where $\vec{a}$ = average acceleration during time Δt, the average acceleration has the same direction as $\Delta\vec{v}$ and is also directed toward the center as $\Delta t \to 0$, or the instantaneous acc. vector is the limit of the average acc. vector as $\Delta t \to 0$ $\left[\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}\right]$, therefore $\Delta\theta \to 0$ and the acceleration vector is directed to the center of the circle at every point.

To get the magnitude of vector a, for small $\Delta\theta$'s we see that the magnitude of $\Delta\vec{v} = v\,\Delta\theta$ [Equation 1], as a chord $\approx$ arc for very small angles. This results in $a = \lim_{\Delta t \to 0} \frac{v\,\Delta\theta}{\Delta t} = \frac{v\,d\theta}{dt}$ [Equation 2],

Since v = constant magnitude,
$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{R\,\Delta\theta}{\Delta t} = \frac{R\,d\theta}{dt}$ [Equation 3].

Dividing Equation 2 by Equation 3,

$$\frac{a}{v} = \frac{v\frac{d\theta}{dt}}{R\frac{d\theta}{dt}} = \frac{v}{R} \quad \text{and} \quad \boxed{a_c = \frac{v^2}{R}}$$

C. By Newton's 2nd Law for a mass m,

$$\boxed{F_c = ma \quad = \frac{m v^2}{R}}$$

D. There is also another form for the above results.

Since $a_c = \frac{v^2}{R}$ and $v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi R}{T}$, then

$$a_c = \frac{\frac{(2\pi R)^2}{T^2}}{R} = \frac{4\pi^2 R^2}{T^2 R} = \boxed{\frac{4\pi^2 R}{T^2} = a_c}$$

$$\text{and } \boxed{F_c = \frac{m\, 4\pi^2 R}{T^2}}$$

INTRODUCING VECTOR ADDITION

by

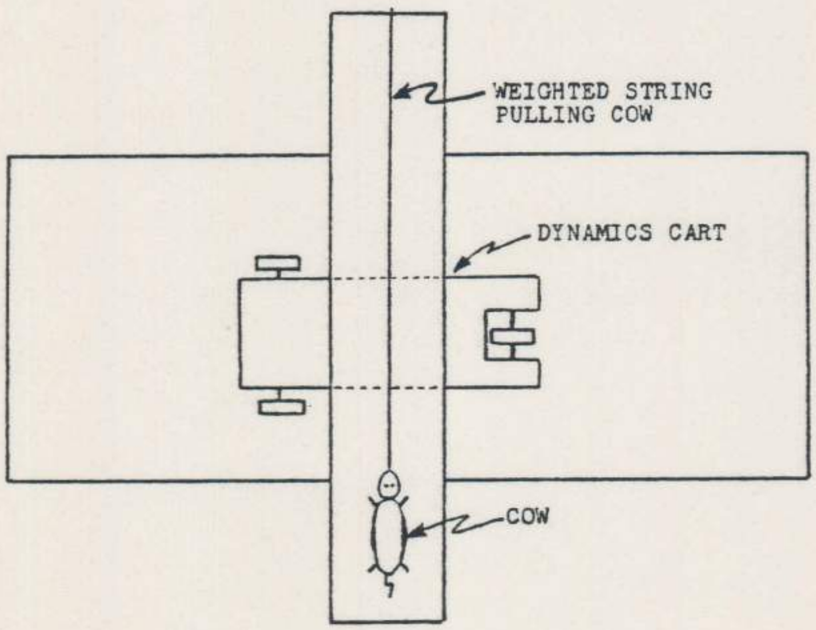
Robert D. Smith
Chester High School

Of all the devices used to illustrate vector addition, a "walking" toy animal has a distinct advantage: appropriate "traveling music" can be supplied while the animal is undergoing its displacement.

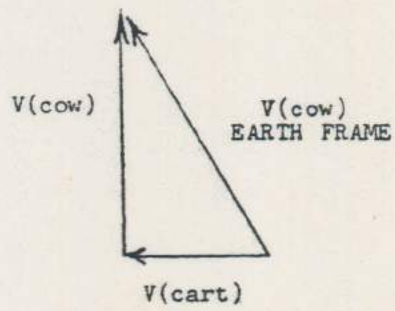
The toy animal we use is a walking cow, and the traveling music is a taped excerpt from "The Light Cavalry Overture." This ludicrous combination of sight and sound removes some of the "heaviness" that students frequently associate with vectors.

The cow is confined to a straight line path by using a board, which had warped to produce a nice concave track. When this board is balanced on a dynamics cart and then pulled across a demonstration table, a displacement at right angle to the cow's motion is obtained. Without previous experience with vector addition, most students can predict correctly the displacement resulting from the two simultaneous displacements at right angles to each other.

Frames of reference can be introduced by placing a small doll on the board and asking students to describe the displacement the doll would see, both when the board is stationary and when the board is displaced at right angle to the displacement of the cow. (Both descriptions would be the same.) A "bird" suspended above the table represents an observer in an Earth frame of reference. After describing the displacements observed by both the doll in the moving frame and the bird in the Earth frame, the question: "Whose description is correct - the doll's or the bird's?" emphasizes that all motion is relative.



TOP VIEW OF DEMONSTRATION TABLE



VECTOR DIAGRAM

Problems pertaining to a boat moving in a stream at right angles to a current, or an airplane flying at right angles to a jet stream become less formidable when the elements of the problem are related to this simple demonstration.

Another use of the walking toy: horizontal and vertical components of force can be introduced in a subtle manner. The cow does not walk off the end of the board when the weighted string pulling it becomes vertical. When the cow reaches the brink - it stops!

Editors' Note: After reading Bob's article we could not resist the temptation to include the following paragraphs from Banesh Hoffmann's book, About Vectors, Dover Publications, Inc., 1975.

THE CURIOUS INCIDENT OF THE VECTORAL TRIBE

It is rumored that there was once a tribe of Indians who believed that arrows are vectors. To shoot a deer due northeast, they did not aim an arrow in the northeasterly direction; they sent two arrows simultaneously, one due north and the other due east, relying on the powerful resultant of the two arrows to kill the deer.

Skeptical scientists have doubted the truth of this rumor, pointing out that not the slightest trace of the tribe has ever been found. But the complete disappearance of the tribe through starvation is precisely what one would expect under the circumstances; and since the theory that the tribe existed confirms two such diverse things as the NONVECTORAL BEHAVIOR OF ARROWS and the DARWINIAN PRINCIPLE OF NATURAL SELECTION, it is surely not a theory to be dismissed lightly.

