# Strength-to-Weight 

## Concepts

cross-sectional area
strength
volume
weight

## Description

A/64 consists of five examples of strength-to-weight ratios for scaled objects. The first segment shows the difference between the support strengths for a small clay arch and a large clay slab. The second example demonstrates the increase in the strength of a cord as its diameter increases. The third segment presents a movie clip of the monster gorilla, King Kong, while the fourth displays the amazing strength-to-weight ratio for a beetle. A beetle can easily push an object five times its weight. The last example explains the leg structure of an animal and a dinosaur relative to their weights, which justifies the hypothesis as to why dinosaurs might have lived in marshy regions.

## Teacher Information

Scaling is a good application not often covered in a standard physics course. This topic has application in engineering fields.

## Classroom Activities

$\sqrt{211} 15$


Discuss each of the five examples with students. The worksheet has five sections, one for each example on the videodisc. Show the video using audio track $1 / \mathrm{L}$. Encourage students to work in small groups and ask them to answer questions as a team. Follow the frame numbers as listed to understand each event.

## A/64: Strength-to-Weight

## Name: ANSWER SHEET

Date: __ Section: $\qquad$

Section 1: The strength of the clay arch is proportional to the cross-sectional area of its supports, while its weight is proportional to its volume.

1. Play frames $41875-41990$ using audio track $1 / \mathrm{L}$. Is the strength of the arch capable of supporting its weight?

The arch is capable of supporting its own weight.
2. Play frames 41991-42235 using audio track 1/L. What is the increase in weight of the larger arch? What is the increase in its support strength? Why did this arch fall? What is the strength-to-weight factor for the smaller arch?
(Note: you are free to use the unit of your choice.)

a. Record the dimensions of the small clay arch:

Length $=10$ unit Width $=1$ unit Height $=1$ unit
Cross-sectional area of clay arch $=I$ unit $^{2}$
Volume of clay arch $=10$ unit $^{3}$
b. Record the dimensions of the large clay slab:

Length $=100$ unit Width $=10$ unit $\quad$ Height $=10$ unit
Cross-sectional area of clay slab $=100$ unit ${ }^{2}$
Volume of clay slab $=10000$ unit $^{3}$
c. Weight is proportional to the volume of an object. The increase in the weight of the larger arch $=\frac{\text { volume of large slab }}{\text { volume of small arch }}=\frac{10000}{10}=1000$
d. Support strength is proportional to the area of an object. The increase in the support strength of the larger arch $=\frac{\text { cross }- \text { sectional area of large slab }}{\text { cross }- \text { sectional area of small arch }}=\frac{100}{1}=100$

The weight of the larger arch is 1000 times while its support strength is 100 times the smaller arch. The larger arch falls because its strength did not increase in the same ratio to volume.

Section 2: The strength of a string is proportional to its cross-sectional area. The weight of the safe is proportional to its volume.
3. Referring to the frames listed below, fill in the data.

| Mass of the toy safe | $=500$ grams (Frame 42236) |
| :--- | :--- |
| String diameter | $=0.01$ inches (Frame 42237) $=\mathrm{d}$ |
| Mass of the large safe | $=500 \mathrm{~kg}$ (Frame 42318), |
|  | $=1000$ times mass of the toy safe |
| Height of large safe | $=10$ times height of the toy safe |
| Volume of large safe | $=1000$ times the volume of toy safe |
| (Note: assume the safe is cubical in shape.) |  |
|  | $=0.1$ inches (Frame 42319) |
| Cord diameter | $=10$ times diameter of the string $=10 \times \mathrm{d}$ |

4. View frames 42320-42551. If you make the diameter of the cord ten times larger, how much does the strength of the cord increase?

$$
\begin{array}{ll}
\text { Cross-sectional area of the string } & =\pi \mathrm{d}^{2} / 4 \\
\text { Cross-sectional area of the cord } & =\pi / 4(10 \times \mathrm{d})^{2} \\
\text { Increase in the strength of the cord } & =\text { area of cord/area of string. } \\
& =100
\end{array}
$$

The strength of the cord increases by 100 because its cross-sectional area increases by 100.
5. View the question on frame 42320 . Will the cord lift the safe?

