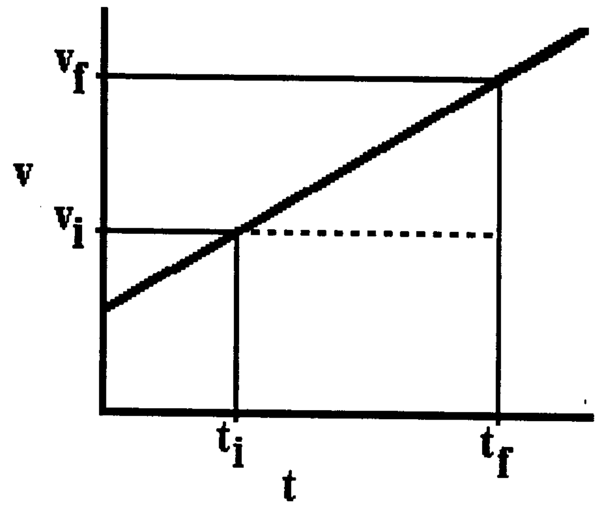
Accelerated Motion Along a Path



There are two things that a velocity - time graph can tell us. The first is the slope of the line represents the acceleration of the object that produced the velocity - time graph.

To obtain the slope one determines the value for the rise and the run and then divides the rise by the run.

Using the triangle shown, the rise can be defined by:

$$v_f - v_i$$

The run can be defined by $t_f - t_i$ which is the time interval which one represents by t. Thus:

(1) $a = (v_f - v_i)/t$ (no d)

The second thing a velocity - time graph can tell us is that the area under a graph will give the distance the object traveled.

The shape of the area is a trapezoid. The formula for finding the area of a trapezoid is multiply one half the sum of the bases by the height.

The bases are v_i and v_f while the height is t. Thus:

(2) $d = 1/2\, t\, (v_i + v_f)$ (no a)

These are all the equations one needs to solve for variables (a, v_i, v_f, d, and t) related to accelerated motion. For in the two equations one finds these five variables. If one know any three, a fourth can be found. To make things easier in the long run one can do some algebraic manipulation. So lets begin.

Notice that the first equation contains a, v_i, v_f, and t while the second one has d, v_i, v_f, and t. The first has no d while the second has no a. Is it possible to find an equation that has a, v_i, d, t and no v_f? It is and can be done by one of two methods. One is to solve one equation for v_f and substitute it into the other equation. A second method is to solve both for v_f and set them equal. If you use one of the two methods, you will obtain:

(3) $d = v_i t + 1/2at^2$ (no v_f)

What does $v_i t$ represent? What does $1/2at^2$ represent? Hint...apples plus peaches do not equal oranges. Hint...What are the units for v_i and for t? Do you get **m/s** and **s**? When you multiply them together what do you get? I hope you get **m** which represents distance. Do the same for $1/2at^2$ and you get the same. Thus both are distances as it should be, for to get distance, one can only add distances.

Now look at the graph above. Where is the distance $v_i t$? It is the area of the rectangle. Where is the area $1/2at^2$? Notice that the area of the triangle above the rectangle is $1/2t(v_f - v_i)$. $v_f - v_i$ is the change (Δ) in velocity. $a = \Delta v/\Delta t$, so $(v_f - v_i)$ can be replaced by $a\Delta t$. Δt is generally represented by t so $(v_f - v_i)$ can be replaced by at. This makes the area of the triangle above the rectangle $1/2at^2$. Thus the graph identifies that equation 3 indeed represents the distance traveled.

Repeating the above process, using the first two equations to eliminate vi gives:

(4) $d = v_f t - 1/2at^2$ (no v_i)

Again using the first two equations to eliminate t gives:

(5) $v_f^2 = v_i^2 + 2ad$ (no t)

Now to solve problems related to an object accelerating along a path, all one has to do is to first list the five variables a, v_i, v_f, d, and t. Using the GUESS method, identify what is given and the unknown, (what one is to find). Then pick one of the 5 equations using the remaining variable that is not used. Solve for the unknown (get it on the left side of the equation by itself), substitute in the variables and finally solve the problem.

That's all there is to it. So with a few practice problems under your belt, you too will be a professional problem solver for accelerated motion along a path.

Equations for Accelerated Motion Along A Path

$$a = \frac{v_f - v_i}{t}$$

$$d = \frac{1}{2}t(v_i + v_f)$$

$$d = v_i t + \frac{1}{2}at^2$$

$$d = v_f t - \frac{1}{2}at^2$$

$$v_f^{\,2} = v_i^{\,2} + 2ad$$