Dr. Bartlett's talk, "THE FORGOTTEN FUNDAMENTALS OF THE ENERGY CRISIS" which was presented at our spring meeting last year, is available on video tape. Send three dollars and 55 minutes worth of one half inch EIAJ or three-quarters inch U-MATIC video cassette tape, color or B&W to:

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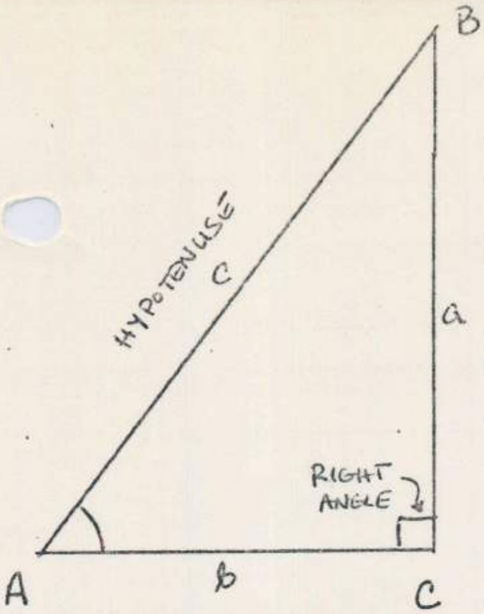


BASIC TRIGONOMETERY FUNCTIONS

IN REFERENCE TO THE ADJACENT RIGHT TRIANGLE, THE BASIC TRIG FUNCTIONS ARE DEFINED AS FOLLOWS:

SINE OF $\angle A = \dfrac{a \text{ (length of side opposite } \angle A)}{c \text{ (length of hypotenuse)}}$

COSINE OF $\angle A = \dfrac{b \text{ (length of side adjacent } \angle A)}{c \text{ (length of hypotenuse)}}$

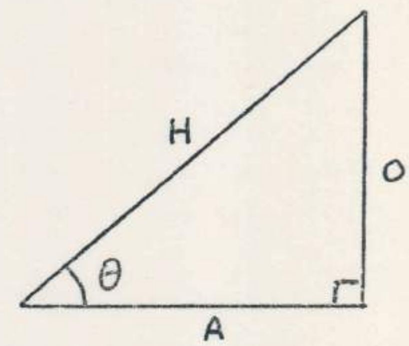
TANGENT OF $\angle A = \dfrac{a \text{ (length of side opposite } \angle A)}{b \text{ (length of side adjacent } \angle A)}$

WRITTEN IN FORMULA FORM:

$$\sin \theta = \frac{O}{H}$$

$$\cos \theta = \frac{A}{H}$$

$$\tan \theta = \frac{O}{A}$$



Since these are definitions, theyshould be memorized. The values for the sin, cos, and tan can be found in the trig tables in your text book and lab book in the back. All of the sides and angles of a right triangle may be determined by using the above information; two quantities must be known, such as the size of the one angle and the length of a side. DO NOT apply these relationships directly to SCALENE triangles. All you will get will be wrong answers!

TWO RELATIONSHIPS THAT MAY BE APPLIED TO ANY TRIANGLE:

SINE LAW: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ Where a, b, and c are sides opposite the corresponding angles

COSINE LAW:

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Where a, b, and c are the length of the sides.

HYPOTENUSE RULE: (PYTHAGOREAN THEOREM) ** USE WITH RIGHT TRIANGLES ONLY.

$$c^2 = a^2 + b^2$$

MOTION OF A PROJECTILE

Background Information

When a ball is thrown horizontally, it follows a curved path in a downward direction until it reaches the ground. The motion may appear to be very complex. However, if we assume that the motion of the ball is actually a combination of two separate component motions occuring at the same time, then the curved motion may be easily understood and we will be able to predict the path of the object (the trajectory) in advance. The two independent components of the projectile's motion may be described as follows:

HORIZONTAL COMPONENT

The hand exerts an unbalanced force (in the horizontal direction) thus causing the ball to accelerate from rest. Once the ball leaves the hand, there is no unbalanced horizontal force acting on the ball. Thus there can be no horizontal acceleration which means that the horizontal component of the velocity must remain constant for as long as the ball is in flight. (Note...We are assuming that the air resistance can be neglected.)

We are now in position to describe the horizontal distance d as varying directly with time.

$$d_h = v_h t$$

VERTICAL COMPONENT

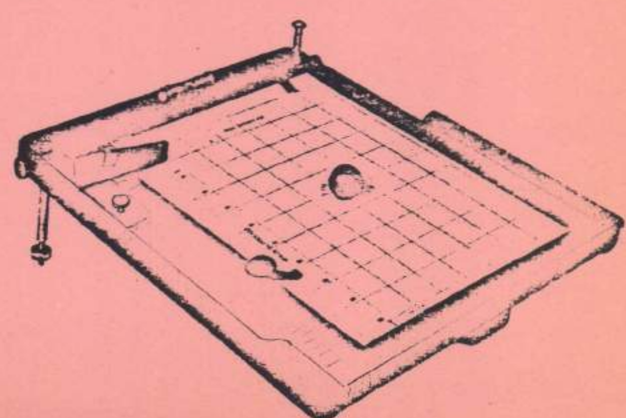
The gravitational attraction (pull of the earth) causes the ball to have a vertical motion that is accelerated downward. Thus for each equal interval of time, the unbalanced force will cause the vertical component of the velocity of the ball to increase. The result is that the vertical displacement d increases during each time interval. To calculate the vertical displacement we will use the 5 kinematic equations identified in chapter 1.

Purpose of this Experiment

In this investigation we will have a rolling ball trace its path (trajectory) on a sheet of paper. At regular spaced time intervals, we will make measurements of the horizontal and vertical displacements of the rolling ball with respect to the starting position. We will then compare the displacements to see how each varies with time.

The Apparatus

Packard's Acceleration Apparatus is a simple device for reducing the acceleration due to gravity thus allowing us to make laboratory measurements. The picture at the right gives an indication of how the apparatus looks in an operating position.



Adjustment

Position the apparatus on the legs such that the angle of the incline is about half the maxium angle. Adjust the leveling screws until the upper edge of the plate is level. Slip the graph paper under the starting trough and spring clips. (One can fasten the graph paper to the apparatus with masking tape if you so wish.) Adjust the paper so that the vertical zero line coincides with the zero marks just above and below the glass plate so that the horizontal "80" line is opposite the "80" mark just to the left of the plate. The carbon paper can then be placed over the graph paper without being clamped when you are ready to make a recorded run. Roughly adjust the position of the starting trough so that when the ball rolls off, it will mark a spot on the horizontal zero line. Next adjust the ball stop clamp until the rolling ball crosses the bottom of the paper at or very near the lower right-hand corner. The final adjustment to the starting trough is now made to assure that the ball strikes the paper on the intersection of the vertical and horizontal zero lines.

Doing the Experiment

With adjustments made so that the mark made by the ball leaving the starting trough is exactly on the zero point, place a sheet of soft carbon paper over the graph paper with carbon side down. Let the rolling ball make a carbon trace on the graph paper.

Change the angle of the incline to about twice that used and then allow the rolling ball to make a carbon trace on the paper. Finally remove the carbon paper and the graph paper from the apparatus.

Analysis of Data

Make three columns of figures, the first, (t) to represent the successive time intervals during which the ball traveled on the paper; the second, (d_h) to represent the total number of spaces that the ball went horizontally from the zero verticial line in time (t); the third, (d_v) to represent the total number of spaces that the ball fell below the zero horizontal line in time (t).

Because the horizontal velocity of the ball was constant, it took the ball equal times to travel the horizontal distance between lines on the paper. It is therefore possible to let the width of these vertical sections represent equal successive time intervals. Each 20th vertical line on the graph paper is numbered. These numbers are proportional to the times taken by the ball to travel across the paper to each line.