The strength of the cord is increased by a factor of 100 while the volume (weight) of the safe is increased by a factor of 1000. The strength-to-weight scale factor for the cord and large safe is smaller than the scale factor for the string and toy safe.
6. View frames 42552-42930. The diameter of the new cord:
$=0.01 \times \sqrt{1000}$ inches
$\approx 0.32$ inches
$=32$ times diameter of string
$=32 \times d$
7. By what factor did the strength of this cord increase over the string lifting the small safe?

$$
\begin{array}{ll}
\text { Cross-sectional area of the new cord } & =\pi / 4 \times 32 \times \mathrm{d}^{2} \\
\text { Increase in the strength of the cord } & =\text { area of cord/area of string } \\
& =1024
\end{array}
$$

The strength of the new cord is increased by 1024 over the string lifting the toy safe, so it can lift the safe that weighs 1000 times the toy safe.
8. Will the two cords, each of diameter $0.01 \times \sqrt{500}$ inches, lift the larger safe?
Diameter of each cord:
$=0.01 \times \sqrt{500}$ inches
$\approx 0.23$ inches
$=23$ times diameter of string
$=23 \times d$
Cross-sectional area of each cord $=\pi / 4 \times 23 \times \mathrm{d}^{2}$


Increase in the strength of the cord $=2 \times$ area of cord/area of string $=1058$.

Yes, the cords will lift the larger safe because their strength is 1058 times the strength of the string.

Section 3: The monster gorilla, King Kong, can support his own weight when the strength-to-weight ratio is the same as the normal gorilla's. Play frames 42931-43631, using audio track $1 / \mathrm{L}$.
9. What is the strength-to-weight ratio for the normal gorilla if we assume his height (or any dimension) to be $h$ ?

Approximate a normal gorilla using a cube:

| Height of the normal gorilla | $=h$ unit |
| :--- | :--- |
| Support area of the normal gorilla | $=h^{2} u^{2}{ }^{2}$ |
| Volume of the normal gorilla | $=h^{3} u^{3}{ }^{3}$ |
| Strength-to-weight factor | $=1 / h$ |


10. If King Kong were fifty times taller than a normal gorilla, what would be his strength-to-weight ratio? Would King Kong be able to support his own weight?

| Height of King Kong | $=50 h$ unit |
| :--- | :--- |
| Support area of King Kong | $=50^{2} h^{2} u^{u_{i t}}{ }^{2}$ |
| Volume of King Kong | $=50^{3} h^{3} u^{3}$ unit $^{3}$ |
| Strength-to-weight factor | $=1 / 50 h$ |

But. $(1 / 50 \mathrm{~h})<(1 / \mathrm{h})$.
Thus, if King Kong's leg bone strength were increased by $50^{2}-2500$ times, then his strength-toweight ratio would be less than the normal gorilla's. So King Kong would not be able to support his own weight.
11. For King Kong not to crumble under his own weight, how many times larger than a real gorilla's legs should his legs be?
For King Kong not to crumble under his own weight, the ratio must be equal to $(1 / h)$.

$$
\begin{aligned}
& (l h)=\text { cross-sectional area of leg bones } / 50^{3} h^{3} \\
& 5 \sigma^{3} h^{2}=\text { cross-sectional area of leg bones }
\end{aligned}
$$



The cross-sectional area of the leg bones is two-dimensional; therefore, the increased diameter (one dimension) of the leg bone must be $\sqrt{50^{3} h^{2}} \approx 350 h=350$ times larger, not just 50 times larger.
12. In the movie, Honey, I Blew Up the Kid, the producer wanted to depict a child twenty times the size of a normal child. How much bigger than a normal child's leg should the giant child's leg be in order to support his weight?

By performing the same calculations used in the previous example, the diameter of the giant child's leg bone is about 90 times larger than the normal child's bone.

Section 4: A beetle can push an object five times as heavy as its own weight because of its high strength-to-weight ratio. Play frames 43632-43809, using audio track 1/L.

Let us assume that a normal beetle $\approx 2$ inches tall, and a normal person $\approx 70$ inches tall. We will call a beetle which is the size of a person a "monster beetle."

13. What is the increase in weight of the monster beetle?

Each dimension of the beetle has to be increased by factor 35 so that the size of the monster beetle would be like the size of the person. The increase in weight would be $35^{3}=42875$ times the weight of the normal beetle.
14. What would be the strength of this beetle, if it were the size of a person?

The strength of a monster beetle would be $35^{2}=1225$ times the strength of a normal beetle.
15. Could a monster beetle support its own weight if it had the proportions of a normal bectle? If not, what would be its support?

Since the strength-to-weight ratio of a monster beetle would be less than that of a normal beetle, a monster beetle would not be able to support its own weight. The support area dimension would be $\sqrt{42875} \approx 207$ times larger, rather than 35 times larger.


Section 5: To support its own weight, the size of the larger animal's leg bone must increase in the same proportion as the animal's weight. Play frames 44065-44363, using audio track 1/L.
16. What happens to the size of the leg bones as dinosaur size is reached? One hypothesis about dinosaurs is that they lived in marshy regions. What advantage might this have given them?

