A regular column of enchanting curiosities, toys and crowd-pleasing demonstrations.

Little known facts about the common tuning fork

The tuning fork was the basis of the experimental competition in the Third Asian Physics Olympiad held in Singapore on 6–14 May 2002. As examiner for this part of the competition, I conducted a preliminary survey on the students' general knowledge of the tuning fork in order to peg correctly the difficulty level of this exercise. Surprisingly, although this is a very familiar gadget in the teaching laboratory, most students, and even teachers, were unable to answer the following survey questions.

- Why should the tuning fork have *two* prongs, instead of just one?
- Do the two prongs vibrate (a) in the same phase, (b) in antiphase, (c) at 90° phase difference or (d) with no phase relationship at all?
- How can one *measure* the frequency of vibration without touching the prongs (and hence dampening their motion)?
- Is there a *standard* textbook calculation of the frequency of vibration from a knowledge of the Young modulus of elasticity?

At this point the reader is invited to take up the challenge and try to answer the above questions before reading the solutions below.

Solutions

- A tuning fork is designed to vibrate for as long as possible at a fundamental frequency with no harmonics after it is struck. To conserve its vibrational energy, two prongs are needed so that the net force on its holder can be cancelled out. A tuning fork with only one vibrating prong must, by Newton's Third Law of Motion, necessarily exert a non-zero oscillatory force on its holder, thereby leading to very rapid damping of its vibration.
- The two prongs of the fork are symmetrical in every respect. Immediately after striking, the prongs may start to vibrate in many different modes. However, only the *antiphase* mode will survive and all other modes die off very quickly. In the antiphase mode, the prongs always exert exactly equal and opposite forces on the central holder, so that the net force on the holder is always zero. Hence holding it firmly will not cause any undesirable damping. For the same reason the prongs of a tuning fork cannot vibrate in like phase because this will result in an oscillatory force on its holder which would cause the vibration to die away very quickly.
- The most convenient way to experimentally measure the frequency of any object undergoing periodic motion without the need for any direct physical contact is to use a stroboscope (strobe). It is an electronic device consisting of a discharge lamp which can be made to flash for a short duration with a high intensity at highly regular intervals.

  Under the strobe light, students generally find it quite exciting to see for the first time that the prongs are indeed always vibrating in antiphase.

For a more detailed theory on the observations, consider a particle rotating with uniform circular motion being illuminated by a strobe. If the flash frequency is a multiple or sub-multiple of that of the motion, the particle will appear stationary. It follows that the periodicity of the circular motion of the particle can be determined by tuning the frequency of the light flash. Suppose the frequency of rotation of the particle is x Hz, and that of flashing is y Hz. Then, in the time interval of $1/y$ s between two successive flashes the particle would have moved through an angle $2\pi x/y$. If y/x is an irrational number (i.e. it cannot be expressed as a ratio of two integers) then the particle would not appear stationary but would appear to rotate slowly in the forward or backward direction depending on whether y/x is just slightly smaller or larger than some rational number nearby. (This is the same reason why, in cinema pictures, a fast rotating wheel sometimes appears to rotate backwards slowly, instead of forwards.) If $y/x = q/p$ where p and q are integers, then the strobe would flash q times for every p complete cycles. Furthermore, if p and q have no common factors between them, then each flash would show a different position of the particle. Thus the particle will exhibit q stationary images under the strobe flashlight.

Fundamental synchronism is obtained when the lamp flashes once for every cycle of rotation of the object under observation, so that the object appears to stop at one stationary position. However, it will be appreciated that a similar *and indistinguishable* result will also occur when the flash frequency is a sub-multiple (i.e. 1/2, 1/3, 1/4, etc) of the object's frequency of movement. Thus if the latter frequency is totally unknown, when adjusting for fundamental synchronism, a *safe* procedure is to start at a high flash frequency, when multiple images are obtained, and then slowly reduce the flash frequency until the *first* single image appears. This procedure should be adopted in all measurements to check for fundamental synchronism.

The above theory as applied to the rotating particle can be similarly applied to that of a
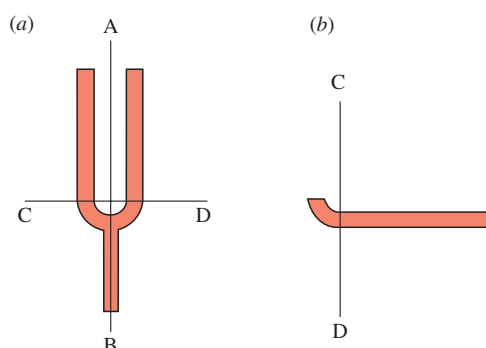
Figure 1. *The approximation of a tuning fork as a cantilever.*

tuning fork vibrating in simple harmonic motion if we regard the vibrational motion as equivalent to the motion of the projection of the rotating particle's position on a given diameter of the circle of motion. However, in this case, because the vibrating object retraces the same path in the opposite direction every half cycle, there is a chance, though very remote, that an image in one half of a vibration cycle coincides with that in the next half cycle. It would result in only one image (but of double the intensity) being recorded, instead of two. This freak coincidence should be guarded against in an experimental observation.

● The theory of the vibrating tuning fork may seem obscure at first sight, but by a simple approximation it can be elegantly shown to be equivalent to that of a standard cantilever.

Figure 1 shows how the approximation is made. In figure 1(*a*), AB represents the plane of symmetry of the tuning fork. Since the prongs always vibrate antisymmetrically, all points of the fork on this plane must always remain stationary. Thus the vibration of one prong, say the right one, is the same as if its linked end is stuck rigidly to a wall occupying the entire left side of AB. Hence, it is possible to consider the vibration of the right prong as if the left one does not exist, and the problem is reduced to that of a single prong without any loss of accuracy or generality.

Next, since the U-bend is usually of much larger cross section than the vibrating arms,

there is very little vibration in the region around the U-bend. Hence little error is introduced if we consider the right prong as protruding from the wall demarcated by line CD instead of AB. The relative orientation of the protruding right prong from the wall CD is now redrawn in figure 1(*b*) to show that it is equivalent to a standard cantilever.

In the case where the vibrational motion of the material in the U-bend cannot be neglected, it is still possible to approximate this section as equivalent to a small effective additional length of the vibrating arm.

For the standard cantilever made of a uniform beam, the period of vibration, *T*, is given by the formula [1]:

$$T^2 = \frac{4\pi^2}{3EI_A}\left(M + \frac{33M_b}{140}\ell^3\right)$$

where E is the Young modulus of the vibrating arm, ℓ the effective length of the arm protruding out of the fixed wall, M the mass of the load at the free end, M_b the mass of the protruding arm, and I_A the second moment of the area of cross section.

In the Olympiad competition, the competitors were asked to use the strobe to measure the frequency of the tuning fork as a function of mass M clamped near the extreme end of each arm. To minimize damping, the two arms must be clamped symmetrically with identical masses.

Reference

[1] Stephenson R J 1969 *Mechanics and Properties of Matter* (New York: Wiley)

P P Ong

Physics Department, National University of Singapore, Kent Ridge, Singapore 117542