Finding the Solution

Find the mathematical relationship of the horizontal displacement (d_h) as a function of time of travel (t). Next find the mathematical relationshiip of the vertical displacement (d_v) as a function of the time of travel (t).

MOTION OF A PROJECTILE

WHAT HAVE WE LEARNED

1. What is the value of the slope in the graph of (d_h) vs (t)? What quantity does the slope represent? Write the mathematical relationship between d_h, t, and the slope of the line.

2. Describe the change of slope in the graph of (d_v) vs (t). What does this change mean?

3. What is the value of the slope in the graph for (d_v) vs (t^2)? What quantity does the slope represent? Write the relationship between d_v, t^2, and the slope of the equation.

4. Which of the following is the only factor common to both the horizontal and vertical motions of a projectile: acceleration, velocity, time, displacement?

5. Why is the effect of friction between the ball and the paper as well as the effects of air resistance disregarded in this experiment?

6. Why does the distance from one vertical line to the next on the trace of the ball's trajectory represent both equal horizontal displacements and equal time intervals?

7. What is the effect of increasing the angle of inclination of the apparatus on both the horizontal and vertical components of the displacement of the ball? Refer to some given point on its trajectory.

8. Describe the motion if the angle of inclination of the plane is zero degrees.

9. Describe the motion for an angle of 90 degrees.

2

4

6

8

10

12

14

16

18

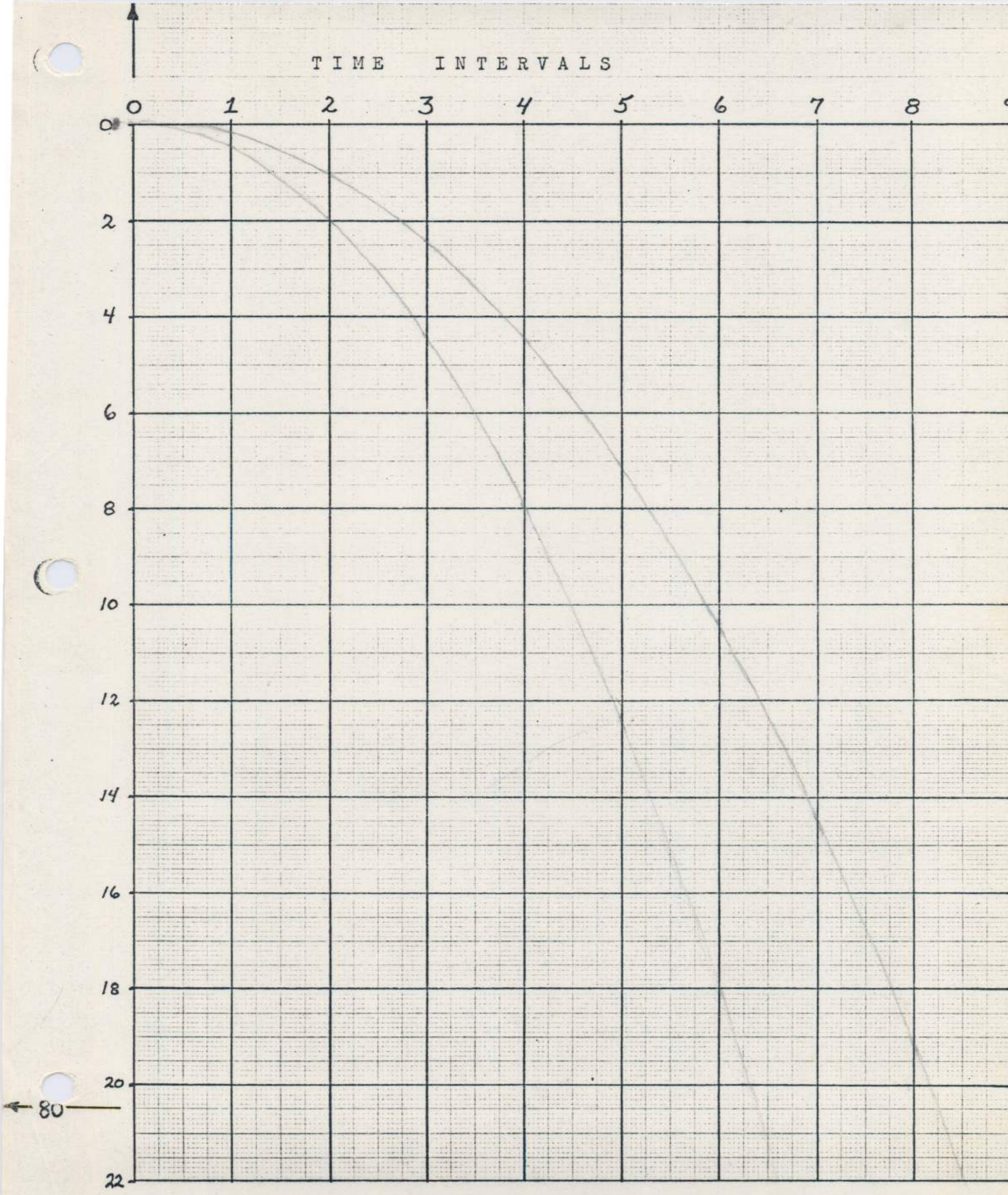
20

— 80 —

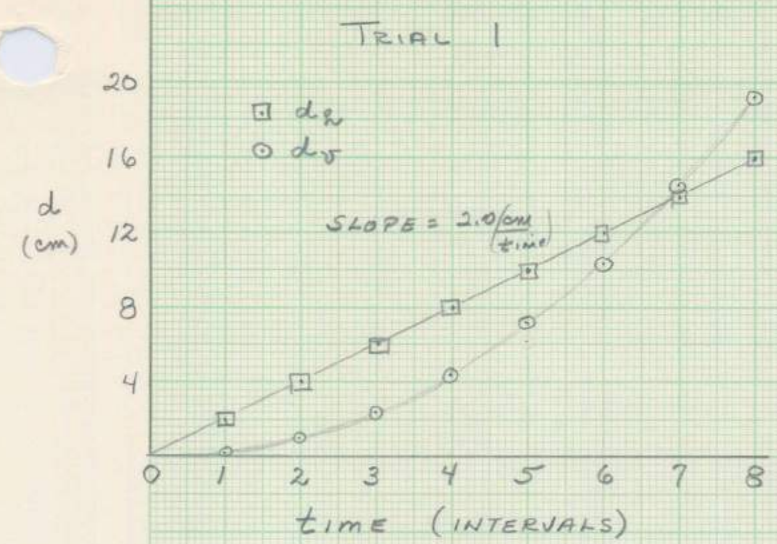
22

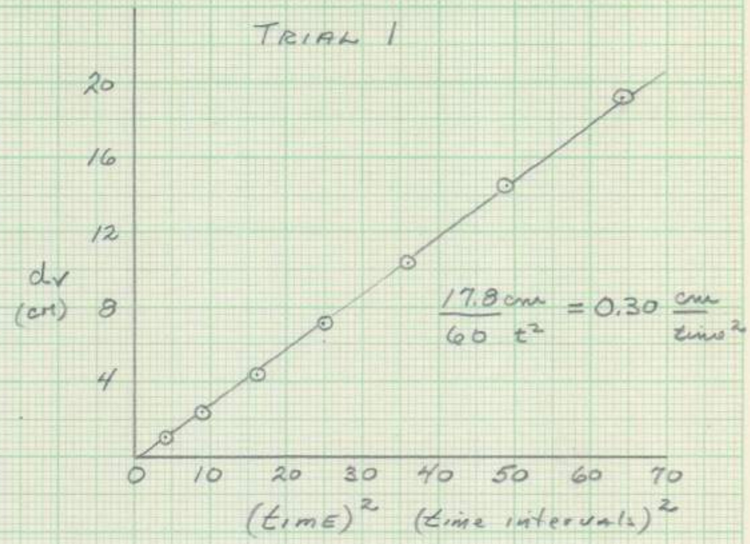
What have we learned?