The weight of the dinosaurs was considerably larger because of the huge size. In order to support their own weight, the leg bone structure had to be massive (large in cross-sectional area). The dinosaur may have benefited from the buoyant force of water to support some of the weight applied to leg bone.

## A/64: Strength-to-Weight

Name:
Date: $\qquad$

## Section:

$\qquad$

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The arch is capable of supporting its own weight.
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(Note: you are free to use the unit of your choice.)

a. Record the dimensions of the small clay arch:

Length $=$ ___ unit $\quad$ Width $=$ unit $\quad$ Height $=$ ____ unit
Cross-sectional area of clay arch $=$ $\qquad$ unit ${ }^{2}$
Volume of clay arch = $\qquad$ unit ${ }^{3}$
b. Record the dimensions of the large clay slab:

Length = $\qquad$ unit

Width $=$ $\qquad$ unit

Height $=$ $\qquad$ unit
Cross-sectional area of clay slab $=$ $\qquad$ unit ${ }^{2}$
Volume of clay slab = $\qquad$ unit ${ }^{3}$
c. Weight is proportional to the volume of an object. The increase in the weight of the larger arch $=\frac{\text { volume of large slab }}{\text { volume of small arch }}=$
d. Support strength is proportional to the area of an object. The increase in the support strength of the larger arch $=\frac{\text { cross }- \text { sectional area of large slab }}{\text { cross }- \text { sectional area of small arch }}$

Section 2: The strength of a string is proportional to its cross-sectional area. The weight of the safe is proportional to its volume.
3. Referring to the frames listed below, fill in the data.

| Mass of the toy safe | ___ grams (Frame 42236) |
| :---: | :---: |
| String diameter | inches (Frame 42237) $=$ d |
| Mass of the large safe | kg (Frame 42318), |
|  | times mass of the toy safe |
| Height of large safe | $=\ldots$ times height of the toy safe |
| Volume of large safe | ___ times the volume of toy safe |
| (Note: assume the safe is cubical in shape.) |  |
| Cord diameter | $=\ldots$ inches (Frame 42319) |
|  | times diameter of the string $=$ |

4. View frames 42320-42551. If you make the diameter of the cord ten times larger, how much does the strength of the cord increase ?

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\begin{array}{ll}
\text { Cross-sectional area of the string } & =\pi d^{2} / 4 \\
\text { Cross-sectional area of the cord } & =\pi / 4( \\
\text { Increase in the strength of the cord } & =\text { area of cord/area of string } \\
& =
\end{array}
$$

5. View the question on frame 42320. Will the cord lift the safe?
6. View frames 42552-42930. The diameter of the new cord:
7. By what factor did the strength of this cord increase over the string lifting the small safe?

Cross-sectional area of the new cord $=$
$\pi / 4 \times$ $\qquad$ $x d^{2}$ Increase in the strength of the cord = area of cord/area of string
8. Will the two cords, each of diameter $0.01 \times \sqrt{500}$ inches, lift the larger safe?
Diameter of each cord =

Cross-sectional area of each cord $=\pi / 4 \times$ $\qquad$ $x d^{2}$


Increase in the strength of the cord $=2 x$ area of cord /area of string $=$ $\qquad$

Section 3: The monster gorilla, King Kong, can support his own weight when the strength-to-weight ratio is the same as the normal gorilla's. Play frames 42931-43631, using audio track $1 / \mathrm{L}$.
9. What is the strength-to-weight ratio for the normal gorilla if we assume his height (or any dimension) to be $h$ ?
Approximate a normal gorilla using a cube:

| Height of the normal gorilla | $=$ | unit |
| :--- | :--- | :--- |
| Support area of the normal gorilla | $=$ | unit $^{2}$ |
| Volume of the normal gorilla | $=$ | $u_{i n}{ }^{3}$ |
| Strength-to-weight factor | $=$ |  |


10. If King Kong were fifty times taller than a normal gorilla, what would be his strength-to-weight ratio? Would King Kong be able to support his own weight?

| Height of King Kong | $=$ | unit |
| :--- | :--- | :--- |
| Support area of King Kong | $=$ | unit $^{2}$ |
| Volume of King Kong | $=$ | unit $^{3}$ |
| Strength-to-weight factor | $=$ |  |

11. For King Kong not to crumble under his own weight, how many times larger than a real gorilla's legs should his legs be?
For King Kong not to crumble under his own weight, the ratio must be equal to $\qquad$ -.
$=$ cross-sectional area of leg bones $/ 50^{3} h^{3}$
$=$ cross-sectional area of leg bones

12. In the movie, Honey, I Blew Up the Kid, the producer wanted to depict a child twenty times the size of a normal child. How much bigger than a normal child's leg should the giant child's leg be in order to support his weight?

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