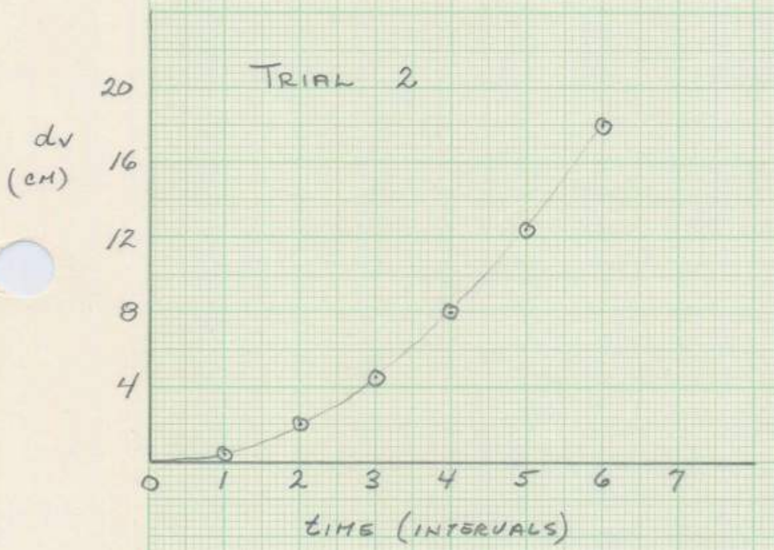
1. What is the value of the slope in the graph of (d_h) vs (t)? What quantity does the slope represent? Write the mathematical relationship between d_h, t, and the slope of the line.
   2.0 cm/t v_h $d_h = v_h t$
2. Describe the change of slope in the graph of (d_v) vs (t). What does this change mean? INCREASING => ACC.
3. What is the value of the slope in the graph for (d_v) vs (t^2)? What quantity does the slope represent? Write the relationship between d_v, t^2, and the slope as an equation.
   0.30 cm/t^2 $\frac{1}{2}$ ACC $d_v = at^2$
4. Which of the following is the only factor common to both the horizontal and vertical motions of a projectile: acceleration, velocity, time, displacement? time
5. Why is the effect of friction between the ball and the paper as well as the effects of air resistance disregarded in this experiment?
   MINIMAL
6. Why does the distance from one vertical line to the next on the trace of the ball's trajectory represent both equal horizontal displacements and equal time intervals? v_h is CONSTANT $\therefore d_h \propto t$
   $d_h = v_h t$
7. What is the effect of increasing the angle of inclination of the apparatus on both the horizontal and vertical components of the displacement of the ball? Refer to some given point on its trajectory.
   HORIZONTAL -> NONE   VERTICAL -> YES
8. Describe the motion if the angle of inclination of the plane is 0°.
   ALL HORIZONTAL
9. Describe the motion for an angle of 90°.
   ALL VERTICAL

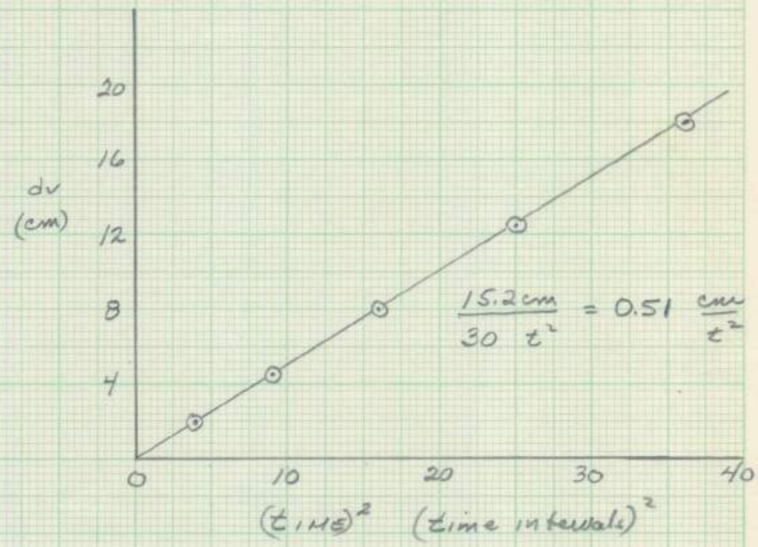


10 Millimeters to the Centimeter









TRIAL 1

t	d_v (cm)	d_h (cm)	t^2
0	0	0	0
1	0.2	2	1
2	1.1	4	4
3	2.4	6	9
4	4.4	8	16
5	7.2	10	25
6	10.4	12	36
7	14.5	14	49
8	19.2	16	64

TRIAL 2

t	d_v (cm)	d_h (cm)
1	0.4	2
2	2.0	4
3	4.5	6
4	8.0	8
5	12.5	10
6	18.0	12
7		
8		

Student Exercise: THE ACCELERATION VECTOR

In your previous studies in this course, you have learned something about acceleration. In chapter 9, section 6, you learned that the average acceleration along the path was expressed by the equation:

$$a_{avg} = \frac{\Delta v}{\Delta t} .$$

In the same section you learned that the instantaneous acceleration along the path had a rather strange expression:

$$a_{path} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} .$$

In chapter 10 you have been studying the vector applications of the same ideas. You have found that:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} ,$$

and that the instantaneous acceleration vector is expressed by the relationship:

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} .$$

Again, that strange mathematical notation! It would seem that, by allowing the time interval between the measurements of two successive instantaneous velocity vectors to approach zero, there would be no measurable Δv, and consequently, no acceleration.

The purpose of this exercise is to convince you that such is not the case; that a real value for the instantaneous acceleration vector exixts, even for a Δt of zero. To examine these ideas, we will deal with a specific situation. The basic facts we uncover about this specific case will also be true in general.

Consider an object traveling with a uniform speed of exactly 60.0 m/sec in a perfectly circular path, making one complete revolution in exactly 60.0 sec. [This motion is called uniform circular motion.] Even though the speed is not changing, you should recognize the fact that acceleration is going on because the velocity vector is constantly changing in direction. In 15 sec the direction of the instantaneous velocity changes by 90^o as seen in Figure 1. Can you justify that the instantaneous velocity vector changes 60^o in 10 sec?

Your first task will be to find the average acceleration over the 15 sec time interval. Knowing that $\vec{a}_{avg} = \Delta\vec{v}/\Delta t$, you must find $\Delta\vec{v}$ which is the vector difference between $\vec{v}_o$ (the velocity at t=0) and $\vec{v}_{15}$ (the velocity at t=15). You should be familiar with the process used for finding the vector difference between the velocities at 0 and 15 seconds, which is:

$$\Delta\vec{v} = \vec{v}_{15} - \vec{v}_0 = \vec{v}_{15} + (-\vec{v}_0).$$

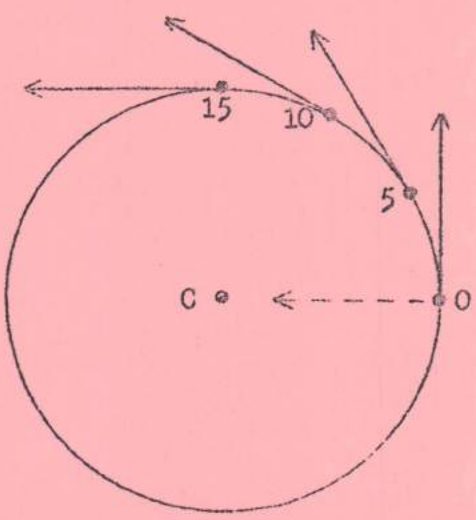


Fig. 1. The magnitude of the velocity vectors (lengths of arrows) are all the same, but their directions are all different. This difference in the directions means that acceleration is taking place. Note...the reference arrow pointing from point 0 towards the center of the circular path. The numbers represent Δt in sec.

This manipulation is indicated in Figure 2.

After determining Δv, one needs only to divide by Δt to determine the average acceleration.

The direction of the average acceleration vector needs to be expressed in terms of the reference direction (that has been chosen for us). Note...that the reference direction is indicated both in Figure 1 and in Figure 2.

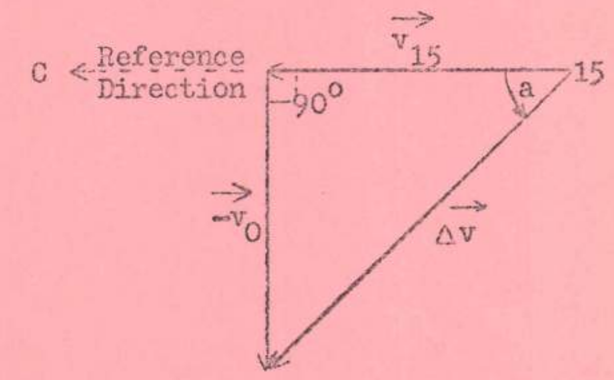


Fig. 2. The angle (a) is measured between the reference direction and the vector Δv. Will it always be half the angle between the two instantaneous velocity vectors? How do you find the average acceleration vector from the Δv vector?

What will happen to the average acceleration vector if we calculate its magnitude and direction over shorter and shorter time intervals? By so doing, we would allow Δt to 'approach zero'. To find out, we will determine its magnitude and direction over the progressively shorter time intervals: 15, 10, 5, 3, 2, 1, and 0.5 seconds. One fairly obvious result is that the value for the average acceleration gets progressively closer to whatever it might be for a Δt of zero. This would be the instantaneous acceleration. Using the indicated method, accumulate the data called for in Table I. Be precise in your calculations; we are after a precise relationship.

Using the data you have collected, construct the following two graphs:

1. THE MAGNITUDE OF THE AVERAGE ACCELERATION AS A FUNCTION OF Δt.

   Expand your acceleration scale as much as possible by reserving nearly all of your acceleration axis for the range of values you obtained.

2. ANGLE (a) AS A FUNCTION OF Δt.

   Carefully extrapolate your average acceleration curve to zero at Δt. What value do you get for the magnitude of your instantaneous acceleration vector? Do the same for your angle (a) curve. What is the direction of your instantaneous acceleration vector?

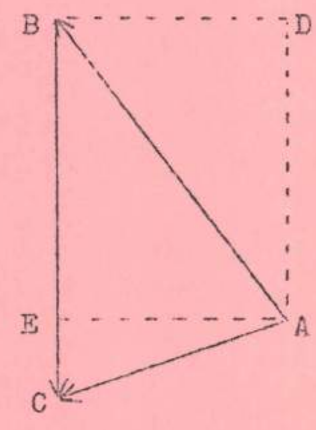
The results you have just obtained gave the instantaneous acceleration vector at the point chosen as O. Any other point on the circular path could have been chosen without materially affecting your results. Write a generalization for the instantaneous acceleration vector for any point on this circular path.

From the information given and the appropriate formula found on page 212 of the text, calculate the radius of the circle. What is the name for this kind of motion? for this kind of acceleration? Using the appropriate formula, again found on page 212, calculate the acceleration. How does it compare to the value obtained by extrapolation on your graph?

Table I. DATA FOR AVERAGE ACCELERATION

Δt (sec)	Δv (m/sec)	a_{avg} (m/sec^2)	a (deg.)
15	84.85	5.66	45
10	60.00	6.00	30
5	31.06	6.21	15
3	18.77	6.26	9
2	12.54	6.27	6
1	6.28	6.28	3
0.5	3.14	6.28	1.5

Alternatives for finding AC and angle EAC, given AB, BC, and angle BAD.



1. Scaling

   Choose an appropriate scale and lay out vectors AB and BC in the given direction. Measure vector AC and using the scale factor, determine the value it represents. Also measure angle EAC.

2. Using Sine and Cosine Law

   Using Cosine Law to find AC

   $$AC = \sqrt{(AB)^2 + (BC)^2 - 2(AB)(BC)(\cos ABC)}$$

   Using Sine Law to find angle EAC

   $$\frac{BC}{\sin BAC} = \frac{AC}{\sin CBA}$$

   Angle EAC = Angle BAC − Angle EAB

3. By resolving vectors into X-Y components.

   Using basic trig. to find X-Y components of AB, which are: BD and AD

   $$\sin BAD = \frac{BD}{BA} \qquad \cos BAD = \frac{AD}{BA}$$

   Finding angle EAC:

   $$\tan EAC = \frac{EC}{EA} \qquad EC = BC - BE \qquad EA = BD$$

   Finding AC.

   $$\sin EAC = \frac{EC}{AC}$$

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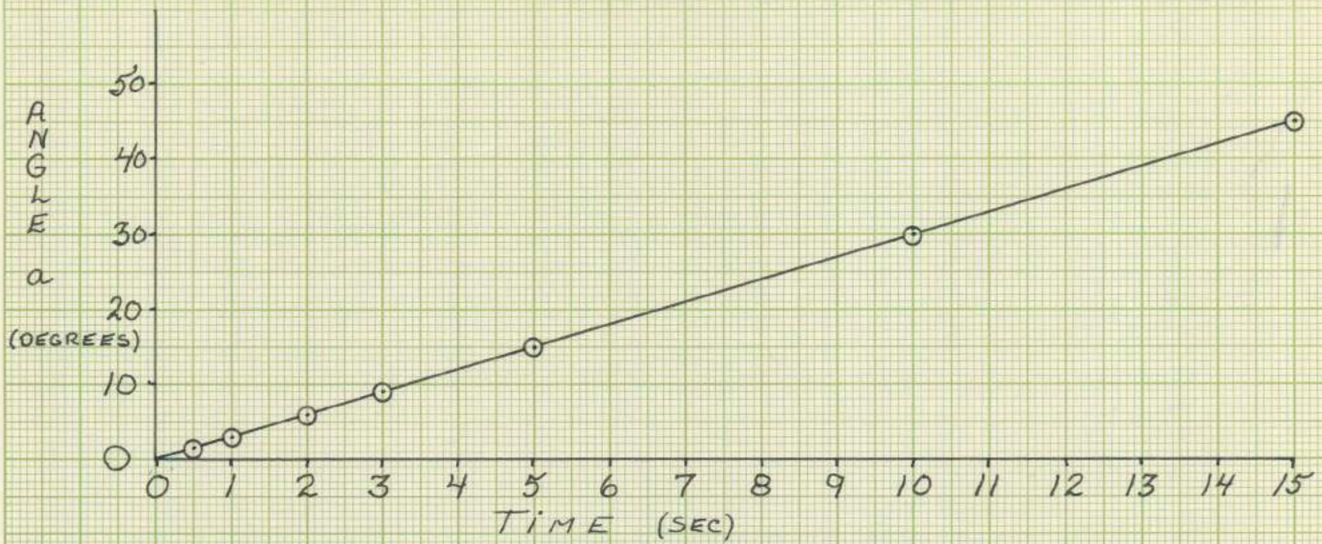
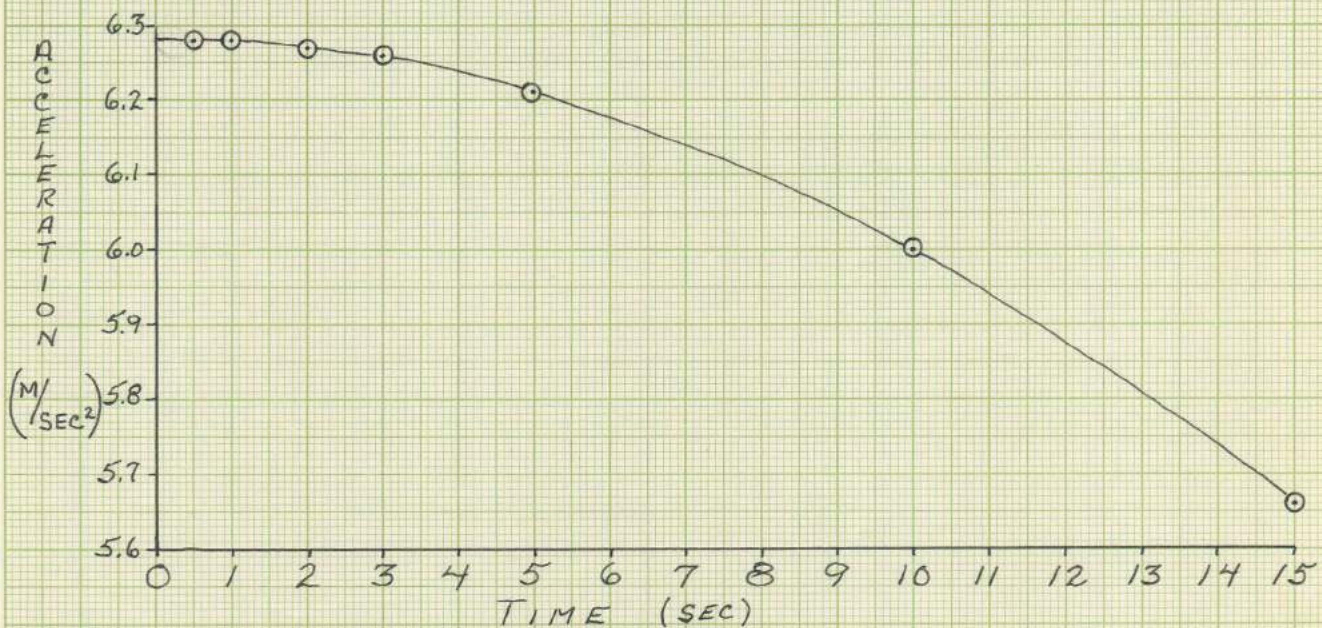
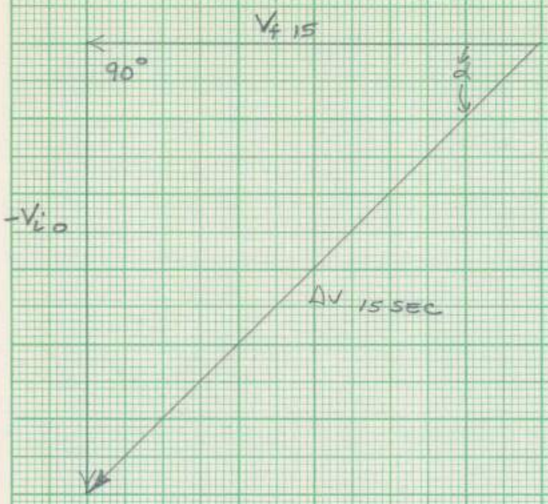
STUDENT EXERCISE INSTANTANEOUS VECTOR





10 Millimeters to the Centimeter



BY SCALING:

$\Delta V = 85\ M/SEC$

$\alpha = 45°$

BY SINE & COSINE LAW

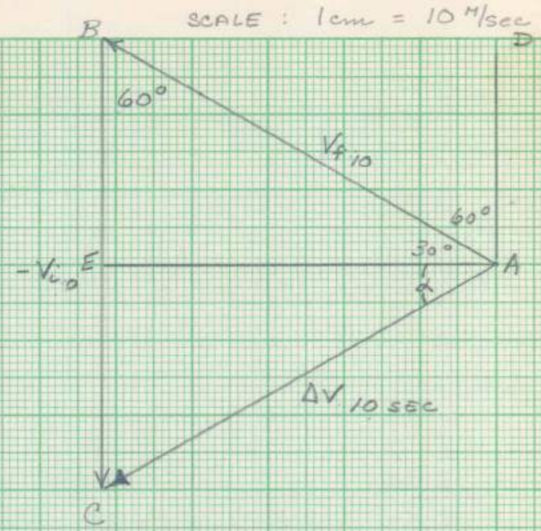
$\Delta V = \sqrt{60^2 + 60^2 - 2 \times 60 \times 60 \times \cos 90°}$

$\Delta V = 84.85\ M/SEC$

$\frac{60}{\sin\alpha} = \frac{84.85}{\sin 90°}$

$\alpha = 45°$

SCALE : 1cm = 10 M/sec



BY SCALING

$\Delta V = 60\ M/SEC$

$\alpha = 30°$

BY SINE AND COSINE LAW

$\Delta V = \sqrt{60^2 + 60^2 - 2 \times 60 \times 60 \times \cos 60°}$

$\Delta V = 60.00$

$\frac{60}{\sin \angle A} = \frac{60}{\sin 60°}$

$A = 60°$ $\alpha = 30°$

BY RESOLVING INTO X-Y COMPONENTS

$\sin 60° = \frac{BD}{60}$ $\cos 60° = \frac{AD}{60}$

$BD = 51.96$ $AD = 30$

$EC = BC - AD = 60 - 30 = 30$

$EA = BD$

$\tan\alpha = \frac{EC}{EA} = \frac{30}{51.96}$

$\alpha = 30°$

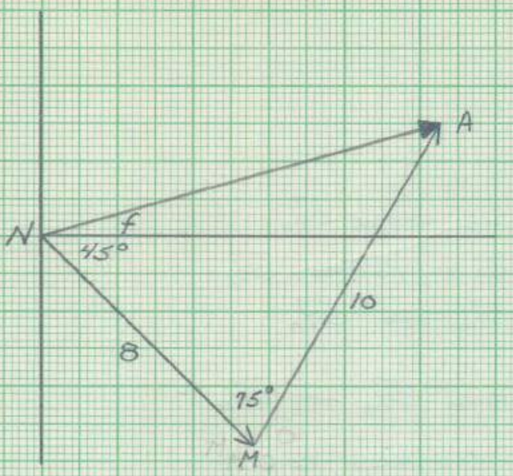
$\sin\alpha = \frac{CE}{CA}$

$CA = 30/\sin 30°$

$CA = 60\ M/sec$

10 Millimeters to the Centimeter

SCALE: 1 CM = 2 UNITS



USING SINE & COSINE LAW

$NA = \sqrt{(NM)^2 + (OA)^2 - 2 \times (NM)(OA)(\cos NMA)}$

$= \sqrt{8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 75^\circ}$

$= 11.07$

$\measuredangle ANM =$

$\dfrac{10}{\sin \measuredangle ANM} = \dfrac{11.07}{\sin 75^\circ}$

$\measuredangle ANM = 60.76$

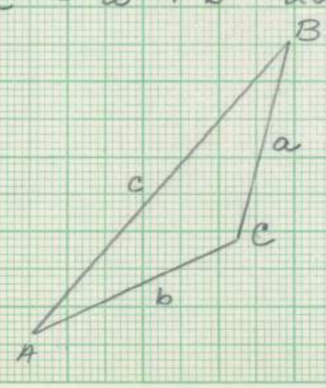
$\measuredangle f = 15.76^\circ$

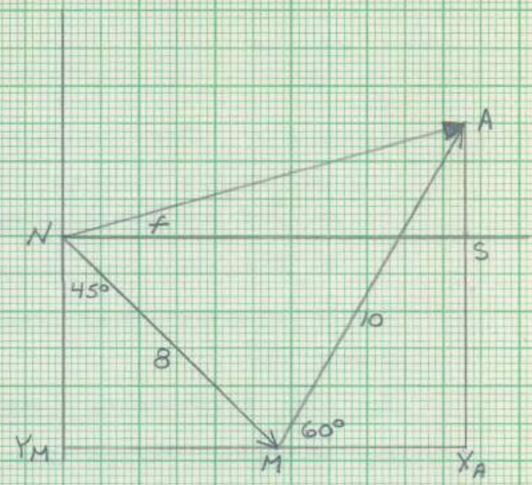
SINE LAW

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

COSINE LAW

$c^2 = a^2 + b^2 - 2ab \cos C$





WORKING WITH $N\text{-}M\text{-}Y_M$

$\sin 45^\circ = \dfrac{MY_M}{8}$ $\quad \cos 45^\circ = \dfrac{Y_M N}{8}$

$MY_M = 5.66$ $\quad Y_M N = 5.66$

WORKING WITH $A\text{-}M\text{-}X_A$

$\cos 60^\circ = \dfrac{MX_M}{10}$ $\quad \sin 60 = \dfrac{X_A A}{10}$

$MX_A = 5.00$ $\quad X_A A = 8.66$

$NS = Y_M M + X_A M = 10.66$

$AS = AX_A - NY_M = 3.00$

$\tan f = \dfrac{3.00}{10.66} = 15.72^\circ$

$\sin f = \dfrac{3.00}{AN}$

$AN = 11.07$

 Name ____________________

1. The vector $\vec{V}_3$ in the diagram at the right is equivalent to

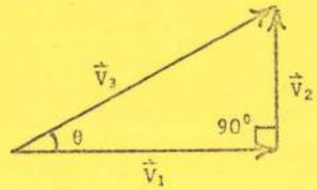
(A) $\vec{V}_1 - \vec{V}_2$

(B) $\vec{V}_1 + \vec{V}_2$

(C) $\vec{V}_2 - \vec{V}_1$

(D) $\vec{V}_1^{\,2} + \vec{V}_2^{\,2}$

(E) $\vec{V}_1 \cos \theta$



2. A plane travelling at 200 m/s north turns and then travels south at 200 m/s. Its change of velocity is

(A) 0 m/s

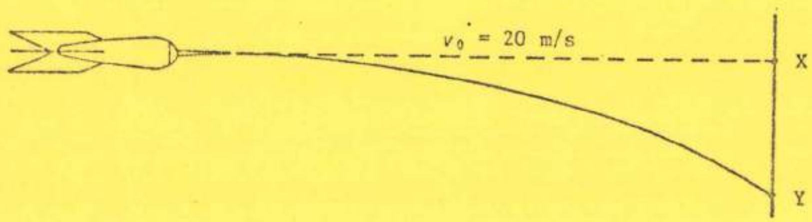
(B) 200 m/s north

(C) 200 m/s south

(D) 400 m/s north

(E) 400 m/s south

3. A dart is thrown horizontally toward X with a speed of 20 m/s. It hits a point Y 0.1 s later.



The distance XY will be approximately

(A) 2 m

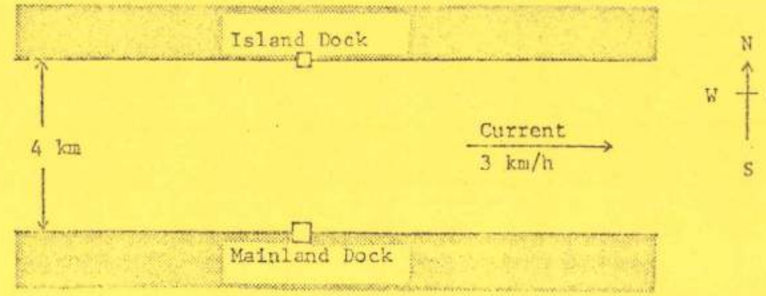
(B) 1 m

(C) 0.5 m

(D) 0.1 m

(E) 0.05 m

A man lives on an island 4 km offshore, across a channel which has a 3 km/h current when the tide is running. On a dark night, with the tide running, he rows from the mainland dock toward the island, keeping his boat aimed directly north. The island dock is due north of the mainland dock. He rows his boat at a speed of 4 km/h relative to the water.



4. Where will he land?

(A) at the dock directly opposite his starting point

(B) 3 km downcurrent from the island dock

(C) 4 km downcurrent from the island dock

(D) 5 km downcurrent from the island dock

(E) 7 km downcurrent from the island dock

5. Because of the current he will have a speed relative to the earth of

(A) 1 km/h (C) 4 km/h (E) 7 km/h

(B) 3 km/h (D) 5 km/h

6. In order to row directly across to the island dock, he should have set his course upcurrent at an angle west of north. The sine of that angle is

(A) $\frac{5}{3}$ (C) $\frac{4}{5}$ (E) $\frac{3}{5}$

(B) $\frac{5}{4}$ (D) $\frac{3}{4}$

Name______________________

7. As a vehicle goes from 3.0 m/s [N 90° E] to 4.0 m/s [N 270° E], the change in velocity is

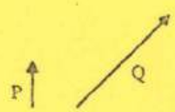
(A) 1.0 m/s, [N 90° E]

(B) 1.0 m/s, [N 270° E]

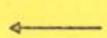
(C) 5.0 m/s, [N 315° E]

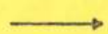
(D) 7.0 m/s, [N 90° E]

(E) 7.0 m/s, [N 270° E]



8. Given the vectors $\vec{P}$ and $\vec{Q}$ above, which one of the vectors shown below best represents the vector $2\vec{P} - \vec{Q}$?

(A) 

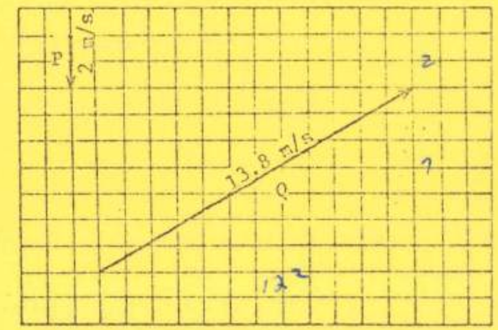
(B) 

(C) 

(D) 

(E) 

Vectors P and Q in the grid below represent velocities. Each side of each little square in the grid represents a speed of 1 m/s.



9. Let vector P represent one velocity component of a particle. Let vector Q represent the other velocity component of the same particle. The resultant speed of the particle is

(A) 11.8 m/s

(B) 13 m/s

(C) 13.8 m/s

(D) 15 m/s

(E) 15.8 m/s

10. Let vectors P and Q represent the initial and final velocities respectively of a particle over a 4.0 s time interval. Then the magnitude of the average acceleration during this interval is

(A) 3.25 m/s²

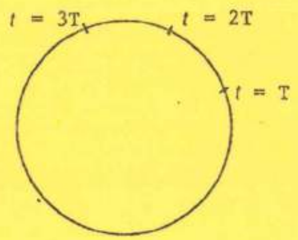
(B) 3.45 m/s²

(C) 3.75 m/s²

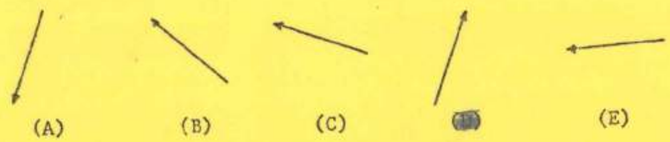
(D) 13 m/s²

(E) 15 m/s²

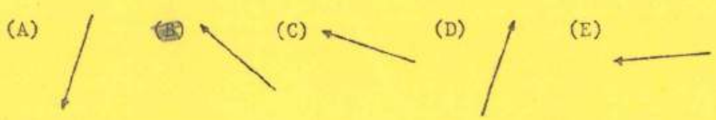
A proton situated in a magnetic field is observed to travel in a circular path. The positions of the proton at times T, 2T and 3T are shown in the figure below.



11. Which vector best represents the displacement of the proton from the centre of the circle at t = 2T?

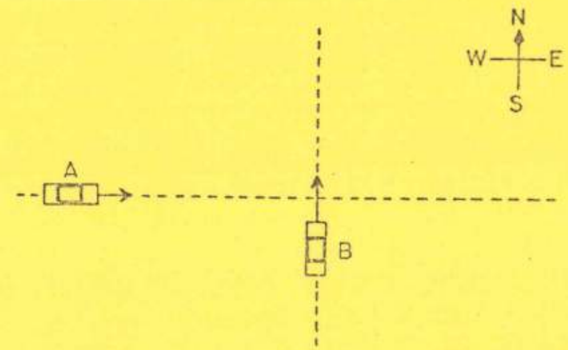


12. Which vector best represents the average velocity of the proton between t = T and t = 2T?



13. Which vector best represents the instantaneous velocity of the proton at t = 2T?

Two cars approach an intersection at right angles. Car A is travelling with a velocity of 30 km/h east, and car B is travelling with a velocity of 30 km/h north.



14. The velocity of B relative to A is

(A) 42 km/h SE

(B) 42 km/h NW

(C) 60 km/h NW

(D) 60 km/h SE

(E) 42 km/h NE

15. A ship is travelling 8.0 m/s due west in still water. A passenger is walking along the deck at 3.0 m/s toward the back end of the ship. He throws an apple core north at 12 m/s. The velocity of the apple core relative to the water is

(A) $\sqrt{265}$ m/s in a direction between north and northwest

(B) 13 m/s in a direction between north and northwest

(C) 13 m/s in a direction between west and northwest

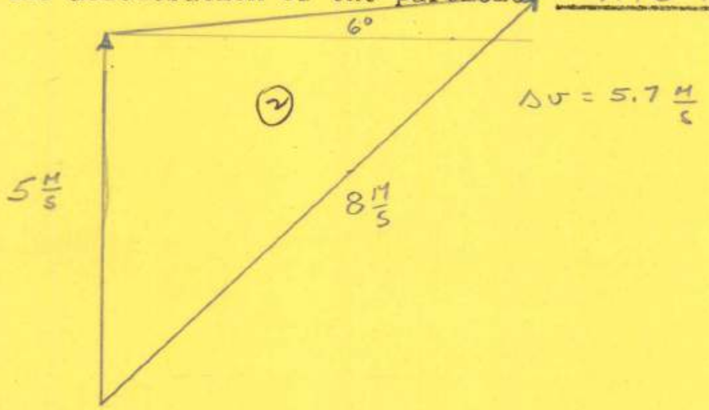
(D) 17 m/s in a direction between north and northwest

(E) $\sqrt{265}$ m/s in a direction between west and northwest

16. The velocity of a particle changes from 5 m/sec north to 8 m/sec N 45° E in 3 seconds. Draw vectors representing, and state the magnitude and direction of:

a. the change in velocity, 5.69 m/s (2) $\theta = 6.6^\circ$ (2)

b. the acceleration of the particle, 1.90 m/s² (2) $\theta = 6.6^\circ$ (2)

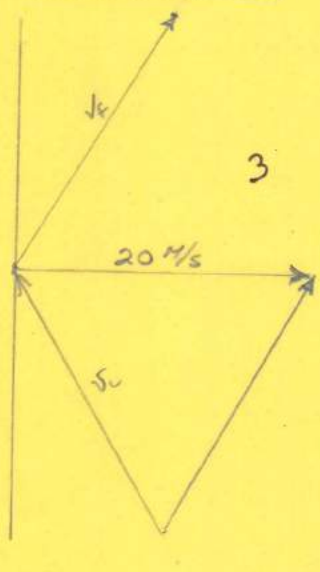


8

17. A billiard ball hits a cushion at a velocity of 20 cm/sec at 30° relative to the edge of the cushion and rebounds without loss of speed so that the angle of reflection equals the angle of incidence. What has been the change in:

a. speed, 0 3

b. velocity. 20 M/s → 2 2



3

arrows

10

A 1 +2
B - 6 - 3
C - 2 + 1
D - 1 + 2
E - 5 - 2

1. B 2. E 3. E 4. B 5. D 6. D 7. E 8. A

9. B 10. C 11. D 12. B 13. C 14. B 15. B